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SRA MATHEMATICS LEARNING SYSTEM TEXT 5

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SRA MATHEMATICS LEARNING SYSTEM TEXT

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SRA

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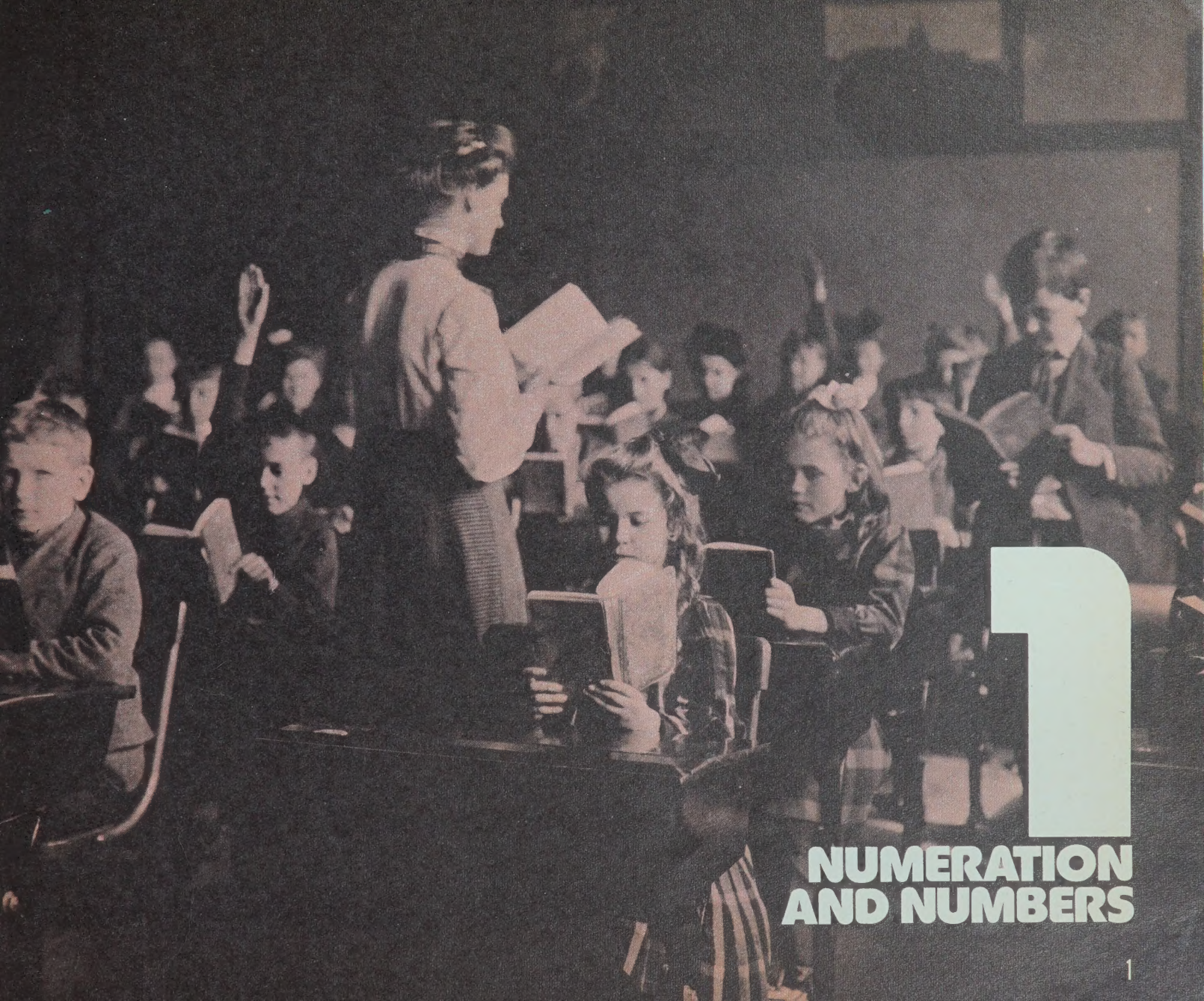

1 NUMERATION AND NUMBERS

An Abstract Number is not applied to a particular thing or quantity; as, 4, 6, 20.

A concrete number is composed of concrete units; and an abstract number, of abstract units.

ART. 6. A **Problem** is a question proposed for solution.

ART. 7. An **Example** is a problem used to illustrate a process or a principle.

ART. 8. A **Rule** is a general description of a process.

ART. 9. In the **Written Solution** of a problem, the results of the successive steps are written.

In the **Mental Solution** of a problem, the results of the successive steps are not written.

NUMERATION AND NOTATION.

ORAL EXERCISES.

1. How many tens and how many units in 37? 65? 84? 90? 75? 18? 60? 80?
2. Read 29; 47; 85; 70; 77; 90.
3. How many hundreds, tens, and units in 368? 427? 549? 608? 680? 600? 806? 860? 800?
4. Read 452; 506; 560; 600; 784; 690; 900; 909.
5. How many ten-thousands and how many thousands in 48500? 83250? 50400? 60070? 82405?
6. Read 37500; 84250; 70840; 92080; 90900.
7. How many hundred-thousands, ten-thousands, and thousands in 456048? 707803? 680435? 700450? 650048? 805347? 170480?

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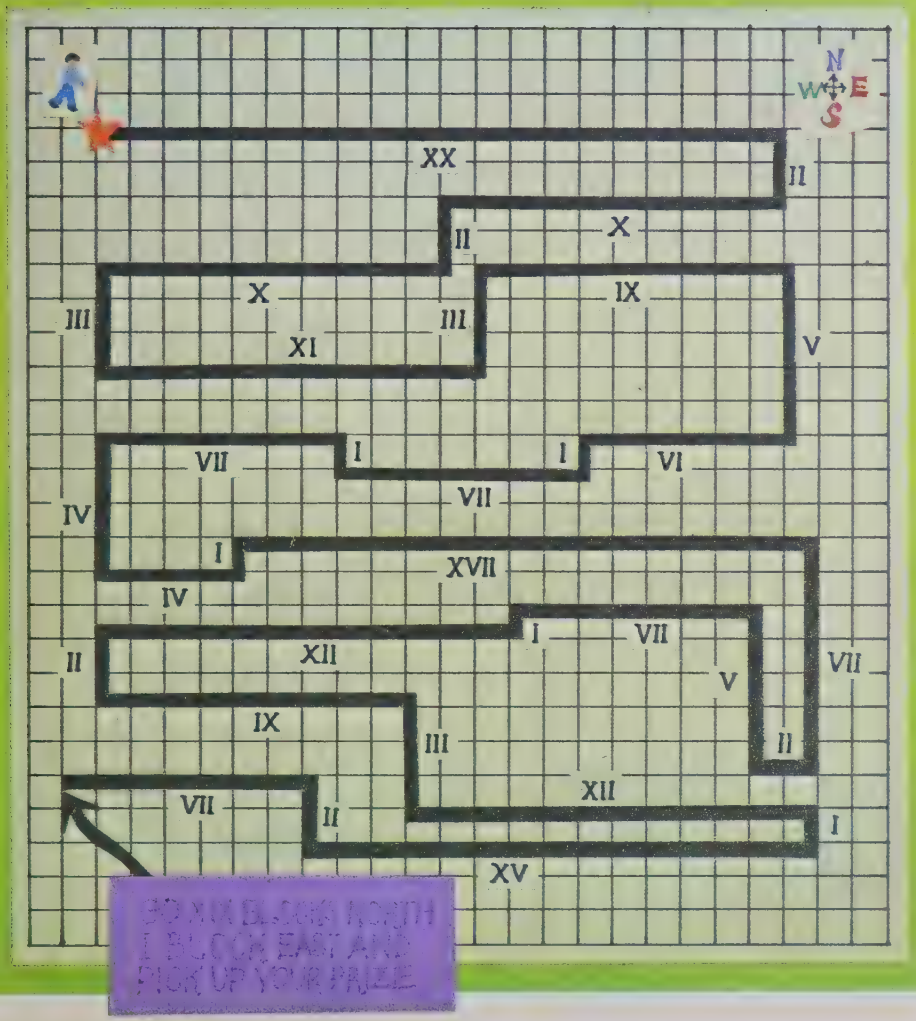
Your goal is to know at least as much about numbers as people your age in 1883. Maybe a lot more!

Here is a copy of two pages from a math book used in 1883. It was written for people your age.

1. About how old is this book?
2. This page is marked *v*. What does *v* stand for?
3. How would you have done back in those days? Answer the problems on the page.



A radio station in Roma had a treasure hunt. The treasure map told the direction to go. There was also a code to tell how far. Sam decided to join the hunt. He walked all over the city. He started at the star and walked XX blocks east. Can you figure out how many blocks is XX? Then he walked II blocks south. How many blocks is II? Then he walked X blocks west. How many blocks is X?

How far did Sam walk to get to the directions? Decode the directions. Follow Sam to the prize.

How far did poor Sam walk in all? Where did he pick up his prize?

1. What does IV stand for? IV
   V
   VI
   VII
   } What is the pattern here?

2. What does IX stand for? IX
   X
   XI
   } What is the pattern here?

   What do you think XII stands for?

3. What does XIX stand for? XIX
   XX
   XXI
   XXII
   } What is the pattern here?

4. What does X stand for? XX? XXX?

5. The code name for 50 is L. The code name for 40 is XL.
   XLI
   XLII
   XLIII
   XLIV
   XLV
   } What is the pattern here?

6. What does LX stand for? LXX? LXXX?

7. The code name for 100 is C. What is the code name for 90? for 95? 99? 101? 200?

8. The code name for 500 is D. The code name for 400 is CD.

Have you found the master code yet? There is one.

1. Look for the pattern here.

CD
CDX
CDXX
CDXXX
CDXL

2. What is the code name for—

a 450? **b** 499? **c** 500? **d** 501? **e** 600? **f** 700? **g** 800?

3. The code name for 900 is CM. What is the code name for 950? 999?

4. What is the code name for 1000?

5. The code that we've been using is called the *roman numeral* system. Thousands of years ago the Romans used it for their arithmetic.

Copy and complete the following roman numeral chart.

standard numeral →	1	2	3	4	5
roman numeral →	I	II	?	?	V

6	7	8	9	10
VI	?	?	IX	?

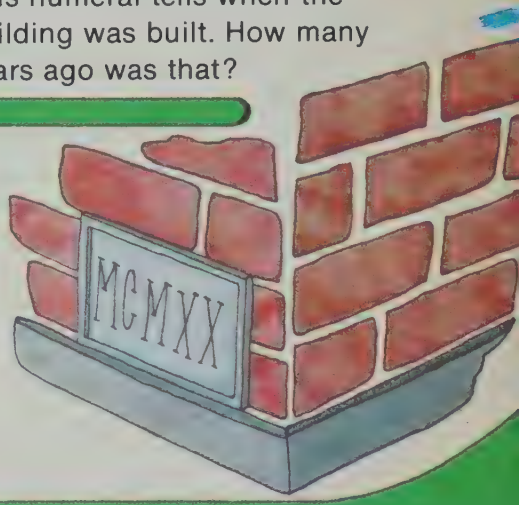
11	12	13	14	15
XI	?	?	?	?

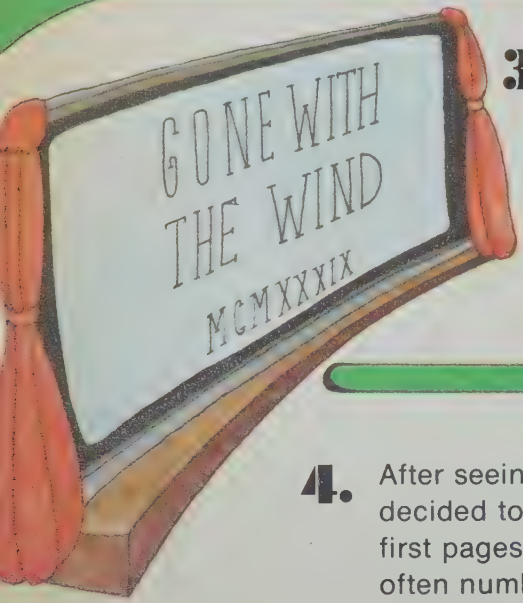
49	50	51	90	99	100	101	400	450	460
XLIX	?	LI	?	?	C	CI	CD	CDL	?

499	500	501	900	999	1000	1001
?	D	DI	?	?	M	?

1. Rosa went to a movie. On the way she passed the jewelry shop. She noticed that some of the clocks had roman numerals on their faces. What time is it when the big hand points to II and the little hand points to IX?

2. The building the theater was in had a cornerstone. On it was the roman numeral MCMXX. This numeral tells when the building was built. How many years ago was that?





3. The movie Rosa saw was *Gone with the Wind*. At the beginning of the film, at the bottom of the credits, she saw the roman numeral MCMXXXIX. This tells when the film was made. When was *Gone with the Wind* made?

4. After seeing the movie, Rosa decided to read the book. The first pages in a book are often numbered with roman numerals written with little letters. What page comes just before page *xix*?

5. Write each standard numeral as a roman numeral.

a 7 **b** 15

c 29 **d** 105

e 1982

Our number system can be thought of as a code too. A visitor from another planet might try to figure it out the way you tried to figure out the Romans' system.

They would look at this pattern.

0	10	20	30	40	100
1	11	21	31	41	101
2	12	22			102
3	13	23			
4	14	24			
5	15	25			
6	16	26			
7	17	27			
8	18	28			
9	19	29			

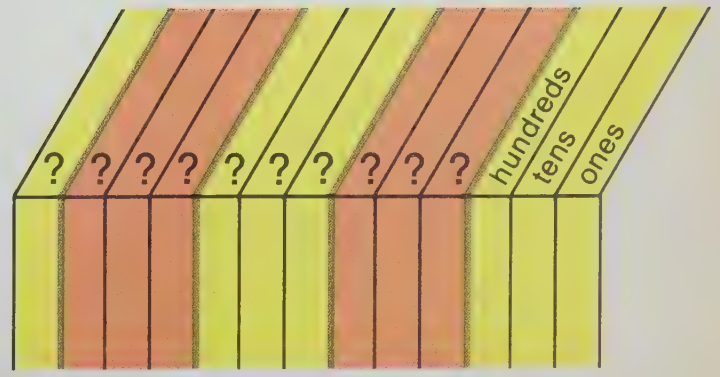
They would get to 100 and think, "Aha! a new key to the code."

What would be the next numeral that would help them break our code?

If our visitor were very smart, he might start to organize his information like this.

Copy the chart. Complete as many of the headings as you can.

The chart can be called a place-value chart.



1. Your chart can hold a very large number. How large?
Write the largest numeral that will fit, using only the digit 9.

2. Pretend that is the number of dollars you have in the bank.
Can you read that number?

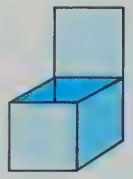
3. Is that number the largest number in the world?

4. Could the chart be made bigger? Where would you add columns?

Big numbers are a part of our lives. We see
them when we read about the— size of the national debt
distances travelled in outer space
amount of energy generated in a power plant
population explosion

And you're full of big numbers too. When you cut yourself,
there is proof that you have blood inside you. How much?
An adult has about 5000 millilitres in his body.

Now think about a tiny box—very tiny. **Think** about a box that is
the size of a cubic centimetre. Now think about a smaller box.
So small that 1000 of them would fit into the cubic centimetre box.
Fill one of these 1000 boxes with a speck of blood.



Do you know there are about **5 400 000** red cells in that box?

An adult has about 35 000 000 000 000 red cells in all.

Each of us has a lot of a lot of things.

Numbers like 35 000 000 000 000 are so large they really don't mean anything to us. It's hard to imagine actually counting that number of things. We can get some idea of "how many" if we study our number code. A place-value chart helps.

Here is part of a place-value chart. It has places for thousands.

thousands			ones		
hundred	ten	one	hundreds	tens	ones
		6	(4)	2	3
(9)	2	1	0	6	0
6	(5)	4	0	0	0
7	0	(9)	6	0	0

What is the value of 4 in 6423?
What is the value of 9 in 921 060?
What is the value of 5 in 654 000?
What is the value of 9 in 709 600?

millions			thousands			ones		
hundred	ten	one	hundred	ten	one	hundreds	tens	ones
		5	6	4	2	0	0	0
2	0	9	4	5	6	2	7	3
	2	6	1	4	4	0	0	0

How is this chart different?

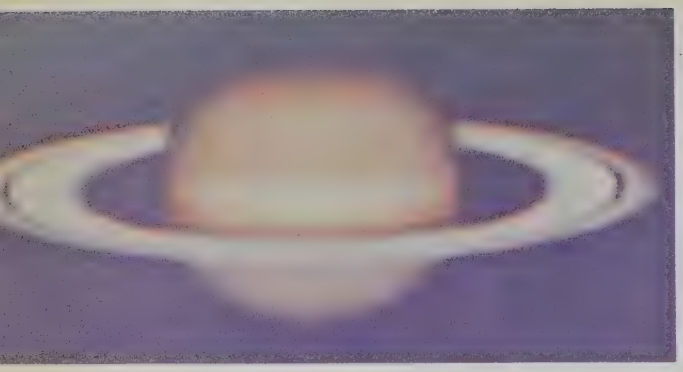
What is the value of 5?
What is the value of 7?
What is the value of 2?

1. Draw a place-value chart with nine columns. Put each digit of the following numerals in the correct column.

a 1 000 000 **b** 792 245 000 **c** 601 001

2. Put the following numerals into the same chart.

a Six million, eight hundred thousand, twenty-five
b Ten thousand, one hundred
c Twenty-one million, three hundred thousand, four
d Seventy-two
e One million, one hundred thousand, one hundred



Mercury is about **58 000 000** km from the sun,
Venus is about **107 000 000** km from the sun,
and Jupiter is about **768 000 000** km from the sun.

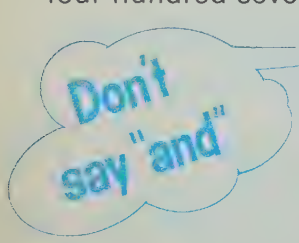
Notice the spaces in the numerals.
What place-value groups do the spaces separate?

Spaces make numerals for large numbers easier to read.
Spaces separate millions from hundred-thousands,
and thousands from hundreds.
Spaces are seldom used in a 4-digit numeral.

How many spaces would be needed for each of the following numerals?

820069423 **6124971** **97431275**

When we place numerals in a place-value chart, we must have a digit in every column. But when we read numerals aloud, we don't always say every place-value label. For example, we read 475 607 000 as "four hundred seventy-five million six hundred seven thousand."



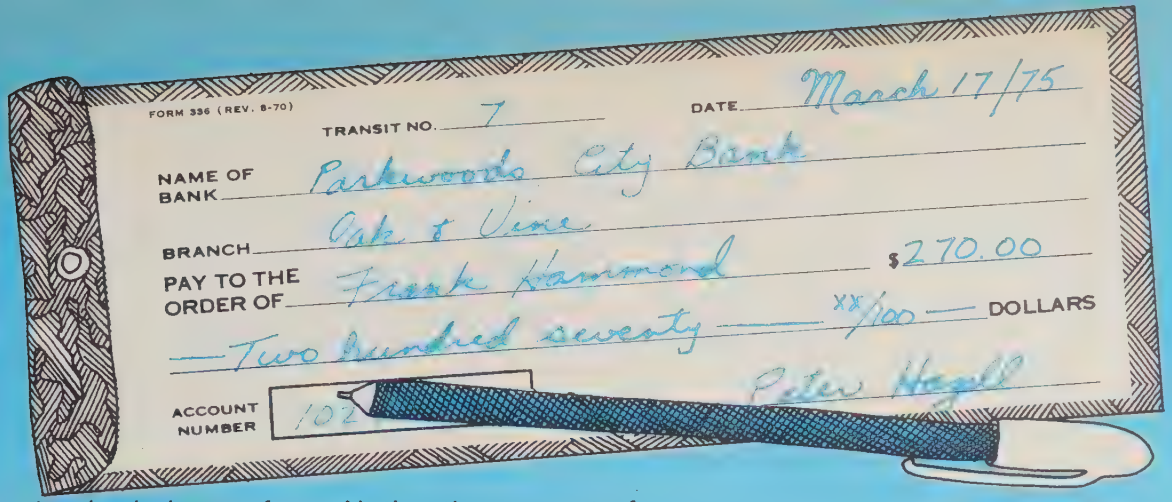
Write standard numerals for the following:

1. Sixty-two million, four hundred seventy thousand, seven hundred.
2. Three hundred ninety million, one hundred thirty-one thousand, fifteen.
3. Four million, eight hundred sixty thousand, five hundred two.

Be able to read the following numerals.

4. 620 975 000 **5.** 66 300 700 **6.** 2 030 500

About the only time you need to write the words for numerals is when you write a cheque to pay for something.
Then it may be very important that you know how. Why?



This is a standard cheque form. Notice the amount of the cheque has been written in words and with a numeral. Why do you think this is? The $\frac{xx}{100}$ means no cents. But a cheque for $1.25 would be written out as One and $\frac{25}{100}$ dollars.

Suppose you had inherited a million dollars. You put the money in the bank. Now suppose you wanted to buy some of the things listed below. Draw cheque forms and write out some cheques. Do not spend more than $1 million.

Food for everyone in town from the Foodery Co. $119 000.00	Island from Gilligan Water Products $36 595 000.00	New clothes from Emperor's Emporium $29.67
A new home from Jiff Construction Co. $499 427.00	Five golden rings from Partridge Pear Co. $463.56	Ten shoes for the right foot from the Prince Shoe Co. $65.58
Stereo and records from the Dip-Disc Record Co. $3095.00	Accountant from Lax Tax & Company $10 270.00	TV knobs from Base TV Service $4.75

Numbers have many names. The focus on this page is on a number's place-value name.

What does 2345 mean? One thing it means is $2000 + 300 + 40 + 5$. It could be written this way:

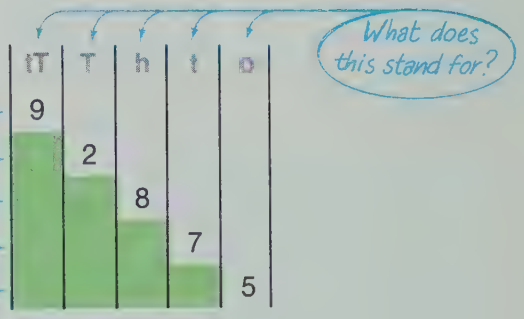
$$\begin{array}{r} 2000 \\ 300 \\ 40 \\ +\ \ 5 \\ \hline 2345 \end{array}$$

and we are back where we started.

1. Look at another number and one name for it.

92 875

a How many ten-thousands?
b How many thousands?
c How many hundreds?
d How many tens?
e How many ones?



2. One more time.

a 8000 – How many thousands?

Get another name.

b 8000 – How many hundreds?

Get still another name.

c 8000 – How many tens?

And another.

d 8000 – How many ones?

T	h	t	o
8			
8	0		
8	0	0	
8	0	0	0

3. Make a | T | h | t | o | chart of your own. Answer these questions.

a How many thousands in 1000?
b How many hundreds in 1000?
c How many tens in 1000?
d How many ones in 1000?

Now let's really get rolling.
The place-value names will be spelled out this time.

How many names are there for—	thousands	hundreds	tens	ones
3254 →	3	2	5	4
Is this a correct name?	2	12	5	4
Is this a correct name?	3	1	15	4
How about this one?	2	12	4	14

If you wonder, take time to add them.
Add the last one, for example.

$$\begin{array}{r} {\scriptstyle 2} \\ 1\ 2 \\ \quad\ 4 \\ +\quad 1\ 4 \\ \hline ?\ ?\ ?\ ? \end{array}$$

GREAT!

PROGRESS CHECK

It's your turn. Make a chart with the word names for each place value. Find at least two other place-value names for each of these.

(1.) 5279 (2.) 2621 (3.) 4836 (4.) 7065

You use this sort of renaming a lot when you subtract. Here's a really hard one to use as an example.

*(5.)
$$\begin{array}{r} 2000 \\ -\ 789 \\ \hline \end{array}$$

There is no doubt about it— 2000 must be renamed. Can you get the job done? Try it.

1. Rename 342 for subtraction.

a 342 − 134 — What name should be used for 342? ? hundreds ? tens ? ones

b 342 − 161 — What name should be used for 342? ? hundreds ? tens ? ones

c 342 − 274 — What name or names should be used for 342? ? hundreds ? tens ? ones

2. In this problem, 63 has been renamed as 5 tens ? ones.

$$\begin{array}{r} {\scriptstyle 5\ 13} \\ \not{6}\not{3} \\ -\ 2\ 4 \\ \hline 3\ 9 \end{array}$$

3. Which numbers in the following problems require renaming?

a	**b**	**c**	**d**	**e**	**f**
65 − 24	82 − 49	92 − 81	56 − 17	48 − 39	96 − 95

4. Look at these renamed numbers. Where did the 9s come from?

$$\begin{array}{r} {\scriptstyle 3\ 9\ 14} \\ \not{4}\ 0\ \not{4} \\ -\ 3\ 8\ 5 \\ \hline ?\ ?\ ? \end{array} \qquad \begin{array}{r} {\scriptstyle 0\ 9\ 9\ 10} \\ \not{1}\ 0\ 0\ 0 \\ -\ \ 4\ 2\ 5 \\ \hline ?\ ?\ ?\ ? \end{array}$$

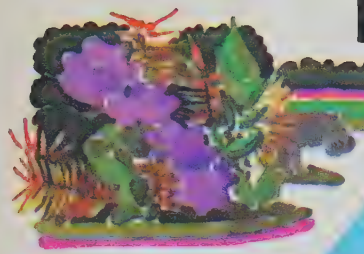
Do these subtraction problems.

5.	**6.**	**7.**	**8.**	**9.**	**10.**
965 − 49	847 − 49	5486 − 677	1000 − 737	900 − 86	1000 − 999

If you can figure out this renaming, you'll probably never have a problem with subtraction. Think about what you are doing as you compute.

$$\begin{array}{r} 1605 \\ -\ 957 \\ \hline \end{array}$$

Be honest with yourself. Was it a very hard problem for you? If so, you may need some special help.



PROGRESS CHECK

Subtract.

	a	b	c	d	e
Group (1)	67 − 4	58 − 23	239 − 29	356 − 56	482 − 162
Group (2)	20 − 8	38 − 9	65 − 47	173 − 54	184 − 78
Group (3)	295 − 76	368 − 59	526 − 88	540 − 93	505 − 27
Group (4)	671 − 184	506 − 254	701 − 355	800 − 262	400 − 367
Group (5)	5273 − 648	4672 − 569	3598 − 1489	6042 − 5256	9004 − 1425

Do you remember how to check a subtraction problem?

$$\begin{array}{r} 523 \\ -\ 179 \\ \hline 344 \end{array} \qquad \begin{array}{r} 344 \\ +\ 179 \\ \hline ? \end{array}$$

Subtraction is "take away"; so to check, just "put it back."

Look at the relation between subtraction and addition again.

$$\begin{array}{r} {\scriptstyle 5\ 13} \\ 763 \\ -\ 149 \\ \hline 614 \end{array}$$

Renaming was required.

$$\begin{array}{r} {\scriptstyle 1} \\ 614 \\ +\ 149 \\ \hline 763 \end{array}$$

Renaming happened here too.

Here is a model of the renaming situation when we *add*.

tens	ones
2	9
+ 2	8
?	?

tens	ones
2	9
+ 2	8
?	17

17 → to be renamed 1 ten and 7 ones

$$\begin{array}{r} {\scriptstyle 1} \\ 29 \\ +\ 28 \\ \hline ?\ 7 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 29 \\ +\ 28 \\ \hline 57 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 4639 \\ +\ 587 \\ \hline 5226 \end{array}$$

16 is renamed ? ten ? ones.

120 is renamed ? hundred ? tens.

1200 is renamed ? thousand ? hundreds.

ADD

1. $\begin{array}{r} 28 \\ +\ 61 \\ \hline \end{array}$ **2.** $\begin{array}{r} 58 \\ +\ 37 \\ \hline \end{array}$ **3.** $\begin{array}{r} 99 \\ +\ 89 \\ \hline \end{array}$ **4.** $\begin{array}{r} 156 \\ +\ 72 \\ \hline \end{array}$ **5.** $\begin{array}{r} 238 \\ +\ 98 \\ \hline \end{array}$

6. $\begin{array}{r} 107 \\ +\ 59 \\ \hline \end{array}$ **7.** $\begin{array}{r} 164 \\ +\ 136 \\ \hline \end{array}$ **8.** $\begin{array}{r} 573 \\ +\ 427 \\ \hline \end{array}$ **9.** $\begin{array}{r} 4327 \\ +\ 894 \\ \hline \end{array}$ **10.** $\begin{array}{r} 6367 \\ +\ 3975 \\ \hline \end{array}$

The secret of accuracy in addition is knowing the addition facts.

```
  5276
+ 4512
```

- that's 6 + 2 ones
- that's 7 + 1 tens
- 2 + 5 hundreds
- 5 + 4 thousands

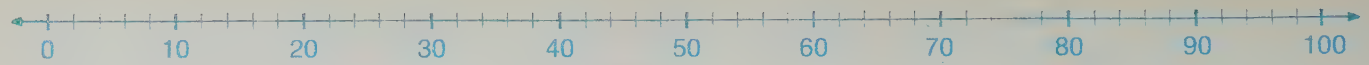
If you have to rename, there is still no problem for you.

```
  6879
+ 5624
```

- 9 + 4 = 1 3 ones
- (7 + 2) + 1 = 1 0 tens
- (8 + 6) + 1 = 1 5 hundreds
- (6 + 5) + 1 = 1 2 thousands

PROGRESS CHECK

Add.	a	b	c	d	e
Group 1	62 + 7	54 + 25	438 + 61	506 + 91	729 + 160
Group 2	58 + 9	83 + 7	76 + 18	257 + 25	562 + 418
Group 3	156 + 76	627 + 99	518 + 95	231 + 59	472 + 28
Group 4	664 + 187	743 + 298	526 + 634	462 + 336	275 + 795
Group 5	1467 + 294	2527 + 695	2762 + 9258	5514 + 2486	8006 + 1994

1. 

a Is 86 closer to 80 or 90? So round 86 to ■.

b Is 38 closer to 30 or 40? So round 38 to ■.

c Is 72 closer to 70 or 80? So round 72 to ■.

d Is 24 closer to 20 or 30? So round 24 to ■.

e Is 65 closer to 60 or 70? It's halfway. Agree to round up to the nearest ten.

2.

a Is 240 closer to 200 or 300? So round 240 to ■.

b Is 779 closer to 700 or 800? So round 779 to ■.

c Is 516 closer to 500 or 600? So round 516 to ■.

d Is 991 closer to 900 or 1000? So round 991 to ■.

e Is 350 closer to 300 or 400? It's halfway. So round 350 to ■.

3.

a Is 3500 closer to 3000 or 4000? So round 3500 to ■.

b Is 5750 closer to 5000 or 6000? So round 5750 to ■.

c Is 1470 closer to 1000 or 2000? So round 1470 to ■.

d Is 9612 closer to 9000 or 10 000? So round 9612 to ■.

4. Round each of these to the nearest ten.

a 53 **b** 15 **c** 46 **d** 97 **e** 55

5. Round to the nearest hundred.

a 138 **b** 670 **c** 465 **d** 851 **e** 949

6. Round to the nearest thousand.

a 2901 **b** 7499 **c** 4099 **d** 8610 **e** 1490

The ability to round numbers will increase your ability to estimate. Ability to estimate will increase your computation skills, **and** increased computation skills will make everybody happy.

If you have trouble imagining a number line, try this method:

Round 87 to the nearest ten.
You want a zero in the ones place.
8(7) – Look at the ones digit.
Five or more?
Round up to 9 tens.

Round 664 to the nearest ten.
You still want a zero in the ones place.
66(4) – Look at the ones digit. Five or more?
Round down to 66 tens.

Now round 664 to the nearest hundred.
Tens and ones places have zeros.
6(64) – Look at the tens and ones. Fifty or more?
Round up to 700.

Round 949 to the nearest hundred.
You want zeros in the ones and tens places.
9(49) – Look at the tens and ones. Fifty or more?
Round down to 900.

Round 950 to the nearest hundred.
9(50) – Fifty or more? Round up to 10 hundreds or 1000.

Numbers can be rounded to the nearest
multiple of 10, of 100, of 1000, of 10 000,
of 100 000, of 1 000 000, and so on.
For example –
Round 15 401 to the nearest thousand.
15 (401) Look at the digits to the right of the
place value you want. Five hundred or more?
Round down to 15 000.

9 (599) Five hundred or more? Round up to 10 000.

Now you should be able to guess
how you can round to the nearest
multiple of 100 000 and 1 000 000.

PROGRESS CHECK

Write the standard numeral.

1. Five hundred fifty-three million, six hundred thousand, twenty-three.
2. Twenty million, nine hundred ninety thousand, nine hundred nine.
3. One million, eight hundred fifty-eight thousand, two hundred five.
4. Thirty-two hundred fifty-one million, seventeen thousand, fourteen.

Round each number to the nearest ten. Then add the rounded numbers.

5. $\begin{array}{r} 194 \\ +\ 62 \\ \hline \end{array}$
6. $\begin{array}{r} 285 \\ +\ 10 \\ \hline \end{array}$
7. $\begin{array}{r} 136 \\ +\ 100 \\ \hline \end{array}$

Round each number to the nearest hundred. Then compute the rounded numbers.

8. $\begin{array}{r} 897 \\ -\ 499 \\ \hline \end{array}$
9. $\begin{array}{r} 569 \\ -\ 389 \\ \hline \end{array}$
10. $\begin{array}{r} 423 \\ +\ 899 \\ \hline \end{array}$

Round each number to the nearest thousand. Then compute the rounded numbers.

11. $\begin{array}{r} 9958 \\ -\ 9857 \\ \hline \end{array}$
12. $\begin{array}{r} 5574 \\ +\ 9857 \\ \hline \end{array}$
13. $\begin{array}{r} 8523 \\ -\ 7000 \\ \hline \end{array}$

Round each number to the nearest million. Then compute the rounded numbers.

*14. $\begin{array}{r} 4\ 950\ 000 \\ +\ 1\ 692\ 400 \\ \hline \end{array}$
*15. $\begin{array}{r} 5\ 271\ 190 \\ +\ 6\ 108\ 000 \\ \hline \end{array}$
*16. $\begin{array}{r} 67\ 809\ 100 \\ -\ 3\ 955\ 000 \\ \hline \end{array}$

City	POPULATION 1891 census	1971 census
Calgary	4 000	403 319
Halifax	38 495	122 035
Montreal	216 650	1 214 352
Ottawa	37 269	602 510
Vancouver	18 229	426 256

You will get reasonable answers if you round these large numbers to thousands. Operate with estimated answers.

1. Which of the cities on the 1971 list has the greatest population? Which has the least?

2. By how much did the population increase in Calgary between 1891 and 1971?

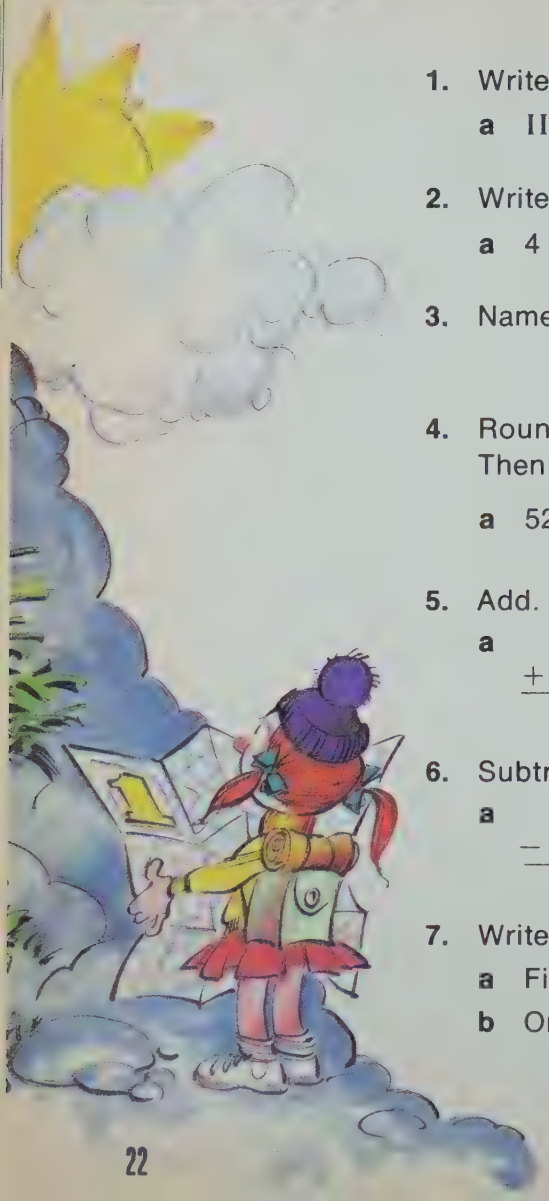
3. How many more thousands of people lived in Halifax than in Vancouver in 1891?

4. Was the difference between the population of Halifax and the population of Vancouver greater in 1891 than in 1971?

5. If someone who lived in Ottawa in 1971 said the population of his city was about 600 000 people, would he be reporting correctly?

6. If a person in Vancouver in 1971 said the population of his city was about 1 000 000, would he be reporting correctly? What if he said the population was about 500 000? What would be the best rounded number for him to report? Do you suppose there was a period of one month when the population was exactly 426 256?

CHECKOUT



1. Write the standard numeral for each of these roman numerals.

   a III **b** IV **c** XII **d** XX **e** C

2. Write a roman numeral for each of these standard numerals.

   a 4 **b** 10 **c** 15 **d** 30 **e** 100

3. Name the value of any underlined digit in this numeral.

   $\underline{1}\,52\underline{8}\,\underline{346}$

4. Round each number to the nearest ten.
   Then round each to the nearest hundred.

   a 526 **b** 162 **c** 354 **d** 856 **e** 965

5. Add.

a	**b**	**c**	**d**	**e**
539	678	515	864	6826
+ 93	+ 102	+ 385	+ 479	+ 1275

6. Subtract.

a	**b**	**c**	**d**	**e**
598	653	400	6728	8005
− 49	− 186	− 261	− 1619	− 1216

7. Write each of these as a standard numeral.

   a Five hundred thousand, six hundred seventy-two

   b One thousand, one hundred one

2 WHOLE NUMBERS × AND ÷

Your goal is to be able to multiply and divide just about any two numbers. And you will get the right answer, too!

HOW ACCURATE IS YOUR WORK?

Don't copy the multiplication problem itself. Put a sheet of paper under the first row of problems. Write only your answer. When you finish the row, fold your answers under so that you have a clean surface. Do the next row. Fold your sheet again to have writing space for the last row.

1. 34×2	**2.** 43×3	**3.** 211×6	**4.** 423×3	**5.** 46×5	**6.** 38×3
7. 213×7	**8.** 524×7	**9.** 368×18	**10.** 194×26	**11.** 507×59	**12.** 680×47

You will have to copy these problems and then compute.

13. $6\overline{)126}$ **14.** $7\overline{)342}$ **15.** $9\overline{)5783}$ **16.** $60\overline{)2227}$

Why do you think you were asked to do these problems *before* you started your study of multiplication and division?

An array is a picture of rows and columns of objects. The picture usually forms a rectangular shape.

This array has 2 rows and 3 columns. The number of rows and the number of columns represent the factors of a multiplication problem. The number of squares in all represents the product.

1. Use squares or blocks. See if you can form an array that has more than one row for any of these numbers. 1, 3, 5, 7, and 9

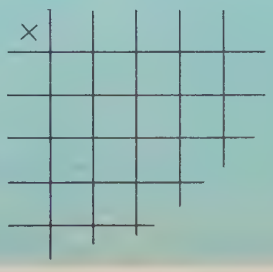
2. The numbers you have just worked with are the odd numbers between 0 and 10. You should have been able to make an array for only one of the odd numbers listed above. Which one?

3. Look at the factors of the odd number that formed an array. Are the factors odd numbers?

4. An even number is 2. All multiples of 2 are also even numbers. In other words, every even number has a factor of 2. Write the even numbers between 2 and 10.

5. Make a multiplication table for factors 1 through 9. Use three different colors in the table.

   Use one color for the grid.
   Use the second color for every even number that you write.
   Use the third color for every odd number that you write.



Use the MULTIPLICATION TABLE you just made for these questions

1. Are there more odd numbers or more even numbers in your table?
2. Is the product **of** two even numbers an odd number? an even number?
3. Can every **even** number be divided by 2?
4. Is the product of two odd numbers an odd number? an even number?
5. Can every odd number be divided by 1? by 3?
   (It shouldn't take you long to answer that.
   The answer is in the first row of products)
6. Is the product of every odd and even number an even number?
7. What pairs of factors name each of these products?

   a 4 **b** 9 **c** 16 **d** 25
   e 36 **f** 49 **g** 64 **h** 81
8. Are all other products named by more than one pair of factors?
   How many pairs of factors?
9. Is 11 a product of more than one pair of factors?
   Is 13? Is 17? Is 19?

The numbers **2, 3, 5, 7, 11, 13, 17,** and **19** are called prime numbers. There are more, but these are all you need for now.

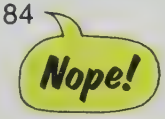
A prime number is a whole number greater than 1 that has only the factors of 1 and itself.

All products in your multiplication table except 1, 2, 3, 5, and 7 are called composite numbers. The word *composite* means "made up of parts." A composite number is made up of parts called factors. Every composite number has at least one pair of factors besides 1 and itself.

82 is the next-largest number outside your table. It is a composite number. You know that because it is an even number. All even numbers have 2 as a factor.

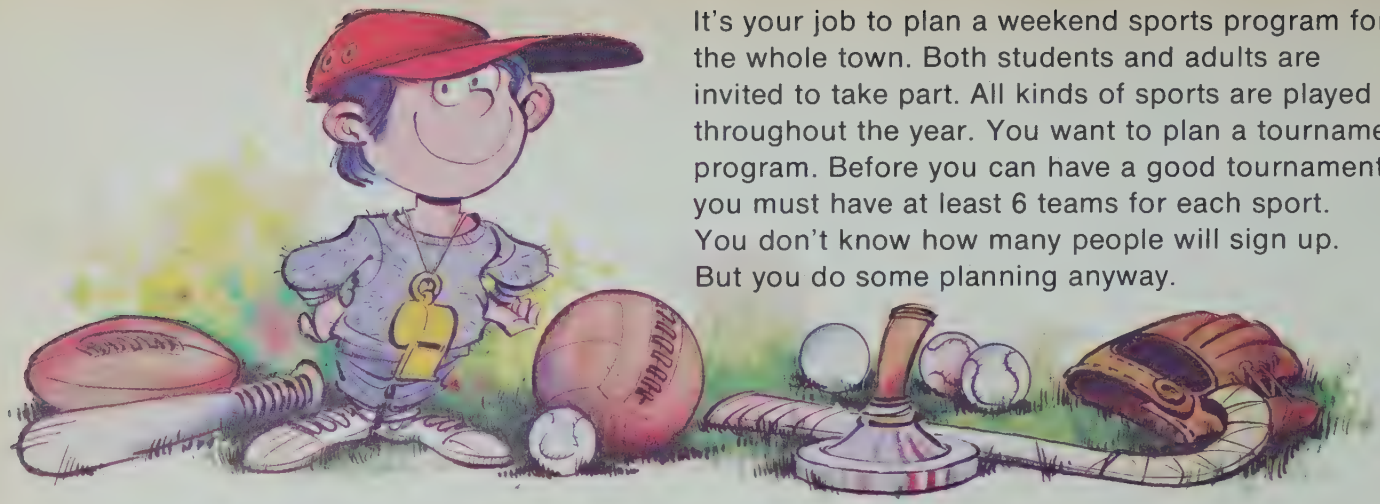
83 is the next number to consider.
Is it prime or composite?
It's an odd number, so 2 is not a factor.
How many 3s in 83?
There are more than 10, because $10 \times 3 = 30$.
There are more than 20, because $20 \times 3 = 60$.
There are less than 30, because $30 \times 3 = 90$.
Try 25 threes. $25 \times 3 = 75$ There are more than 25.
$26 \times 3 = 78$ There are more than 26.
$27 \times 3 = 81$ There are more than 27.
$28 \times 3 = 84$ Too many! 3 is not a factor of 83.

Go on. Investigate the next prime factor. 83 does not end in 5 or 0, so you know 5 is not a factor.
How about 7? How many 7s in 83?
There are more than 10, because $10 \times 7 = 70$.
Try 11 sevens. $11 \times 7 = 77$
$12 \times 7 = 84$



7 is not a factor of 83. So 83 must be a prime number.

There are more prime numbers. But multiplication skills must be developed first.
And we really need division to do this work efficiently.
We will stop for now. We'll come back to primes later.



It's your job to plan a weekend sports program for the whole town. Both students and adults are invited to take part. All kinds of sports are played throughout the year. You want to plan a tournament program. Before you can have a good tournament, you must have at least 6 teams for each sport. You don't know how many people will sign up. But you do some planning anyway.

Copy and complete the charts.

1 For baseball

Number of teams	1	6	7	8	9	10
Least number of people	9	?	?	?	?	?

2 For volleyball (Same number as needed for women's basketball.)

Number of teams	1	6	7	8	9	10
Least number of people	6	?	?	?	?	?

3 For men's basketball

Number of teams	1	6	7	8	9	10
Least number of people	5	?	?	?	?	?

4 For field hockey (Same number as needed for football.)

Number of teams	1	6	7	8	9	10
Least number of people	11	?	?	?	?	?

5 For curling (If you don't know what this ice game is, look it up.)

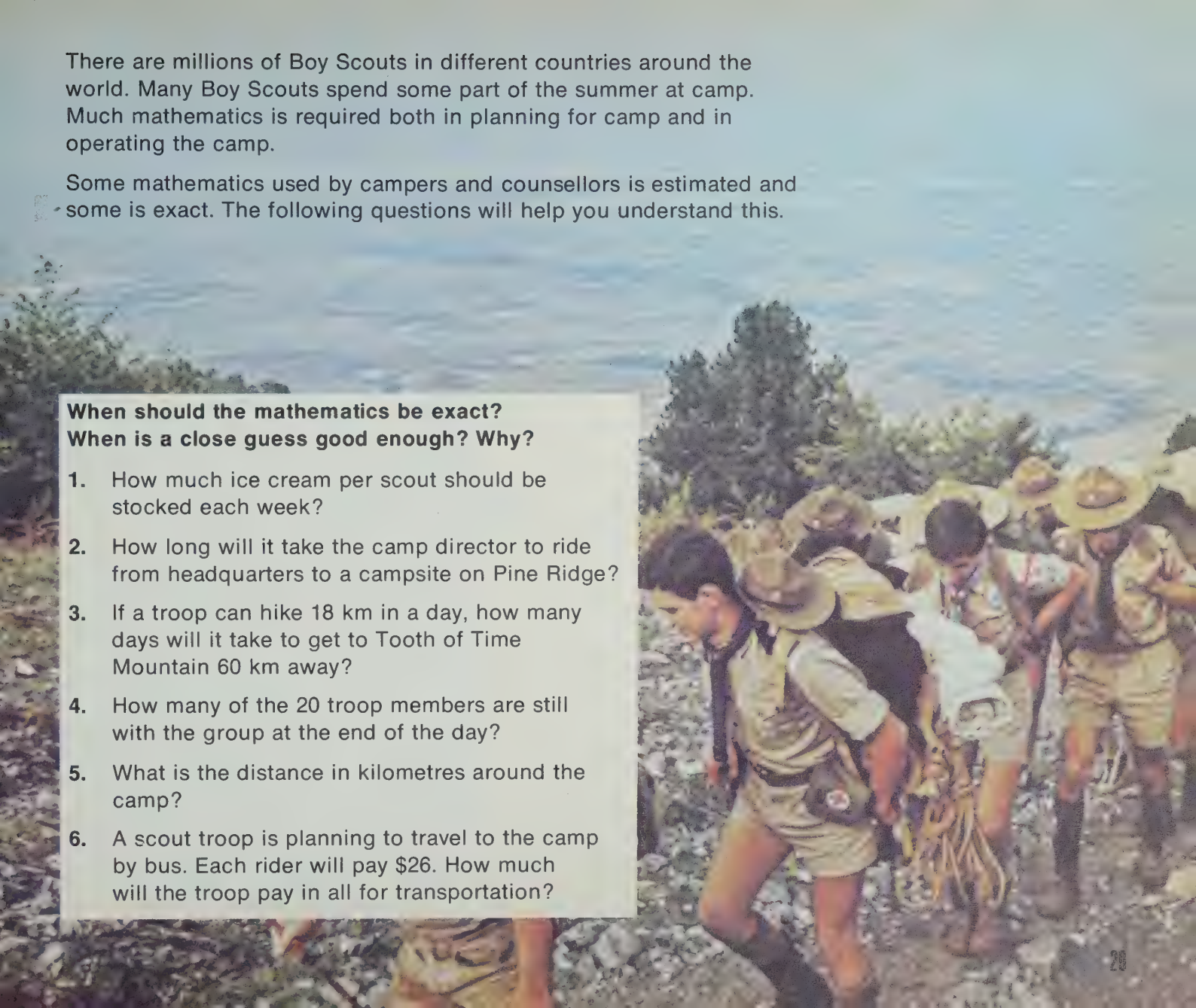
Number of teams	1	6	7	8	9	10
Least number of people	4	?	?	?	?	?

6 For rugby (This is another game you might have to check on.)

Number of teams	1	6	7	8	9	10
Least number of people	15	?	?	?	?	?

There are millions of Boy Scouts in different countries around the world. Many Boy Scouts spend some part of the summer at camp. Much mathematics is required both in planning for camp and in operating the camp.

Some mathematics used by campers and counsellors is estimated and some is exact. The following questions will help you understand this.



When should the mathematics be exact? When is a close guess good enough? Why?

1. How much ice cream per scout should be stocked each week?
2. How long will it take the camp director to ride from headquarters to a campsite on Pine Ridge?
3. If a troop can hike 18 km in a day, how many days will it take to get to Tooth of Time Mountain 60 km away?
4. How many of the 20 troop members are still with the group at the end of the day?
5. What is the distance in kilometres around the camp?
6. A scout troop is planning to travel to the camp by bus. Each rider will pay $26. How much will the troop pay in all for transportation?

A good guess can be called an estimate.
An estimate is based on some thinking.

1. The camp wants to develop 6 new trails. About 22 scouts will be needed to work on each trail. About how many scouts will be working on all 6 trails?
Think 22 is close to 20. 20×6 is 120.
An answer of 120 scouts would be a good estimate.
Would there be more or less than 120 scouts?

2. The camp bus averages about 70 kilometres per hour (km/h). It travels for at least $2\frac{1}{2}$ hours to reach camp. About how many km does it go?
Think $2\frac{1}{2}$ hours is close to 3 hours.

In 1 hour it goes about 70 km .
In 3 hours it should go about 3×70, or 210 km .
Would it be more or less than 210 km?

3. Estimate these.

a $3 \times 33 = \underline{?}$ Think $3 \times 30 = 90$, so 90 would be a good estimate. Would 3×33 be more than or less than 90?

b $2 \times 87 = \underline{?}$ Think $2 \times 90 = 180$.
Would 2×87 be more or less than 180?

c $5 \times 24 = \underline{?}$ Think $5 \times 20 = 100$.
Would 5×24 be more or less than 100?

d $3 \times 412 = \underline{?}$ Think $3 \times 400 = 1200$.
Would 3×412 be more or less than 1200?

e $9 \times 279 = \underline{?}$ Think $9 \times 300 = 2700$.
Would 9×279 be more or less than 2700?

You have been estimating products. Even if you have to find the exact product, an estimate should be made. An estimate will help you avoid silly mistakes.

You will usually round one number before you estimate. You will round so that you can estimate in your head. An estimate is just more work if you have to figure it out with pencil and paper.

1. Compute these numbers in your head.

a 2×40	**b** 4×80	**c** 5×20	**d** 6×70
e 8×50	**f** 3×60	**g** 7×90	**h** 8×30
i 2×200	**j** 5×400	**k** 8×300	**l** 4×600

2. Tell if the exact product will be more or less than the estimate that is written.

a	2×38	more than / less than	2×40	**b**	4×59	more than / less than	4×60
c	6×18	more than / less than	6×20	**d**	7×22	more than / less than	7×20

A ⍰ will be in the place of the words *more than / less than.* But you will give the same kind of answer as you did before.

e 9×47 ⍰ 9×50	**f** 8×63 ⍰ 8×60
g 5×393 ⍰ 5×400	**h** 4×715 ⍰ 4×700

These two examples show an estimated product and an exact product for 3×32.

ESTIMATE

$3 \times 30 = 90$

EXACT

$$\begin{array}{r} 32 \\ \times\ \ 3 \\ \hline 96 \end{array}$$

Round the two-digit factor and estimate the product. Then find the exact product. How does the exact product compare with the estimate?

1. 2×42 **2.** 3×13 **3.** 2×24 **4.** 4×21

5. 2×23 **6.** 4×12 **7.** 3×22 **8.** 6×11

These two examples show an estimated product and an exact product for 4×28. Notice that the answer in both cases has three digits.

ESTIMATE

$4 \times 30 = 120$

EXACT

$$\begin{array}{r} 28 \\ \times\ \ 4 \\ \hline 112 \end{array}$$

Write an estimate. Then find the exact product.

9. 3×26 **10.** 7×39 **11.** 4×17 **12.** 5×38

13. 2×18 **14.** 9×48 **15.** 6×16 **16.** 8×25

Are your answers reasonable? How do you know?

17. Having trouble estimating? Practise rounding skills on these.

a Is 42 closer to 40 or 50?

b Is 39 closer to 30 or 40?

c Is 33 closer to 30 or 40?

d Is 68 closer to 60 or 70?

e Is 54 closer to 50 or 60?

f Is 94 closer to 90 or 100?

g Is 76 closer to 70 or 80?

h Is 96 closer to 90 or 100?

i Is 25 closer to 20 or 30? It's halfway. We round up to 30.

These two examples show estimated and exact products. The problem is 3×123. To what number should 123 be rounded?

ESTIMATE

$3 \times 100 = 300$

EXACT

$$\begin{array}{r} 123 \\ \times \quad 3 \\ \hline 9 \\ 60 \\ 300 \\ \hline 369 \end{array} \quad \text{or} \quad \begin{array}{r} 123 \\ \times \quad 3 \\ \hline 369 \end{array}$$

(You can use either form of multiplication.)

Estimate. Then find the exact products.

1. 2×214 **2.** 3×312 **3.** 2×443 **4.** 3×232

5. 3×221 **6.** 2×124 **7.** 4×411 **8.** 3×302

Are the answers reasonable? How do you know?

The next examples show estimated and exact products for 3×286. To what number is 286 rounded?

ESTIMATE

$3 \times 300 = 900$

EXACT

$$\begin{array}{r} 286 \\ \times \quad 3 \\ \hline 18 \\ 240 \\ 600 \\ \hline 858 \end{array} \quad \text{or} \quad \begin{array}{r} 286 \\ \times \quad 3 \\ \hline 858 \end{array}$$

Estimate. Then find the exact products.

9. 3×237 **10.** 6×456 **11.** 5×216 **12.** 4×462

13. 8×564 **14.** 2×765 **15.** 7×839 **16.** 9×278

17. 5×483 **18.** 3×514 **19.** 6×973 **20.** 9×596

Are the exact answers close to your estimates?

PROGRESS CHECK

Estimate these products. Do *not* compute the exact product.

1. 4×65	2. 6×28	3. 9×47
4. 3×59	5. 7×31	6. 2×18
7. 8×71	8. 5×94	9. 1×16
10. 8×39	11. 7×77	12. 5×89

Estimate these products. Do *not* compute the exact product.

13. 3×382	14. 7×745	15. 2×621
16. 8×914	17. 9×381	18. 7×861
19. 6×298	20. 5×109	21. 1×580
22. 4×475	23. 8×659	24. 9×344

Find the exact products.

25. $\begin{array}{r} 914 \\ \times\ 8 \\ \hline \end{array}$	26. $\begin{array}{r} 621 \\ \times\ 2 \\ \hline \end{array}$	27. $\begin{array}{r} 774 \\ \times\ 7 \\ \hline \end{array}$	28. $\begin{array}{r} 382 \\ \times\ 3 \\ \hline \end{array}$
29. $\begin{array}{r} 109 \\ \times\ 5 \\ \hline \end{array}$	30. $\begin{array}{r} 298 \\ \times\ 6 \\ \hline \end{array}$	31. $\begin{array}{r} 475 \\ \times\ 4 \\ \hline \end{array}$	32. $\begin{array}{r} 381 \\ \times\ 9 \\ \hline \end{array}$

Do you think your answers are correct?
Are they reasonable?

Write the answers.

1.
$3 \times 20 = ?$
$30 \times 20 = ?$
$300 \times 20 = ?$

2.
$6 \times 30 = ?$
$60 \times 30 = ?$
$600 \times 30 = ?$

3.
$7 \times 40 = ?$
$70 \times 40 = ?$
$700 \times 40 = ?$

4.
$5 \times 40 = ?$
$50 \times 40 = ?$
$500 \times 40 = ?$

What is the pattern? Would the pattern help you find the answer to $600 \times 40 = ?$

5.
$8 \times 70 = ?$
$60 \times 70 = ?$
$300 \times 70 = ?$

6.
$3 \times 50 = ?$
$50 \times 50 = ?$
$400 \times 50 = ?$

7.
$9 \times 80 = ?$
$60 \times 80 = ?$
$300 \times 80 = ?$

8.
$4 \times 90 = ?$
$30 \times 90 = ?$
$200 \times 90 = ?$

What differences are there between this set of problems and the first set of problems?

9.
$8 \times 40 = ?$
$50 \times 20 = ?$
$200 \times 80 = ?$

10.
$6 \times 30 = ?$
$20 \times 40 = ?$
$500 \times 60 = ?$

11.
$3 \times 70 = ?$
$80 \times 30 = ?$
$800 \times 70 = ?$

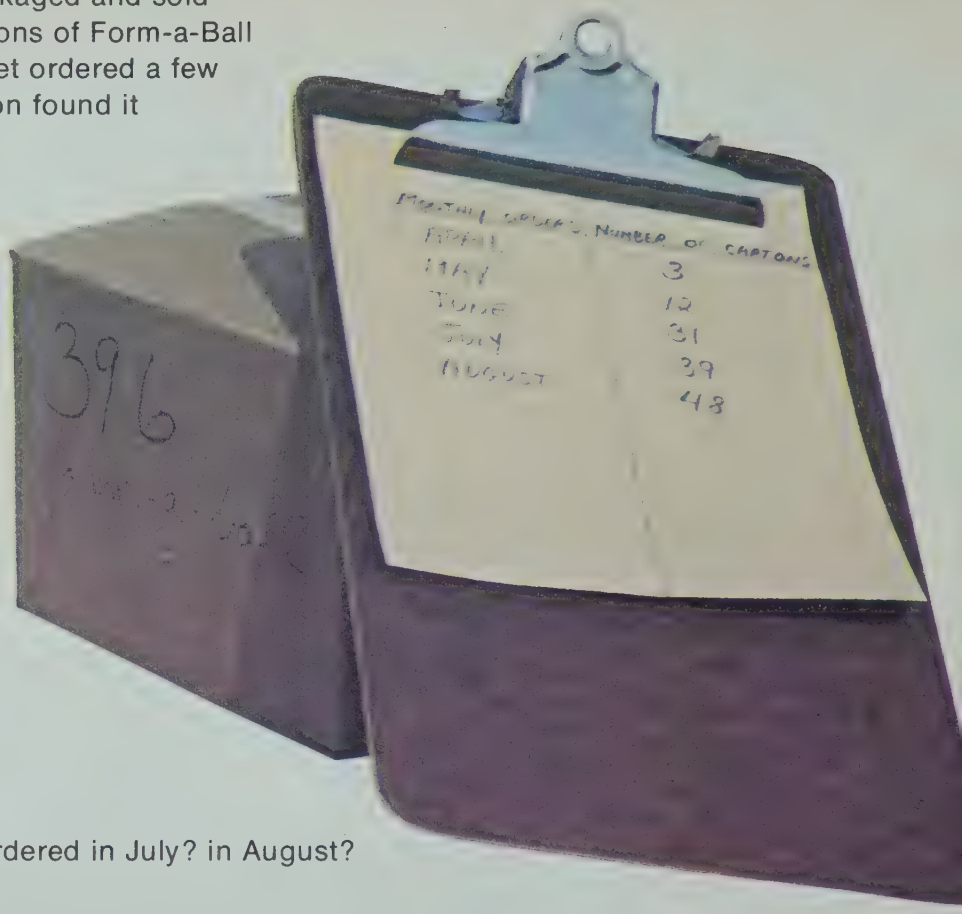
12.
$2 \times 60 = ?$
$40 \times 60 = ?$
$600 \times 20 = ?$

13.
$4 \times 60 = ?$
$10 \times 90 = ?$
$800 \times 50 = ?$

14.
$6 \times 40 = 2\ 4\ ?$
$70 \times 80 = 5\ 6\ ?\ ?$
$300 \times 90 = 2\ 7\ ?\ ?\ ?$

Have you found the way to do these quickly? Hint–find a number fact. Then count the zeros.

A small puttylike ball of material is packaged and sold by the Redi-Play Game Company. Cartons of Form-a-Ball contain 396 balls. The Shoppers' Market ordered a few cartons for early in the season, but soon found it necessary to order many more.



About how many Form-a-Balls were ordered in April?
396 is about 400, to the nearest hundred. 3 cartons of about 400 is about 1200.

$3 \times 400 = 1200$

About how many were ordered in May? To the nearest ten, 12 is 10. 10 cartons of 400 is 4000.

$10 \times 400 = 4000$

How many were ordered in June? To the nearest ten, 31 is 30. 30 cartons of 400 is 12 000.

$30 \times 400 = 12\,000$

About how many Form-a-Balls were ordered in July? in August?

Estimate the products.

1. 19×412	**2.** 267×91	**3.** 30×689
4. 586×59	**5.** 83×728	**6.** 61×163
7. 41×832	**8.** 342×78	**9.** 907×22

634 cartons are stacked in a warehouse.
Each contains 40 bottles of shampoo.
How many shampoo bottles are there all together?
One estimate is $40 \times 600 = ?$

$$\begin{array}{r} 634 \\ \times\ 40 \\ \hline 160 \\ 1200 \\ 24000 \\ \hline 25360 \end{array}$$

40×4
40×30
40×600

or

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 634 \\ \times\ 40 \\ \hline 25360 \end{array}$$

Why do we write 1 above the 3?
What does it represent?

Is the product close to the estimate?

What numbers replace the question marks?

$$\begin{array}{r} 745 \\ \times\ 30 \\ \hline ? \\ ? \\ ? \\ \hline ? \end{array}$$

$\times 5$
$\times 40$
30×700

or

$$\begin{array}{r} 745 \\ \times\ 30 \\ \hline ? \end{array}$$

An estimate of the product is $30 \times 700 = 21\,000$.
How close is this to the actual product?

Estimate. Then find the exact products.

1. 30×61 **2.** 80×92 **3.** 70×49
4. 60×897 **5.** 50×643 **6.** 90×357

Compare each exact product with an estimate.

Think of at least two situations where multiplication is needed to solve a problem. Are exact answers needed in both situations?

$32 \times 48 = \underline{?}$

The product will be about $30 \times 50 = 1500$.
Can you explain why?

When both factors are 2-digit numbers, you are really doing two multiplication problems. Then you put the two products together.

$$\begin{array}{r} 48 \\ \times\ 32 \\ \hline 96 \\ 1440 \\ \hline 1536 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 2 \\ \hline 96 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 30 \\ \hline 1440 \end{array}$$

Here is another example.

$$\begin{array}{r} 76 \\ \times\ 29 \\ \hline ? \quad 9 \times 76 \\ ? \quad 20 \times 76 \\ \hline ? \end{array}$$

What are the two partial products?

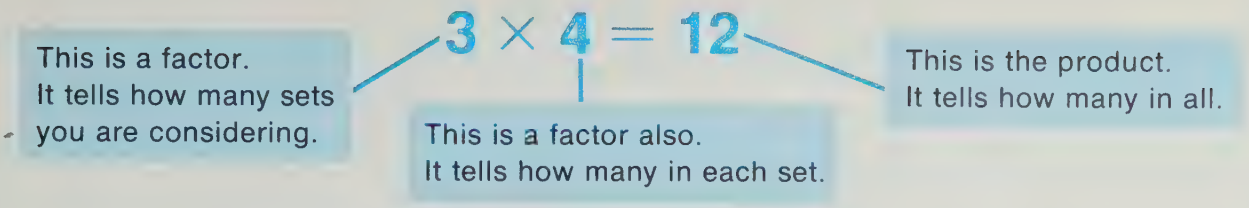
What is the final product?

PROGRESS CHECK

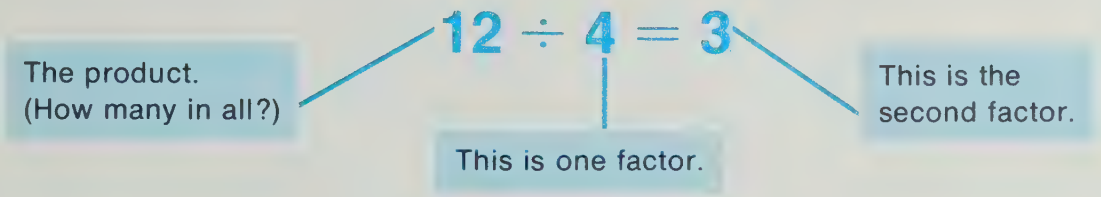
Estimate the answers. Complete the multiplication.
Compare your estimates with the exact answers.

1. $\begin{array}{r} 23 \\ \times\ 7 \\ \hline \end{array}$	2. $\begin{array}{r} 87 \\ \times\ 6 \\ \hline \end{array}$	3. $\begin{array}{r} 256 \\ \times\ 9 \\ \hline \end{array}$	4. $\begin{array}{r} 709 \\ \times\ 2 \\ \hline \end{array}$	5. $\begin{array}{r} 184 \\ \times\ 10 \\ \hline \end{array}$
6. $\begin{array}{r} 95 \\ \times\ 20 \\ \hline \end{array}$	7. $\begin{array}{r} 414 \\ \times\ 80 \\ \hline \end{array}$	8. $\begin{array}{r} 806 \\ \times\ 60 \\ \hline \end{array}$	9. $\begin{array}{r} 55 \\ \times\ 83 \\ \hline \end{array}$	10. $\begin{array}{r} 80 \\ \times\ 19 \\ \hline \end{array}$

Multiplication and division are related operations.


3 × 4 = 12
This is a factor.
It tells how many sets
you are considering.
This is a factor also.
It tells how many in each set.
This is the product.
It tells how many in all.

Compare this multiplication to a related division equation.



If you can't remember 54 ÷ 6, you can **Think** What number × 6 = 54?

If you can't remember 36 ÷ 4, you can **Think** What number × 4 = 36?

It saves a lot of time if you can remember the division facts in the first place. How many division facts give you trouble? Find out. See how fast you can write the answers.

	a	b	c	d	e	f	g
1.	42 ÷ 7	24 ÷ 6	64 ÷ 8	32 ÷ 4	49 ÷ 7	54 ÷ 6	45 ÷ 9
2.	48 ÷ 6	72 ÷ 9	18 ÷ 6	28 ÷ 7	24 ÷ 3	72 ÷ 8	36 ÷ 6
3.	54 ÷ 9	36 ÷ 4	35 ÷ 7	27 ÷ 3	40 ÷ 8	63 ÷ 9	28 ÷ 4
4.	36 ÷ 6	21 ÷ 3	30 ÷ 5	56 ÷ 8	63 ÷ 7	42 ÷ 6	81 ÷ 9

When last seen, the number 83 had just been tagged as a prime number.
What number comes next in the never ending search for prime suspects?

Number 84 is in the lineup.
But it's not even a suspect.
84 is an even number.
You know 2 is a factor of any even number.

85 can also be dismissed.
The last digit is 5,
so 5 must be a factor.

86? No luck.
Here is another even number.
But 87 looks suspicious.
Is 87 a composite number?
Use division to find out.

$$\begin{array}{r} 29 \\ 9 \\ \hline 20 \\ 3\overline{)87} \\ 60 \quad 20 \times 3 \\ \hline 27 \\ 27 \quad 9 \times 3 \\ \hline 0 \end{array}$$

It is a composite number.
3 is a factor.

89 is the next number to check.

$$\begin{array}{r} 29 \text{ (R2)} \\ 9 \\ \hline 20 \\ 3\overline{)89} \\ 60 \quad 20 \times 3 \\ \hline 29 \\ 27 \quad 9 \times 3 \\ \hline 2 \end{array}$$

The remainder tells you
3 is not a factor.

So find out if 7 is a factor.

$$\begin{array}{r} 12 \text{ R5} \\ 2 \\ \hline 10 \\ 7\overline{)89} \\ 70 \quad 10 \times 7 \\ \hline 19 \\ 14 \quad 2 \times 7 \\ \hline 5 \end{array}$$

The remainder tells you
7 is not a factor.

89 must be a PRIME NUMBER

1. Use division to find the other primes between 90 and 100 (if there are any).

2. How many more primes do you think there are?

Remainders in regular computation are not a problem.

Remainders in actual division can be handled in different ways.
Sometimes common sense tells you what to do.

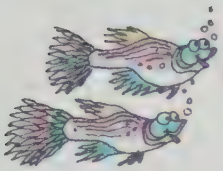
1. 34 people want to go. Only 6 people could go in each car. How many cars have to go?

How many 6s in 34? $6 \times 5 = 30$
$6 \times 6 = 36$

$$\begin{array}{r} 5\ \text{R4} \\ 6\overline{)34} \\ \underline{30} \\ 4 \end{array}$$

6 cars have to go. Or how many people have to stay home?

2. There were 38 guppies. 8 people were going to divide them.



How many 8s in 38? $4 \times 8 = 32$
$5 \times 8 = 40$

$$\begin{array}{r} 4\ \text{R6} \\ 8\overline{)38} \\ \underline{32} \\ 6 \end{array}$$

What are you going to do with the remaining 6?

3. 28 watermelons were ripe. 3 families are going to divide them.

How many 3s in 28? $9 \times 3 = 27$
$10 \times 3 = 30$

$$\begin{array}{r} 9\ \text{R1} \\ 3\overline{)28} \\ \underline{27} \\ 1 \end{array}$$

What can be done with the remaining watermelon?
If it were divided, how much would each get?

4. The candy was 2 for 15¢.
How much will 1 bar cost?

$$\begin{array}{r} 7\ \text{R1} \\ 2\overline{)15} \\ \underline{14} \\ 1 \end{array}$$

You can't pay $7\frac{1}{2}$ cents.
What will you pay?

$5\overline{)91}$

How many 5s in 91?
There are more than 10, because $10 \times 5 = 50$.
There are less than 20, because $20 \times 5 = 100$.

$$\begin{array}{r} 18\text{ R1} \\ 5\overline{)91} \\ \underline{50} \quad 10 \times 5 \\ 41 \\ \underline{40} \quad 8 \times 5 \\ 1 \end{array}$$

You know the answer is reasonable.
The estimate was a number
more than 10 and less than 20.

Try These

	a	b	c	d
1.	$2\overline{)62}$	$4\overline{)79}$	$3\overline{)78}$	$6\overline{)68}$
2.	$7\overline{)88}$	$3\overline{)51}$	$5\overline{)64}$	$4\overline{)41}$
3.	$2\overline{)89}$	$6\overline{)84}$	$9\overline{)96}$	$5\overline{)78}$

If you want to make sure your answers are reasonable *and* correct, you can check your division.

$3\overline{)76}$

There are more than 20: $20 \times 3 = 60$
There are less than 30: $30 \times 3 = 90$

$$\begin{array}{r} 25\text{ R1} \\ 3\overline{)76} \\ \underline{60} \quad 20 \times 3 \\ 16 \\ \underline{15} \quad 5 \times 3 \\ 1 \end{array}$$

Multiply to check division.

$$\begin{array}{r} 25 \\ \times\ 3 \\ \hline 75 \\ +\ 1 \\ \hline 76 \end{array}$$

Add the remainder.

Choose one of the three estimates given.

1.	How many 9s in 160?	**a**	10 nines	**b**	20 nines	**c**	30 nines

Why did you pick the estimate that you did?

2.	How many 7s in 480?	**a**	50 sevens	**b**	60 sevens	**c**	70 sevens
3.	How many 5s in 390?	**a**	60 fives	**b**	70 fives	**c**	80 fives
4.	How many 6s in 900?	**a**	90 sixes	**b**	100 sixes	**c**	200 sixes
5.	How many 3s in 420?	**a**	90 threes	**b**	100 threes	**c**	200 threes
6.	How many 4s in 952?	**a**	100 fours	**b**	200 fours	**c**	300 fours
7.	How many 2s in 875?	**a**	300 twos	**b**	400 twos	**c**	500 twos

Put your estimating skills to work.

$$\begin{array}{r} 87\text{ R4} \\ 7\overline{)} \\ 80 \\ 6\overline{)526} \\ 480 \quad 80 \times 6 \\ \hline 46 \\ 42 \quad 7 \times 6 \\ \hline 4 \end{array}$$

You could have estimated 60. It would be O.K.
Or you could have estimated 70.
What would have happened if you had tried 90?

Try These

	a	b	c	d	e
8.	$6\overline{)126}$	$7\overline{)154}$	$8\overline{)431}$	$4\overline{)565}$	$7\overline{)623}$
9.	$9\overline{)375}$	$5\overline{)785}$	$7\overline{)201}$	$5\overline{)433}$	$6\overline{)732}$
10.	$8\overline{)597}$	$9\overline{)700}$	$9\overline{)452}$	$6\overline{)541}$	$3\overline{)906}$

Study this example. $5\overline{)644}$

How many 5s in 644?
$10 \times 5 = 50$ Oops!
$100 \times 5 = 500$ That's about right.
$200 \times 5 = 1000$ That's too many!
There are between 100 and 200 fives in 644.

$$
\begin{array}{r}
128\text{ R4} \\
\hline
8 \\
20 \\
100 \\
5\overline{)644} \\
500 \quad 100 \times 5 \\
\hline
144 \\
100 \quad 20 \times 5 \\
\hline
44 \\
40 \quad 8 \times 5 \\
\hline
4
\end{array}
$$

The answer is reasonable. Is it right? Check to make sure.
Multiply 128×5. Then add the remainder of 4.
Is the answer correct?

Your turn. Try these.

	a	b	c	d	e
1.	$4\overline{)565}$	$3\overline{)824}$	$7\overline{)741}$	$5\overline{)918}$	$8\overline{)896}$
2.	$3\overline{)755}$	$4\overline{)911}$	$5\overline{)840}$	$7\overline{)762}$	$6\overline{)963}$

Why not start thinking really big!

How many 6s in 1200?
If you answered 20, you were right.
But there are a LOT more than 20.
If you answered 200, you were really right.
$200 \times 6 = 1200$
If you answered 2000, you were thinking too BIG!
$2000 \times 6 = 12\,000$

Estimate. A How many 8s in 6500? B How many 4s in 2500?

C How many 5s in 4100? D How many 7s in 2900?

E How many 9s in 2750? F How many 6s in 4830?

G How many 2s in 1652? H How many 3s in 2563?

One problem like this takes up a lot of paper. **Are you ready?**

$3\overline{)8227}$

How many 3s in 8227?
$1000 \times 3 = 3000$ Not enough.
$2000 \times 3 = 6000$ That's better.
$3000 \times 3 = 9000$ Too many!
The answer will be between 2000 and 3000.

"WOW!" That's a lot of threes.

```
   2742 R1
   ----
      2
     40
    700
   2000
3)8227
   6000   2000 × 3
   ----
   2227
   2100   700 × 3
   ----
    127
    120   40 × 3
   ----
      7
      6   2 × 3
   ----
      1
```

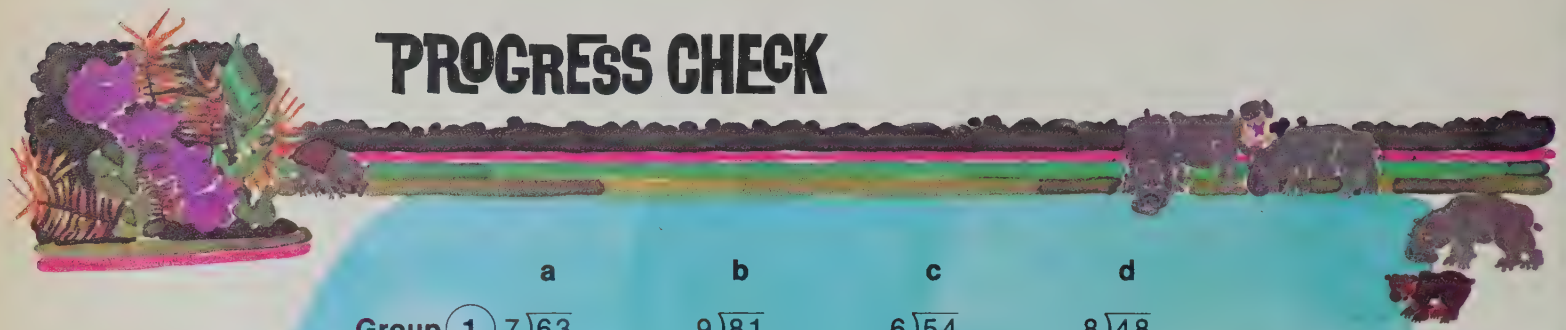
These problems are for very brave people only. Have a lot of paper ready.

1. $4\overline{)4558}$ **2.** $6\overline{)7821}$ **3.** $5\overline{)9426}$ **4.** $2\overline{)5632}$ **5.** $2\overline{)1089}$

If you got those done, you are a hero. Want to try these?

6. $8\overline{)4347}$ **7.** $7\overline{)4927}$ **8.** $2\overline{)8641}$ **9.** $3\overline{)3003}$ **10.** $4\overline{)9416}$

You have come a long way in division. Below you will find an example of each type of division that you have done. Use these problems to check your progress.



PROGRESS CHECK

	a	b	c	d
Group (1)	$7\overline{)63}$	$9\overline{)81}$	$6\overline{)54}$	$8\overline{)48}$
Group (2)	$6\overline{)76}$	$5\overline{)63}$	$4\overline{)81}$	$3\overline{)45}$
Group (3)	$7\overline{)497}$	$6\overline{)612}$	$9\overline{)990}$	$8\overline{)648}$
Group (4)	$5\overline{)623}$	$7\overline{)715}$	$8\overline{)968}$	$9\overline{)949}$

And maybe you tackled these too.

Group (5)	$8\overline{)2048}$	$6\overline{)6720}$	$5\overline{)6267}$	$4\overline{)4820}$

If you missed only one or two, you should feel very proud.

If you missed more than one in group ① or group ②, you'd better get someone to help you. You need to get problems like these right or you'll be getting very discouraged.

If you missed more than one in group ③ or group ④, go back and look at your work. Can you find your mistakes? Get someone to help you if you can't find any mistakes.

1. How many 4s in 40? in 400? in 4000?
2. How many 8s in 160? in 1600? in 16 000?
3. How many 2s in 40? in 400?
4. How many 20s in 40? in 400?

LOOK OUT!

5. How many 3s in 6? in 60? in 600?
6. How many 30s in 60? in 600?
7. How many 12s in 24? in 240? in 2400?
8. How many 9s in 360? in 3600?
9. Are there a hundred 20s in 2794?
   Think $100 \times 20 = 2000$
10. Are there a hundred 30s in 2416?
    Think $100 \times 30 = 3000$
11. Tell if the answer will be more than or less than the estimate given.

a	$1722 \div 40$	more than / less than	100	**b**	$4562 \div 40$	more than / less than	100
c	$1953 \div 30$	more than / less than	100	**d**	$6345 \div 30$	more than / less than	100

Pick the right symbol.

e	$9342 \div 90 >$ or < 100	**f**	$2178 \div 20 >$ or < 100
g	$1586 \div 10 >$ or < 100	**h**	$6051 \div 30 >$ or < 100

$30\overline{)810}$ The quotient is less than 100, because $100 \times 30 = 3000$.
The quotient is greater than 10, because $10 \times 30 = 300$.
So how many 30s?

THINK $20 \times 30 = 600$
$30 \times 30 = 900$

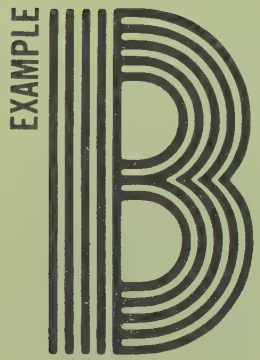
The quotient is between 20 and 30.

How many 30s in 810?
THINK $20 \times 30 = 600$
How many 30s in 210?
THINK $7 \times 30 = 210$

$$\begin{array}{r} 27 \\ 30\overline{)810} \\ 600 \\ \hline 210 \\ 210 \\ \hline 0 \end{array}$$

CHECK

$$\begin{array}{r} 30 \\ \times 27 \\ \hline 210 \\ 600 \\ \hline 810 \end{array}$$



$20\overline{)1380}$ Is the quotient between 10 and 100?
How many 20s? Estimate.
$40 \times 20 = 800$
$50 \times 20 = 1000$
$60 \times 20 = 1200$
$70 \times 20 = 1400$ Too big!
The quotient is between 60 and 70.
Copy the problem and finish it.

How many 20s in 1380?
THINK $60 \times 20 = 1200$
How many 20s in 180?
THINK $9 \times 20 = 180$

$$\begin{array}{r} 60 \\ 20\overline{)1380} \\ 1200 \\ \hline 180 \\ \blacksquare\blacksquare\blacksquare \\ \hline \blacksquare \end{array}$$

Check your answer with multiplication.

70)3426 Is the quotient between 10 and 100?

THINK How many 70s in 3426?

$20 \times 70 = 1400$ Too small.

$40 \times 70 = 2800$ That's better.

$50 \times 70 = 3500$ Too big.

The quotient is between 40 and 50.

How many 70s in 3426?

THINK $40 \times 70 = 2800$

How many 70s in 626?

THINK $8 \times 70 = 560$

How many more 70s?

What is the remainder?

$$\begin{array}{r} 48\text{R}66 \\ 8 \\ 40 \\ \hline 70\overline{)3426} \\ 2800 \\ \hline 626 \\ 560 \\ \hline 66 \end{array}$$

CHECK

$$\begin{array}{r} 70 \\ \times\ 48 \\ \hline 560 \\ 2800 \\ \hline 3360 \\ +\ \ 66 \\ \hline 3426 \end{array}$$

Your turn. Divide and check.

1. $30\overline{)510}$	**2.** $60\overline{)3660}$	**3.** $40\overline{)2640}$	**4.** $80\overline{)3920}$
5. $70\overline{)4970}$	**6.** $50\overline{)5050}$	**7.** $20\overline{)2000}$	**8.** $40\overline{)8080}$
9. $80\overline{)6488}$	**10.** $30\overline{)3933}$	**11.** $60\overline{)1206}$	**12.** $90\overline{)2709}$

Here is another time when remainders in division problems have to be treated with common sense.
A bus could hold 40 passengers.

80 wanted to go. How many buses?
90 wanted to go. How many buses?
160 wanted to go. How many buses?
200 wanted to go. How many buses?

Be an operator.
Copy and complete this chart.

	×3	×5	×8	×9	×10	×100
4	?	?	?	?	?	?
6	?	?	?	?	?	?
7	?	?	?	?	?	?
9	?	?	?	?	?	?

There are 7 days in 1 week. Copy and complete these charts too.

Number of days	42	63	84	105	126	147	182	365
Number of weeks	?	?	?	?	?	?	?	?

There are 3 apples in one tray.

Number of apples	3	6	12	20	50	100	500	1000
Number of trays	?	?	?	?	?	?	?	?

There are 6 oranges in one bag.

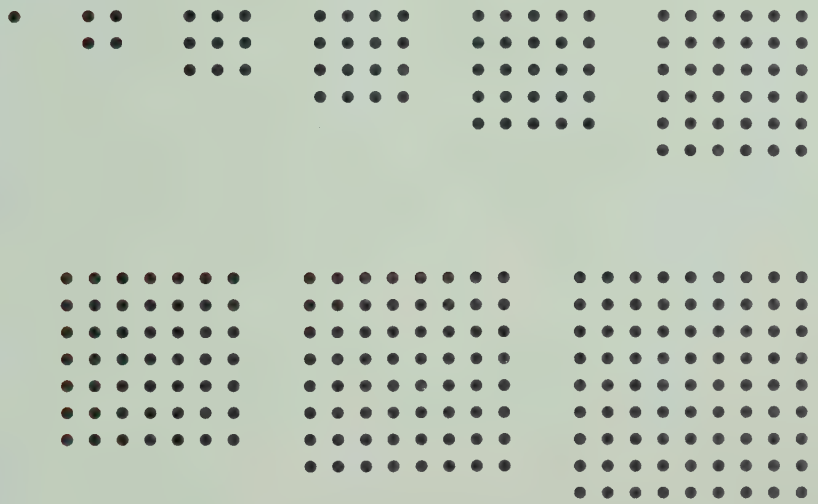
Number of bags	?	?	?	?	?	100	150	260
Number of oranges	6	12	24	48	120	?	?	?

There are 12 eggs in 1 dozen.

Number of eggs	12	24	?	120	?	?	300	600
Number of dozens	?	?	5	?	15	20	?	?

Just for Fun

The diagonal on your multiplication table contains the products 1, 4, 9, 16, 25, 36, 49, 64, and 81. What pairs of factors give these results? These products are called *square numbers*. What might be the reason they are called square numbers? Do the following dot patterns help you to tell why?



What would be the next three square numbers?

CHECKOUT

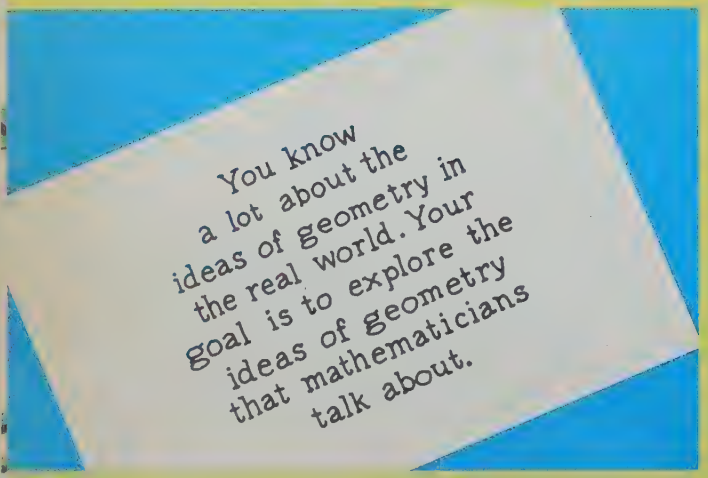
Write an estimate. Then find the exact answer.

	a	b	c	d
1.	420×5	107×9	596×6	125×19
2.	424×48	213×94	178×82	890×71
3.	$9\overline{)891}$	$7\overline{)714}$	$8\overline{)900}$	$6\overline{)606}$
4.	$4\overline{)8328}$	$3\overline{)1209}$	$20\overline{)645}$	$70\overline{)7210}$

5. Think of some good answers for these.
   - **a** If you buy a package of nails or a package of pins, one size will say "Approximately 100." What does that mean? Why is it printed on the package? Do you think there will be more or less than 100? How many more or less?
   - **b** Chocolates are not always sold by the box. Do you have any way of knowing how many might be in 1 kg? If you want to make sure you have 50 chocolates, what could you do?

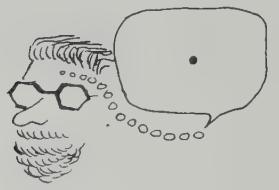
Suppose you dipped a sponge ball in ink. If you touched it to a piece of paper, what kind of mark would it make?

If you rolled the ball a little way, what kind of mark would it make?



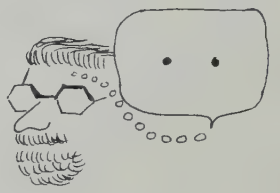
Imagine a ball such as that rolling straight, never starting or stopping, and never running out of ink. What kind of mark would it make?

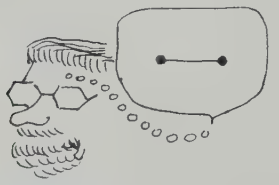
Can you imagine nothing? It's pretty hard, isn't it?



Now try to think of just one point in space. That's easy, isn't it?

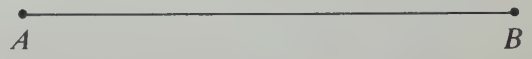
Think of another point near the first one. (Dots can be used to show points.)





Now think of these two points and a path going straight from one to the other. You're thinking about a figure called a *line segment*. The points at its ends are called *endpoints*.

To help us talk about a line segment, we usually give letter names to its endpoints—A and B, for example.



Then we name the line segment by naming its endpoints. The diagram shows *line segment* AB.

Do you know how many points are in a line segment?

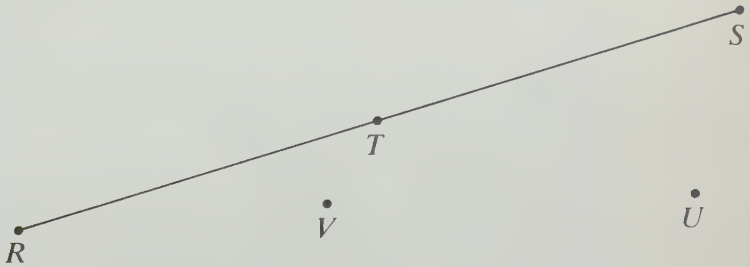
Point C is halfway between point A and point B.

1. Copy and label points A, B, and C.

2. Now mark a point halfway between point A and point C. Label this point D. Mark a new point halfway between point A and point D. Label it E. See if you can mark another point halfway between point A and point E.

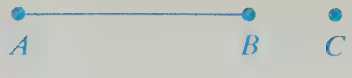
3. If you had a very sharp pencil and a microscope, could you keep on marking these halfway points? You can also mark halfway points between C and B, and so on. How many points do you think there are in a line segment? Could you mark all the points with dots?

4. A line segment consists of two points and all the points between them.



a) Do you think point T is on line segment RS?
b) Is point U?
c) Is point V on line segment RS?

1



Look at line segment AB and point C.
Point B is between point A and point C.

a Can you extend the line segment to get a line segment whose endpoints are A and C?

b Of course you can. Now suppose you mark point D so that point C is between point B and point D.

c Could you extend the line segment again so that the endpoints are point A and point D? Do you think that you could keep on extending the line segment in this way?

2

In the diagram above, the arrowhead shows that the figure extends without end. This figure is called a *ray*.

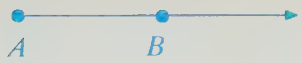
a Look at the following ray:

This ray starts at point J, passes through point K, and continues without end. Point J is thought of as the "front end" point of the ray. But it is simply called the *endpoint*.
How many endpoints does a ray have?

b What is the endpoint of this ray?

The ray at the left is called ***ray ST***. We always say the name of the endpoint first.

What is the endpoint of ray AB? Ray AB begins at A, passes through B, and continues without end.



B is also the endpoint of a ray that passes through point A and continues without end.

Now consider the following diagram.

Point B can be thought of as the endpoint of two rays that go in opposite directions.
We call this set of points a *line*.

Here's another way to think of a line.

Start with line segment AB.
Now extend it so that its new endpoints are point C and point E.

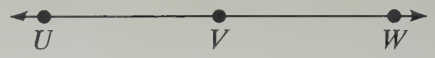
Could you keep on extending the line segment to new endpoints? Could the line segment be extended in both directions without end?

We name a line after any two points on it. For example, this diagram shows line RS or line SR or line ST.
The order of the letters doesn't matter in naming a line.

We have seen several pictures of lines.
Is it possible to draw an entire line?
Not really—because a line goes on without end.
We indicate this idea with arrowheads.

We draw a picture of a line to express the *idea* of a line. In the same way, we draw pictures of points, line segments, and rays to express the ideas of points, line segments, and rays.

1. Name this line in six different ways.



2. Draw and label line XZ that includes point R.

3. Draw and label line BC that includes points D and E.

4. Draw and label line RS that includes points U, V, Y, and W.

5. Draw two lines that include the same point.

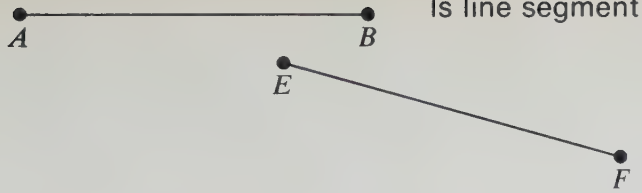
6. Mark a point. Label it B. Draw a line through B. Draw another line through point B. Is there a limit to the number of lines that pass through point B?

7. Mark two points. Label them C and D. Draw a line through point C and point D. If you can, draw another line through points C and D. How many lines are there through points C and D?

8. Mark a point. Label it E. Draw two line segments that pass through point E. Draw a line that contains both line segments, if you can.

9. Mark and label two points, F and G. Draw two line segments that pass through both point F and point G. Draw a line that contains both line segments, if you can.

10. Mark and label a point H. Draw two rays that have H as the endpoint. Draw a line that contains both rays, if you can.



Is line segment AB congruent to line segment EF?

???What does "CONGRUENT" mean???

When we say two segments are congruent, we mean that if you place one segment on the other, the segments will match exactly.

How do you do that?

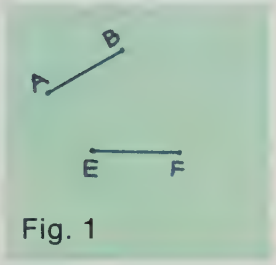


Fig. 1

Figure 1 shows two line segments.

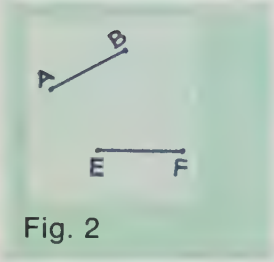


Fig. 2

One way to tell if they are congruent is to place a piece of paper over them.

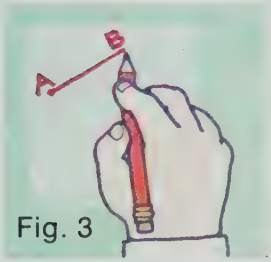


Fig. 3

Trace one of the line segments.

Fig. 4

Then turn the paper so that the line segment you traced is directly over the other.

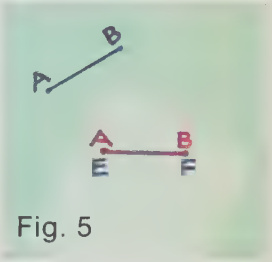
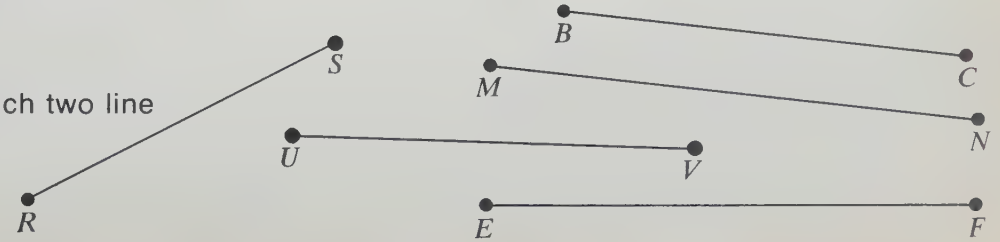


Fig. 5

This way you can easily see if the two line segments are congruent.

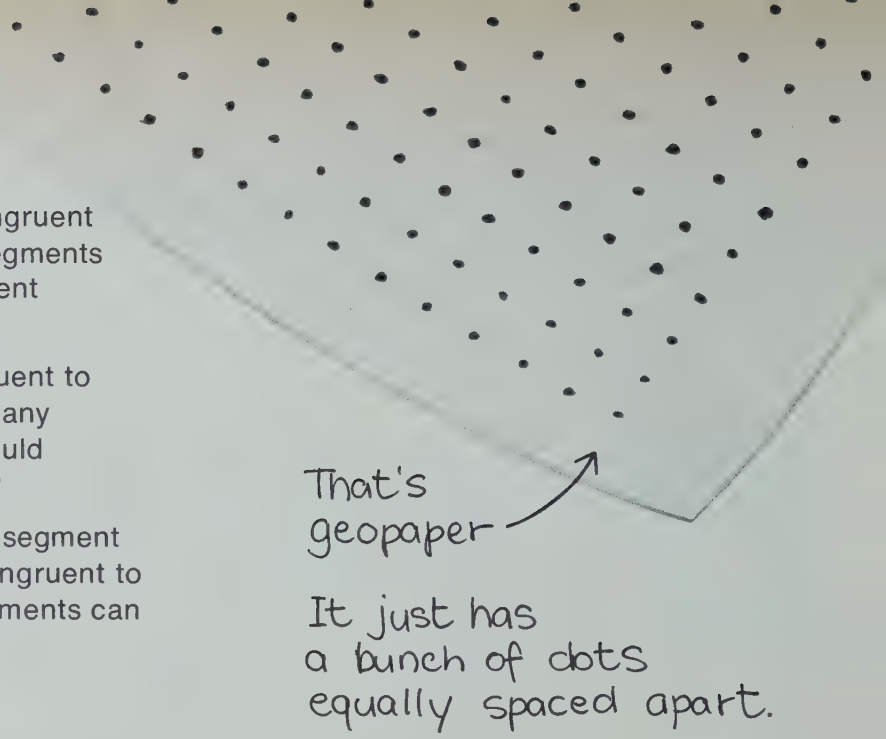
1. If two line segments are congruent, are they the same length?

2. Determine by tracing which two line segments are congruent.



The following exercises can be done on geopaper.

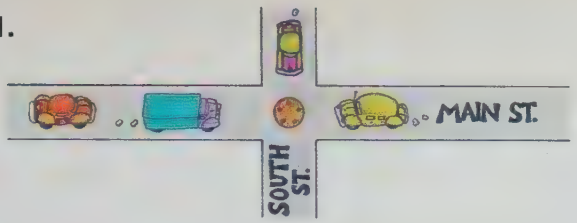
1. Draw and label a line segment BC. Draw another line segment that is congruent to line segment BC. How many line segments could you draw that would be congruent to line segment BC?
2. Draw a line segment that is *not* congruent to line segment BC of exercise 1. How many line segments could you draw that would *not* be congruent to line segment BC?
3. Mark and label a point D. Draw a line segment with point D as an endpoint that is congruent to line segment BC. How many such segments can you draw?



Think about these questions.

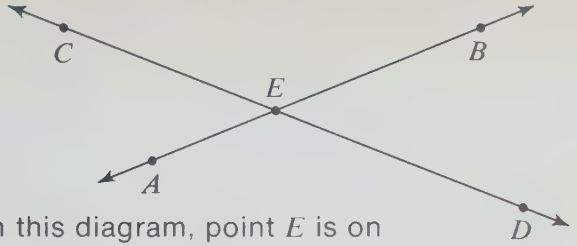
4. What would a table look like if opposite edges were not congruent?
5. What kinds of problems would you have raising or lowering a venetian blind if opposite sides were not congruent?
6. Are the opposite edges of a book congruent? What makes you think so?
7. What would happen if the four legs of a table were not congruent?
8. Could you slip the lid of a box down over the bottom of a box if the lid and the bottom were congruent?

1.



In the picture a manhole cover is in the middle of an intersection. Which street does it lie on?

2.



In this diagram, point E is on how many lines?
These two lines are said to intersect.
E is the *point of intersection.*

3. What can you think of that reminds you of two intersecting lines?

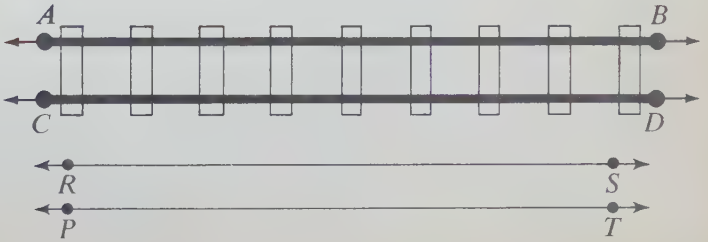
4. Do the rails of a straight track appear to intersect?

5. If you walked along the tracks, could you find the intersection? Do the rails really intersect?

If you looked down on a railway track from above, you might see something like this.

The rails represent two lines that do not intersect.

Line RS and line PT do not intersect.
Such lines are said to be *parallel* to each other.



Parallel lines are lines that do not intersect and are always the same distance apart.

Find things in a room that suggest two parallel lines.

Two line segments or rays are parallel if they are contained in two parallel lines. Find parallel line segments in a room.

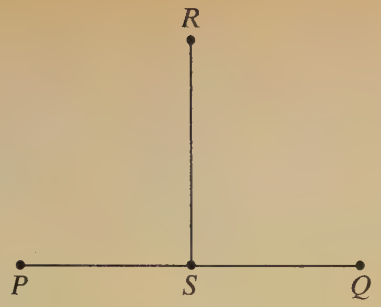
1. Which line segments look parallel?

2. Which line segments intersect?

3. Why are opposite sides of most boxes parallel?

4. Can you think of a building that does not suggest at least one pair of parallel lines?

5. Think of things of nature. What kind of lines do you see most often? parallel? straight? curved?



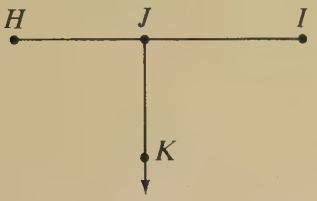
Line segments PQ and RS intersect.
What do you notice about them?

The two line segments form square corners.
These square corners are called *right angles*.

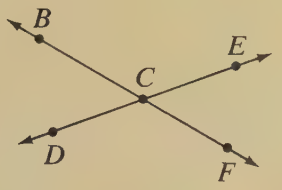
Two lines, line segments, or rays that form right angles are said to be *perpendicular* to each other. Above, line segment RS is perpendicular to line segment PQ and line segment PQ is perpendicular to line segment RS.

Which of the following look perpendicular?

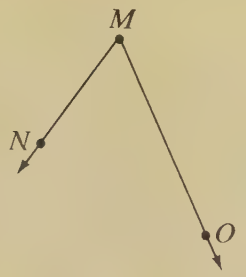
1.



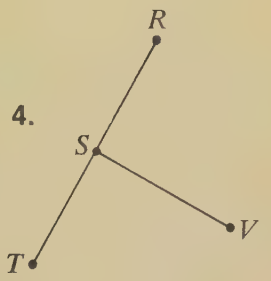
2.



3.

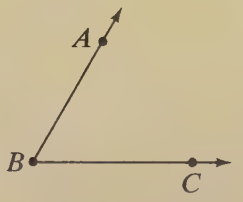


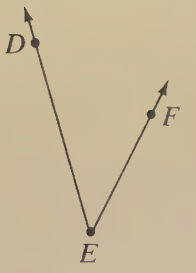
4.



5. Do angle ABC and angle DEF look like right angles?

6. Do they look congruent?





Not all angles are right angles. Some are larger than a right angle, some are smaller.

We can determine if any two angles are congruent by tracing.

7. Trace to tell whether angle ABC is congruent to angle DEF.

Line AB is parallel to line CD.
Line EF is parallel to line HI.
How many angles are formed by these four intersecting lines?

1. Trace angle AOF. Which other angles are congruent to angle AOF? Determine by tracing if there are any other congruent angles. Be sure to check all the angles.

2. Trace the figures. Get two pencils of different colors. With one color shade a little of the inside of angle AOF and of all the angles congruent to angle AOF. With the other color shade a little of the inside of angle FOL and of all the angles congruent to angle FOL.

   You should not need more than two colored pencils.
   Do you see a pattern? Try to describe the pattern.

THE CONTINUING ADVENTURES OF

RICK TRACE-AND-SEE

H-M-M-M

The Case of The Congruent Cowboy

HINT FOR ROOKIES: GET OUT YOUR TRACING PAPER

FLASH: SYRUS BACKBITER, WEALTHY PLAYBOY, PHONES OUR HERO...

MY BUTLER HAS STOLEN MY WINDOW!

H-M-M-M

RICK RUSHES TO THE SCENE OF THE CRIME TO FIND:

I ARRIVED HOME THIS MORNING FROM BERMUDA TO DISCOVER MY PRIZE PLATINUM WINDOW (WORTH $95000 $21.95) HAD BEEN RIPPED OFF. MY BUTLER'S FOOTPRINTS ARE IN THE SOFT CLAY.

HM-M-M

NO! I'M BEING FRAMED.. I DIDN'T DO IT!

AND HERE'S THE CRIMINAL (SHUT UP, CUR!) WHO STOLE MY WINDOW. SEE HIS COWBOY BOOTS! I DEMAND HE BE ARRESTED!

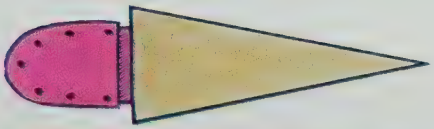
HM-M-M – LET ME LOOK AROUND THE PLACE FIRST, MR. BACKBITER.

UPSTAIRS, INSIDE SYRUS'S CLOSET, TRACE-AND-SEE FINDS A PAIR OF COWBOY BOOTS BELONGING TO – YOU GUESSED IT – SYRUS BACKBITER.

OUR HERO MAKES A TRACING OF THE ANGLE OF THE TOE-MARKS THAT WERE IN THE GARDEN.

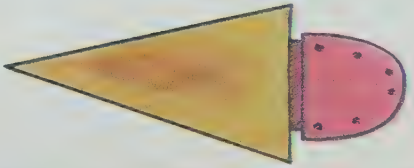
HM-M-M

O.K. JR. ROOKIES, WHOSE BOOTS MADE THE FOOTPRINTS IN THE GARDEN?

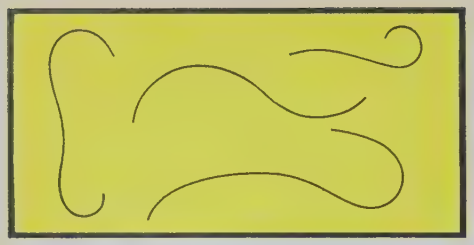


BOTTOM OF BUTLER'S COWBOY BOOT

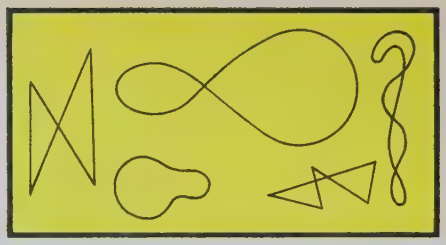
?



BOTTOM OF SYRUS'S COWBOY BOOT



These sets of points are called *curves*.

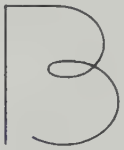


These sets of points are called *closed curves*.
Why do you think they are called "closed"?

These sets of points are called *simple closed curves*.
What do you think is the difference between a simple closed curve and a closed curve?

1. Which of the following are curves?

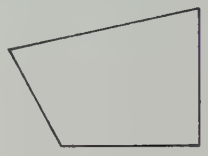
a

b 

c

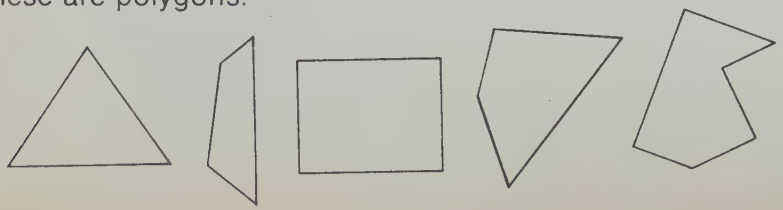
d

e

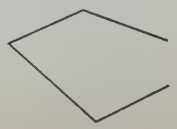
f 

2. A closed curve is a curve that doesn't stop or start at any particular point. If you were tracing a closed curve, you'd eventually end up where you began.
Which of the curves above are closed curves?

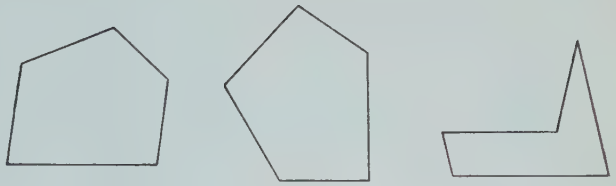
Simple closed curves made up of line segments are *polygons*.
All these are polygons.



This is not a polygon.



You might think of polygons as being a family. The members of the polygon family are all made up of line segments. If polygons could talk and someone said "Hey, polygon," all of them would answer. If you wanted a specific one, you would have to call it by its first name. (The first names generally give a hint about the number of sides.)



Polygons with five sides are called *pentagons*.

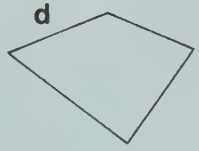
1. Which of the following are pentagons?

a

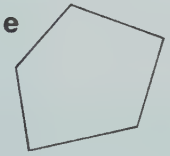
b

c

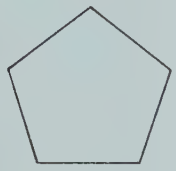
d



e



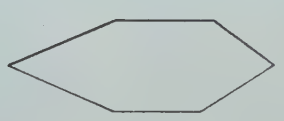
A *regular pentagon* is a pride and joy of the polygon family. It looks like this:



Why do you think it is called regular?

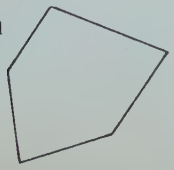
2. Determine by tracing.
Are all sides congruent?
Are all angles congruent?

Polygons with 6 sides are called *hexagons*.



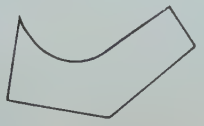
3. Which of the following are hexagons?

a



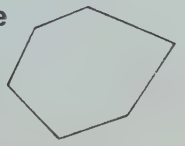
b

c

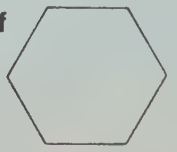


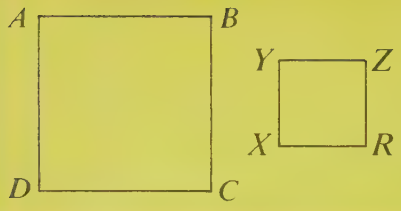
d

e



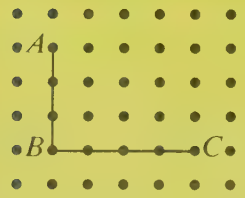
f





Does a square belong to the family of polygons? How do you know?

In order to talk about polygons more easily, we usually label each corner point with a capital letter. The corner point is called a *vertex* of the polygon. Now when we say that square $ABCD$ is larger than square $XYZR$, we know which squares we're talking about.



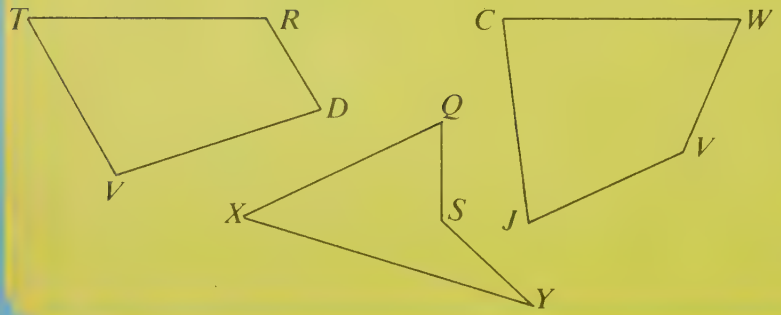
Do the following exercises on geopaper.

1. Start with figure ABC. Complete it so that you have each of the following:

   a triangle **b** rectangle **c** pentagon

   d hexagon **e** polygon **f** square

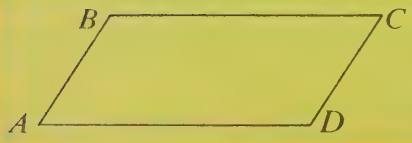
All four-sided polygons are called *quadrilaterals*.



2. Draw a rectangle. Is a rectangle a quadrilateral?

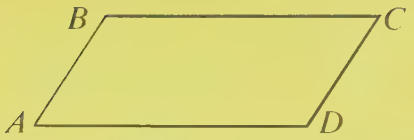
3. Draw a square. Is the square a rectangle? Is it also a quadrilateral?

4. Draw a pentagon. Is the pentagon a quadrilateral? Why?



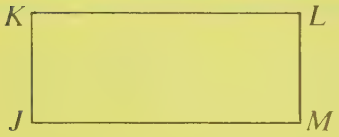
5. This is a quadrilateral. It is a parallelogram. Is this quadrilateral a rectangle? Why?

A parallelogram is a quadrilateral whose opposite sides are parallel.



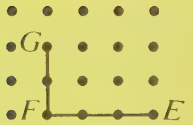
Which side is opposite side BC?
Which side is opposite side BA?
Determine which sides are congruent by tracing.

6. Is this rectangle a parallelogram?
Which side is opposite side KL?
Which side is opposite side KJ?
Determine which sides are congruent by tracing.



7. Answer the same set of questions about a square.
Then try to tell how all parallelograms are alike.

8. You will be using the figure on the right four times.



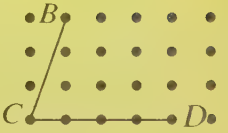
a Copy GFE. Try to draw a quadrilateral that is *not* a parallelogram.

b Copy GFE. Try to draw a parallelogram that is *not* a rectangle.

c Copy GFE. Try to draw a rectangle that is *not* a square.

d Copy GFE. Try to draw a square.

9. You will be using the figure on the right four times also.

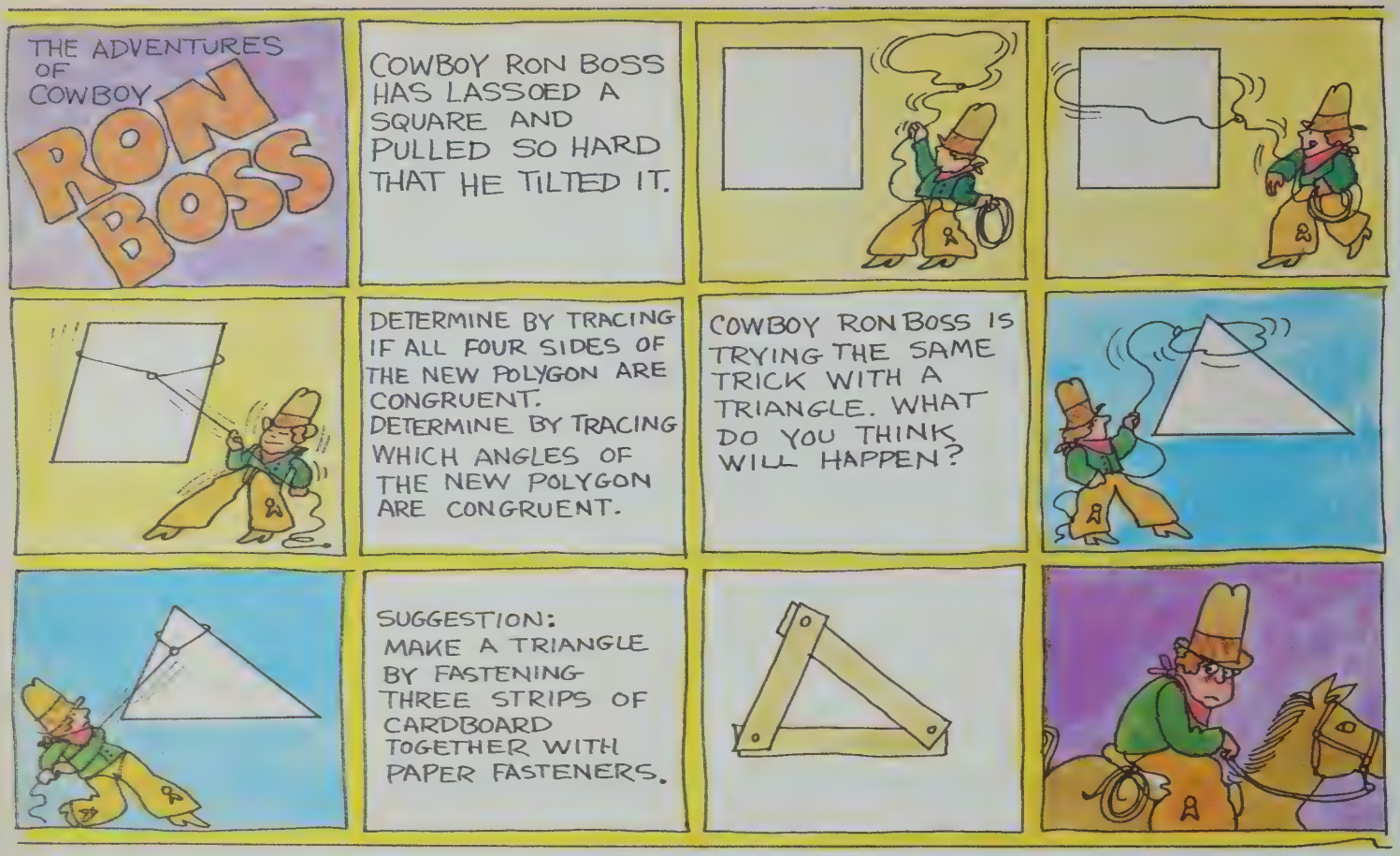


a Copy BCD. Try to draw a quadrilateral that is *not* a parallelogram.

b Copy BCD. Try to draw a parallelogram that is *not* a rectangle.

c Copy BCD. Try to draw a rectangle that is *not* a square.

d Copy BCD. Try to draw a square.


THE ADVENTURES OF COWBOY RON BOSS
COWBOY RON BOSS HAS LASSOED A SQUARE AND PULLED SO HARD THAT HE TILTED IT.
DETERMINE BY TRACING IF ALL FOUR SIDES OF THE NEW POLYGON ARE CONGRUENT. DETERMINE BY TRACING WHICH ANGLES OF THE NEW POLYGON ARE CONGRUENT.
COWBOY RON BOSS IS TRYING THE SAME TRICK WITH A TRIANGLE. WHAT DO YOU THINK WILL HAPPEN?
SUGGESTION: MAKE A TRIANGLE BY FASTENING THREE STRIPS OF CARDBOARD TOGETHER WITH PAPER FASTENERS.

Look around you.

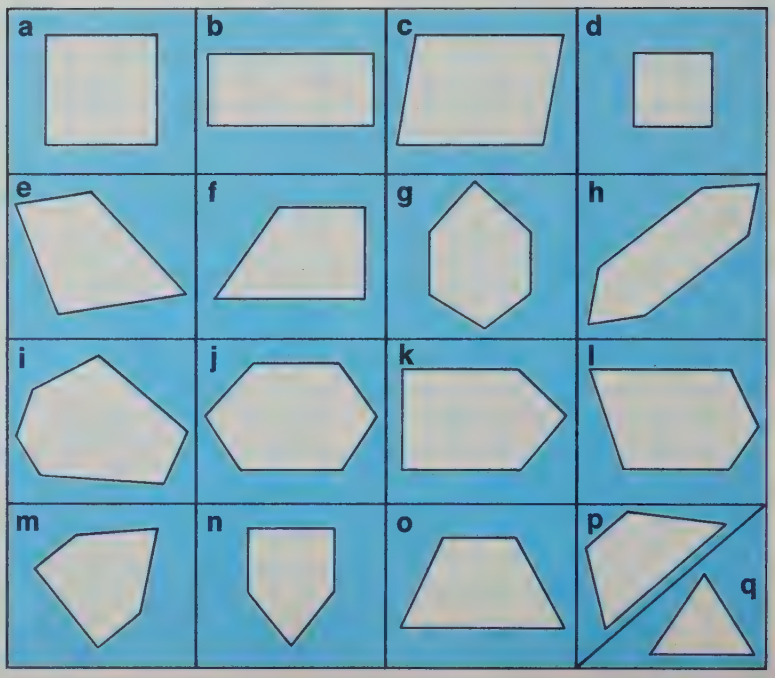
- What is the shape of most dishes?
- What shape do you see most often inside a room?
- What shape are the sides of most boxes?
- What shapes are traffic signs?

What shapes can you find in these pictures?

CHECKOUT

Look at the figures below.

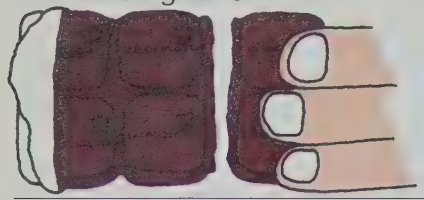
1. Which are quadrilaterals?
2. Which are parallelograms?
3. Which are rectangles?
4. Which are squares?
5. Which are pentagons?
6. Which are hexagons?
7. Which are triangles?
8. Which are regular polygons?



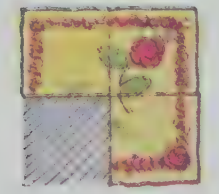
4

FRACTIONS

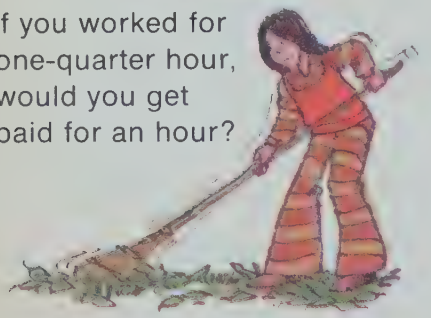
What part of the chocolate bar is being taken?



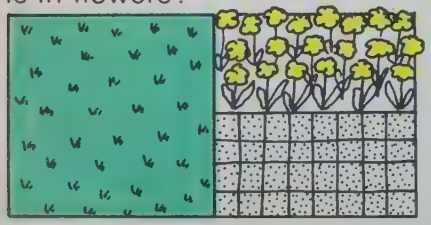
What part of the cake is gone?



If you worked for one-quarter hour, would you get paid for an hour?



What part of the garden is in flowers?



Fractions, fractions, fractions. You use them quite a bit. But you've got to be careful how you use them. By the time you finish this chapter, you will know the answers to these questions. Or do you know the answers now? Find out.

1. Which fraction is the the largest?

   $\frac{3}{5}$ $\frac{2}{3}$ $\frac{3}{4}$

2. Which fraction is the same size as $\frac{3}{4}$?

   $\frac{3}{8}$ $\frac{4}{8}$ $\frac{6}{8}$

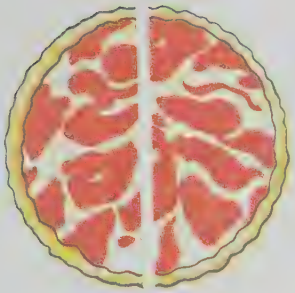
3. What are two other fractions that name the same number as $\frac{8}{12}$?

You will learn much more than the answers to these questions.

Set your goal.
I'm going to learn about the idea of fractions.
I'm going to review how to add and subtract fractions, too.

You have to really think when you are working with fractions.
Pretend there are three pizzas. They are alike in every way.

You get $\frac{1}{2}$ of any one of them.
The half you take won't make any difference in the amount of food you get.



Here you get $\frac{1}{2}$.

Here you get $\frac{1}{2}$.
That is cut into two pieces.
You get $\frac{2}{4}$.

Here you get $\frac{1}{2}$.
That is cut into three pieces.
You get $\frac{3}{6}$.

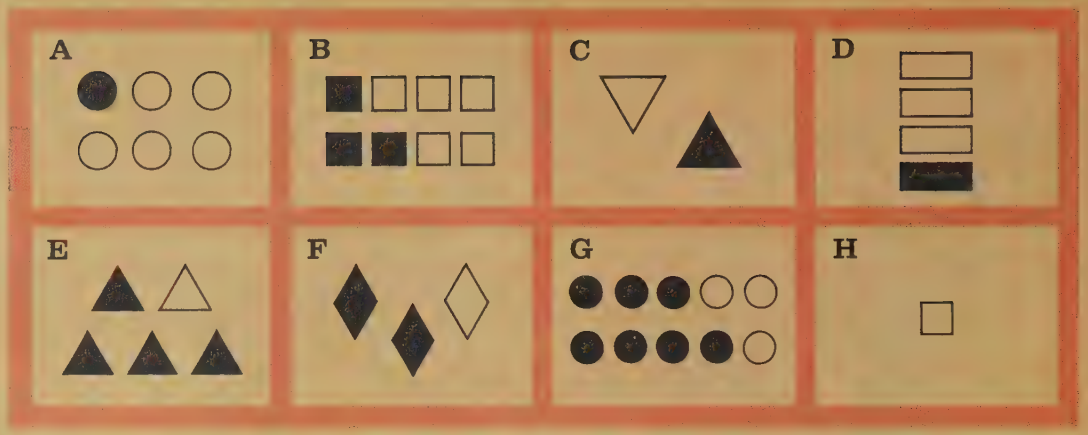
BUT you had better take another look at when $\frac{1}{2} = \frac{1}{2}$.

Let's trade.
I'll trade you—
$\frac{1}{2}$ of my allowance for $\frac{1}{2}$ of your allowance
$\frac{1}{2}$ of my lunch for $\frac{1}{2}$ of your lunch
$\frac{1}{2}$ of my dime for $\frac{1}{2}$ of your dollar

When does $\frac{1}{2} = \frac{2}{4}$ and $\frac{2}{4} = \frac{3}{6}$?
Let's trade again and find out.
I'll trade you—
$\frac{1}{2}$ of my dollar for $\frac{2}{4}$ of your dollar
$\frac{1}{2}$ of my 12 marbles for $\frac{3}{6}$ of your 12 marbles
$\frac{2}{4}$ of my 4 for $\frac{3}{6}$ of your 6

When you compare the fractional parts of the same thing, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equal. When you compare fractional parts of different things, then you had better be careful.

Fractions can be shown by many different models. Here's one kind.



1 Copy and complete the chart to describe the sets.

SET	HOW MANY SHADED?	HOW MANY IN ALL?	WHAT FRACTION SHADED?
A	1	6	?
B	?	8	?
C	?	?	?
D	?	?	?
E	?	?	?
F	?	?	?
G	?	?	?
H	0	?	?

2 Draw sets of objects. Shade enough of the set to show each of the following fractions.

a $\frac{1}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{6}$ **d** $\frac{5}{5}$

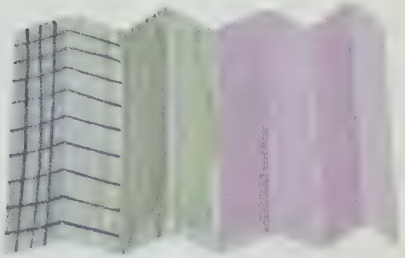
1. Get a sheet of paper. Fold it in half. Crease the fold. Open the paper. Lightly shade the left half of the paper.

2. Refold the paper. Fold it in half again by matching the folded edge to the open edges. Crease this fold.
   - **a** How many parts now?
   - **b** Draw some horizontal lines over the part you have shaded to show $\frac{1}{4}$ of the paper.
   - **c** What part of the paper is not shaded?
   - **d** What part of the paper does not have horizontal lines?
   - **e** Which is more, $\frac{1}{4}$ or $\frac{2}{4}$? $\frac{1}{4}$ or $\frac{3}{4}$? $\frac{1}{2}$ or $\frac{3}{4}$?

3. Refold the paper one more time. Fold it in half again. Crease the fold. Then open the paper.
   - **a** How many parts now?
   - **b** Draw vertical lines over the horizontal lines to show $\frac{1}{8}$ of the paper. (It will look like a checkerboard.)
   - **c** What part of the paper is not checked?
   - **d** What part of the shaded part is not checked?

4. Turn the paper over.
   - **a** Mark $\frac{1}{2}$ on the fold line where $\frac{1}{2}$ stops and the next $\frac{1}{2}$ begins.
   - **b** Mark $\frac{1}{4}$ on the fold line where $\frac{1}{4}$ stops and the next $\frac{1}{4}$ starts. Mark $\frac{2}{4}$ where two $\frac{1}{4}$s stop and the next $\frac{1}{4}$ starts. Finally mark $\frac{3}{4}$ where three $\frac{1}{4}$s stop and the next $\frac{1}{4}$ starts. Is there a fold line to show $\frac{4}{4}$?
   - **c** Mark eighths on the fold lines in the same way.

Open the paper.



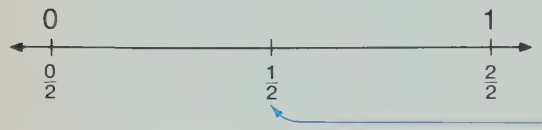
Your folded paper will help you answer these problems.

1. Which is more?

a $\frac{1}{4}$ or $\frac{1}{2}$ **b** $\frac{1}{4}$ or $\frac{1}{8}$ **c** $\frac{1}{2}$ or $\frac{1}{8}$ **d** $\frac{2}{4}$ or $\frac{2}{8}$ **e** $\frac{1}{4}$ or $\frac{4}{8}$ **f** $\frac{4}{8}$ or $\frac{3}{4}$ **g** $\frac{3}{4}$ or $\frac{7}{8}$

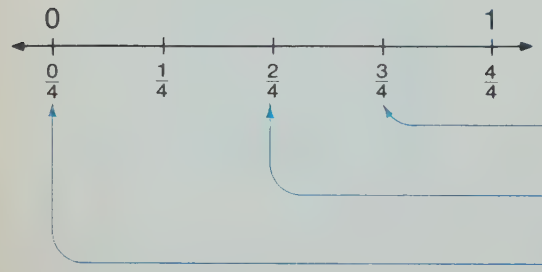
2. A number line can show fractions also.

This is one unit.



The segment from 0 to 1 is now divided into 2 equal parts.

a Why is this point called $\frac{1}{2}$?



Now the segment from 0 to 1 is divided into 4 equal parts.

b Why is this point called $\frac{3}{4}$?

c Does $\frac{2}{4}$ name the same part as $\frac{1}{2}$?

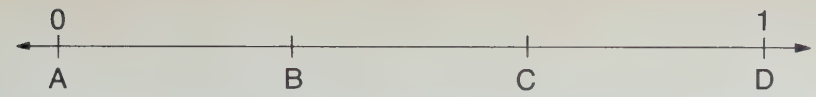
d Why can this point be named $\frac{0}{4}$?

e How many equal parts now?

f Copy this number line. Write a fraction name for each point.

3. List at least four fraction names for the number 1.

4. List at least four fraction names for the number 0.



Stick with the number line for a while. This time the segment has been divided into 3 equal parts. Write a fraction name for each lettered point.

1. Now how many equal parts?

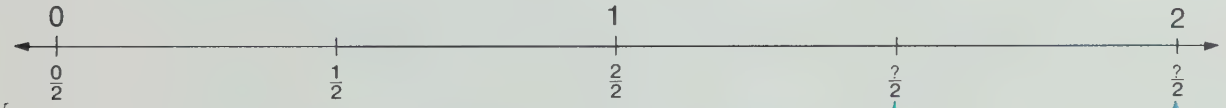
2. And now how many equal parts?

Look out!

3. How many equal parts?

Why name only fractional parts of 1? Extend the number line.

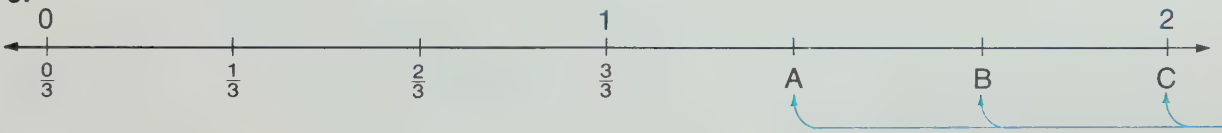
4.



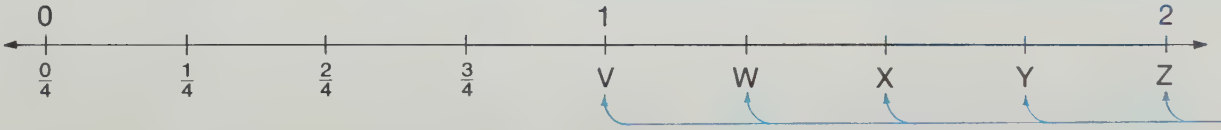
a Name this point. $\frac{2}{2} + \frac{1}{2} =$

b Name this one also. $\frac{3}{2} + \frac{1}{2} =$

5.



6.



7. Could the number line be extended still more? How far?

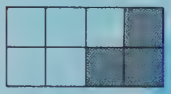
PROGRESS CHECK

Each of these pictures can be named by a fraction.
Match each picture in the left section with a picture in the right section that can be named by the same fraction.

1.

2.

3.

4. 

5.

A

B

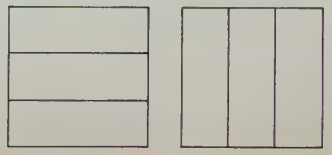
C

D

E

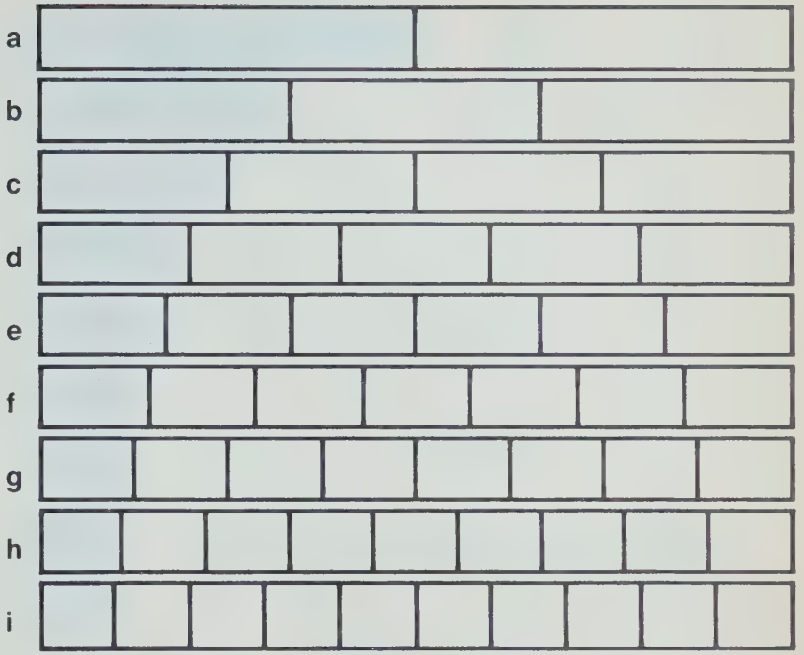
SUPER SLEUTH

Here are two ways of dividing a square region into 3 equal parts. Try to find a third way.



Start getting it all together.

Here's a batch of number strips. Think of a number strip as a combination of a region and a number line. They are arranged in an order. You are to do two things.



1. Tell what fraction of each strip is shaded.

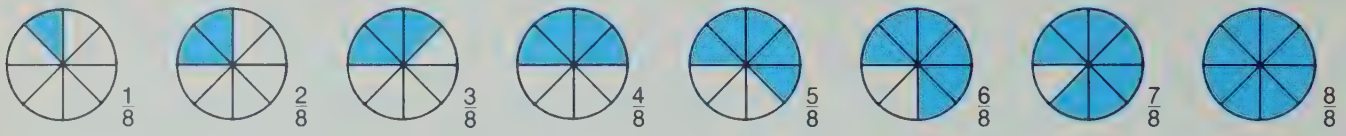
2. Tell what fraction of each strip is NOT.

3. Now use the strips to help you pick the greater of each pair of fractions.

a	$\frac{1}{2}$ or $\frac{2}{3}$	**b**	$\frac{2}{3}$ or $\frac{3}{4}$	**c**	$\frac{3}{4}$ or $\frac{4}{5}$	**d**	$\frac{4}{5}$ or $\frac{5}{6}$
e	$\frac{1}{2}$ or $\frac{2}{10}$	**f**	$\frac{5}{10}$ or $\frac{5}{8}$	**g**	$\frac{5}{8}$ or $\frac{2}{3}$	**h**	$\frac{2}{3}$ or $\frac{3}{6}$
i	$\frac{2}{4}$ or $\frac{3}{5}$	**j**	$\frac{3}{5}$ or $\frac{1}{2}$	***k**	$\frac{3}{6}$ or $\frac{4}{8}$	***l**	$\frac{5}{10}$ or $\frac{1}{2}$

WHAT IS A FRACTION?

numerator ⟶ $\frac{1}{8}$ ⟵ the number of shaded parts
denominator ⟶ $\frac{1}{8}$ ⟵ the number of parts in all



1. Look at the fractions that label the models above.

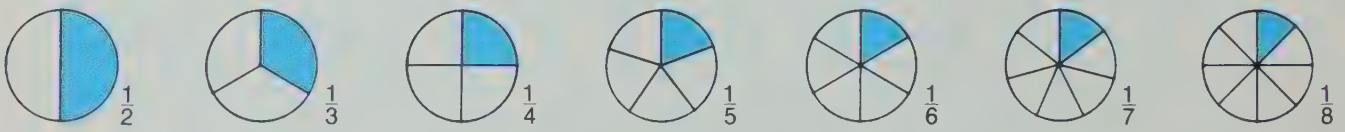
a Why are the denominators alike?

b Why are the numerators different?

c If the denominators are alike, the numerator tells you which fraction is the largest.

d Draw regions. Show fourths. Prove **c** is true.

2. Look out for a change!



How do you know which fraction is the largest if the numerators are the same?

3. Put each set of fractions in order from largest to smallest.

a $\frac{2}{6}, \frac{2}{4}, \frac{2}{5}, \frac{2}{3}$ **b** $\frac{1}{9}, \frac{8}{9}, \frac{2}{9}, \frac{4}{9}$ **c** $\frac{7}{10}, \frac{3}{10}, \frac{9}{10}, \frac{0}{10}$ **d** $\frac{3}{5}, \frac{3}{7}, \frac{3}{4}, \frac{3}{11}$

4. Use the symbol for "is greater than" (>) or the symbol for "is less than" (<) to make each sentence true.

a $\frac{5}{6} ? \frac{5}{8}$ **b** $\frac{3}{4} ? \frac{1}{4}$ **c** $\frac{2}{3} ? \frac{2}{5}$ **d** $\frac{1}{9} ? \frac{1}{8}$ **e** $\frac{1}{10} ? \frac{3}{10}$ **f** $\frac{4}{9} ? \frac{4}{7}$ **g** $\frac{4}{4} ? \frac{1}{4}$ **h** $\frac{3}{5} ? \frac{3}{3}$

1. How many thirds? How many shaded?

2. How many thirds? How many shaded?

3. How many thirds? How many shaded?

a Name the fraction that names the parts shaded.

b Is there another name that could be used?

4. Now how many thirds? How many shaded?

a Is the amount shaded greater than 1?

b Name the fraction that names the parts shaded.

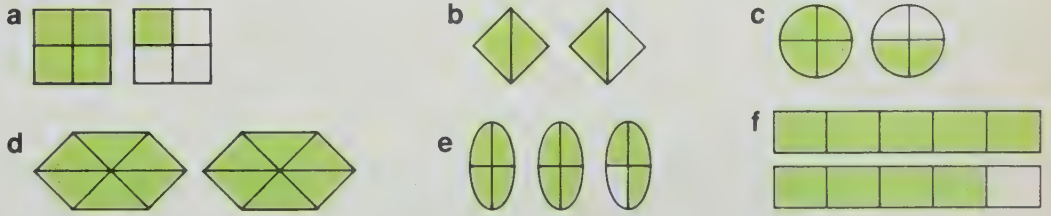
c Does the mixed number $1\frac{1}{3}$ also name the parts of the regions shaded?

5. Now how many thirds? How many shaded?

a Name the fraction that names the parts shaded.

b Name the mixed number that is another name for that fraction.

6. Write the fraction and the mixed number that name the parts shaded.



7. Which pairs show two names that could describe the same region? Draw the regions if you need help.

a $1\frac{2}{3}, \frac{5}{3}$ **b** $3\frac{1}{2}, \frac{7}{2}$ **c** $1\frac{1}{6}, \frac{12}{6}$ **d** $2\frac{1}{4}, \frac{8}{4}$ **e** $5\frac{1}{2}, \frac{11}{2}$

1. Bill used $1\frac{1}{2}$ packages of nails.
Donna used $1\frac{1}{3}$ packages of nails.
Who used more nails?

2. Amy picked $2\frac{1}{2}$ baskets of berries.
Dennis picked $2\frac{1}{4}$ baskets of berries.
Who picked more berries?

3. Roger collected $5\frac{1}{4}$ dozen eggs.
His brother collected $5\frac{1}{3}$ dozen eggs.
Who collected more eggs?

4. Lee did homework for $1\frac{2}{4}$ hours.
Bill did homework for $1\frac{1}{2}$ hours.
Who did homework for a longer time?

5. Use $>$ (greater than) or $<$ (less than) to make each true.

a $2\frac{1}{8} \textcircled{?} 2\frac{1}{3}$ **b** $1\frac{2}{5} \textcircled{?} 1\frac{2}{7}$ **c** $3\frac{5}{6} \textcircled{?} 3\frac{5}{9}$ **d** $1\frac{7}{8} \textcircled{?} 1\frac{7}{9}$ **e** $\frac{5}{2} \textcircled{?} 3$ **f** $4 \textcircled{?} \frac{4}{4}$

6. Name each point on the number line marked with a letter.

PROGRESS CHECK

Complete each math sentence, using $>$ or $<$.
Remember $>$ means "is greater than" and $<$ means "is less than."

1. $\frac{2}{5} \text{ ? } \frac{4}{5}$
2. $\frac{3}{8} \text{ ? } \frac{5}{8}$
3. $\frac{3}{9} \text{ ? } \frac{2}{9}$
4. $\frac{1}{4} \text{ ? } \frac{3}{4}$
5. $\frac{3}{5} \text{ ? } \frac{4}{5}$
6. $\frac{1}{4} \text{ ? } \frac{1}{7}$
7. $\frac{2}{3} \text{ ? } \frac{2}{4}$
8. $\frac{7}{8} \text{ ? } \frac{3}{3}$
9. $1\frac{1}{3} \text{ ? } 1\frac{1}{2}$
10. $2\frac{1}{5} \text{ ? } 2\frac{1}{8}$
11. $5\frac{1}{4} \text{ ? } 5\frac{1}{6}$
12. $3\frac{1}{2} \text{ ? } 3\frac{1}{9}$

Take time to think about these. They are harder.

13. $\frac{1}{3} \text{ ? } \frac{3}{4}$
14. $\frac{7}{8} \text{ ? } \frac{1}{5}$
15. $\frac{1}{6} \text{ ? } \frac{5}{8}$
16. $\frac{1}{10} \text{ ? } \frac{3}{4}$

SUPER SLEUTH

Below is a picture of 4 tables in a restaurant.
Each table can seat 1 person on each side.
How could you rearrange the 4 tables to seat a party of 8?

Rearrange the tables to seat a party of 10.
This is a hard one. Try rearranging the tables to seat 12 people.

Here's $\frac{1}{3}$

shaded parts → $\frac{1}{3}$
parts in all →

What happens if you double the number of parts in all?

The parts in all doubled → $\frac{1 \times ?}{3 \times 2}$ ← What about the shaded parts?

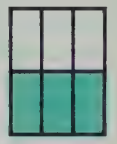
$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ Hope you agree that $\frac{2}{6}$ is another name for $\frac{1}{3}$.

When you multiply the numerator and denominator by the same number, you get another name for the fraction.

Here's $\frac{1}{2}$

Now double the number of parts.
$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ Right?

Start with $\frac{1}{2}$ again.
Make three times as many parts.
$\frac{1 \times 3}{2 \times 3} = \frac{?}{6}$



Start with $\frac{1}{2}$ again.
Make four times as many parts.
$\frac{1 \times 4}{2 \times 4} = \frac{?}{8}$

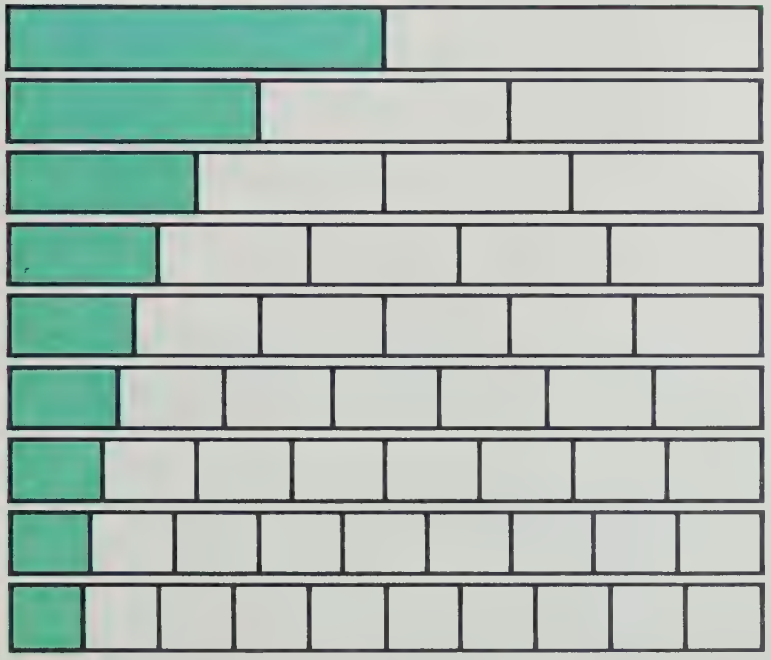
$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all names for the same number. Try to name one more.

Fractions that name the same number are called *equivalent fractions*.

1. Name three equivalent fractions for $\frac{1}{4}$.
2. Now try to name three equivalent fractions for $\frac{2}{3}$.

EQUIVALENT FRACTIONS CAN BE SHOWN BY NUMBER STRIPS, TOO.

You have seen these strips before.



1 Use the number strips to find equivalent fractions.

a $\frac{1}{2}$ **b** $\frac{2}{3}$

c $\frac{3}{4}$ **d** $\frac{4}{5}$

2 The number strips do not show an equivalent for $\frac{1}{7}$. Aren't there any? If you think there are, name one.

3 The number strips do not show an equivalent for $\frac{1}{9}$. There are some. Name at least one.

4 Does $\frac{1}{10}$ name the same number as $\frac{2}{20}$? as $\frac{3}{30}$? as $\frac{10}{100}$?

Your job now is to find an equivalent fraction with the smallest numerator and denominator.

$$\frac{5}{12} + \frac{5}{12} = \frac{5+5}{12} = \mathbf{\frac{10}{12}}$$ ← There has to be a simpler name!

You have multiplied by the same number to get a larger numerator and denominator. You can do the reverse, too. Divide by the same number to get a smaller numerator and denominator.

There is only one rule to follow. The number you use to divide with must be a common factor of the numerator AND the denominator.

$\frac{10}{12}$ → Is 2 a common factor?

$$\frac{10 \div 2}{12 \div 2} = \mathbf{\frac{5}{6}}$$ → Is there a common factor of 5 and 6?
Don't count the common factor 1.
1 is a common factor of every number.
Looks like $\frac{5}{6}$ is the simplest name for $\frac{10}{12}$.

Here's a problem like the ones you will be doing.

$$\frac{1}{10} + \frac{7}{10} = \frac{1+7}{10} = \mathbf{\frac{8}{10}}$$ → Is there a common factor of 8 and 10?

$$\frac{8 \div 2}{10 \div 2} = \mathbf{\frac{4}{5}}$$ → Is there another common factor?
You have the simplest name.

1. Pick out the fractions that don't have their simplest name.

a $\frac{2}{6}$ **b** $\frac{5}{10}$ **c** $\frac{6}{9}$ **d** $\frac{4}{5}$ **e** $\frac{4}{10}$ **f** $\frac{6}{8}$ **g** $\frac{7}{9}$ **h** $\frac{4}{8}$ **i** $\frac{6}{12}$ **j** $\frac{12}{16}$

2. Go back. Find the simplest name for any five of the fractions you picked.

Once in a while you will find a fraction like $\frac{8}{12}$.
No new rules—BUT you have to pay attention.

$\frac{8}{12}$ ← Is 2 a common factor?

Sure it is. BUT 4 is also a common factor.
If you use the greatest common factor,
you will save yourself some work.

This is what happens when you use 2.

$$\frac{8 \div 2}{12 \div 2} = \frac{\mathbf{4}}{\mathbf{6}}$$ Ooops! You are not done.

$$\frac{4 \div 2}{6 \div 2} = \frac{\mathbf{2}}{\mathbf{3}}$$ ← No more common factors. You're done.

This is what happens when you use the greatest common factor.

$$\frac{8 \div 4}{12 \div 4} = \frac{\mathbf{2}}{\mathbf{3}}$$ You're done in only one step.

Don't worry if you take more than one step to find the simplest name.
You get it sooner or later. Be patient.

PROGRESS CHECK

Complete.

(1.) $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \frac{?}{?}$ (2.) $\frac{3}{4} = \frac{3 \times ?}{4 \times 2} = \frac{?}{8}$ (3.) $\frac{2}{3} = \frac{2 \times ?}{3 \times 4} = \frac{?}{?}$

Look for common factors. Then complete.

(4.) $\frac{4}{12} = \frac{4 \div ?}{12 \div ?} = \frac{1}{3}$ (5.) $\frac{4}{10} = \frac{4 \div ?}{10 \div 2} = \frac{?}{5}$ (6.) $\frac{6}{16} = \frac{6 \div ?}{16 \div ?} = \frac{?}{8}$

Complete.

(7.) $\frac{4}{5} = \frac{?}{15}$ (8.) $\frac{7}{10} = \frac{?}{20}$ (9.) $\frac{1}{6} = \frac{?}{24}$ (10.) $\frac{3}{8} = \frac{?}{16}$

Which pairs of fractions are equivalent? How do you know for sure?

(11.) $\frac{1}{4}, \frac{5}{20}$ (12.) $\frac{3}{10}, \frac{3}{30}$ (13.) $\frac{5}{15}, \frac{2}{5}$ (14.) $\frac{2}{3}, \frac{4}{9}$

A man had to work for 8 hours. He looked at his watch and said to himself, only $\frac{1}{8}$ of my work is over. Time passed. He looked at his watch again. He thought, Another $\frac{3}{8}$ of my work is done. How much had he worked in all? Let the number line help.

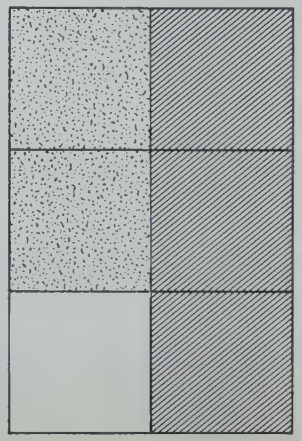
hours being counted → $\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$ = fraction of the workday.
hours in all →

Were the numerators added together?
Were the denominators added together?
Maybe you will want to show your work this way: $\frac{1}{8} + \frac{3}{8} = \frac{1+}{8}$

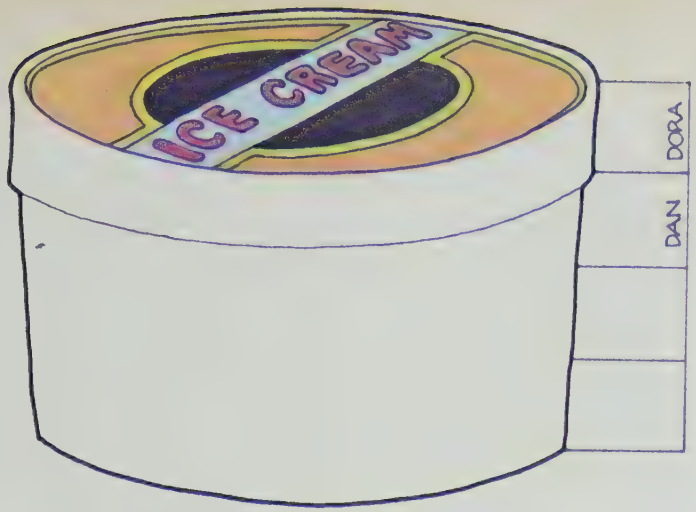
ADD

	a	b	c	d
1.	$\frac{1}{8} + \frac{1}{8}$	$\frac{1}{3} + \frac{1}{3}$	$\frac{1}{4} + \frac{1}{4}$	$\frac{1}{6} + \frac{1}{6}$
2.	$\frac{1}{8} + \frac{2}{8}$	$\frac{1}{3} + \frac{2}{3}$	$\frac{1}{4} + \frac{2}{4}$	$\frac{1}{6} + \frac{2}{6}$
3.	$\frac{1}{10} + \frac{1}{10}$	$\frac{1}{10} + \frac{3}{10}$	$\frac{5}{10} + \frac{1}{10}$	$\frac{5}{10} + \frac{4}{10}$
4.	$\frac{2}{5} + \frac{2}{5}$	$\frac{3}{8} + \frac{3}{8}$	$\frac{4}{9} + \frac{4}{9}$	$\frac{0}{2} + \frac{1}{2}$

5. Go back. Ring each sum that can be renamed.



6. Regions also can show addition of fractional parts.
How many parts are dotted?
How many parts are lined?
How many parts in the whole region?
$\frac{2}{6} + \frac{3}{6} = \frac{2+3}{6} = \frac{?}{?}$



Dora opened a carton of ice cream. She ate about $\frac{1}{4}$. Dan ate about $\frac{1}{4}$. How much ice cream did Dora and Dan eat in all? You don't have a carton of ice cream for an experiment, so here's a picture. Now add: $\frac{1}{4}+\frac{1}{4}=\underline{?}$

When you add fractions such as $\frac{1}{4}$ and $\frac{1}{4}$, do you add the numerators?
Do you add the denominators?

Here's one way to show your work: $\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}$
What is the simplest name for $\frac{2}{4}$?

ADD

	a	b	c	d	e	f	g	h
1.	$\frac{1}{5}+\frac{1}{5}$	$\frac{2}{5}+\frac{2}{5}$	$\frac{1}{7}+\frac{1}{7}$	$\frac{1}{10}+\frac{1}{10}$	$\frac{3}{8}+\frac{2}{8}$	$\frac{1}{5}+\frac{2}{5}$	$\frac{1}{6}+\frac{2}{6}$	$\frac{1}{7}+\frac{3}{7}$
2.	$\frac{1}{12}+\frac{5}{12}$	$\frac{1}{8}+\frac{3}{8}$	$\frac{1}{10}+\frac{7}{10}$	$\frac{3}{4}+\frac{1}{4}$	$\frac{2}{7}+\frac{3}{7}$	$\frac{3}{9}+\frac{4}{9}$	$\frac{0}{2}+\frac{1}{2}$	$\frac{3}{10}+\frac{6}{10}$

3. Go back. Ring each sum that can be renamed.
Then rename as many as you can.

***4.** Try to rename each sum as a whole number or a mixed number.

a	b	c	d	e	f	g	h
$\frac{1}{3}+\frac{2}{3}$	$\frac{2}{5}+\frac{5}{5}$	$\frac{3}{4}+\frac{1}{4}$	$\frac{4}{9}+\frac{5}{9}$	$\frac{4}{8}+\frac{6}{8}$	$\frac{9}{10}+\frac{7}{10}$	$\frac{2}{6}+\frac{7}{6}$	$\frac{3}{8}+\frac{5}{8}$

Here is part of an addition table for fractions.

What's the whole-number name for $\frac{0}{4}$?
What's the whole-number name for $\frac{4}{4}$? for $\frac{8}{4}$?

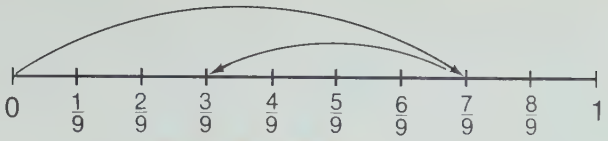
+	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	$\frac{8}{4}$
$\frac{0}{4}$	?	?	?	?	?	?	?	?	?
$\frac{1}{4}$	?	?	?	?	?	?	?	?	
$\frac{2}{4}$	?	?	?	?	?	?	?		
$\frac{3}{4}$	?	?	?	?	?	?			
$\frac{4}{4}$	?	?	?	?	?				
$\frac{5}{4}$	?	?	?	?					
$\frac{6}{4}$	?	?	?	?					
$\frac{7}{4}$	?	?							
$\frac{8}{4}$	?								

Make a chart like this. Complete it.

Here are some questions to help you find patterns on the chart.

1. Find the fraction combinations that equal $\frac{4}{4}$, or 1. List them.
2. Find the fraction combinations that equal $\frac{8}{4}$, or 2. List them.
3. What's another name for 3 on your chart? Is the pattern of fraction addition combinations for 3 the same as the pattern for 2?
4. Does the order in which you add two fractions change the answer?
5. Are there any other whole numbers named by fractions on the chart?

The wallpaper had 9 repeating patterns on each roll.
$\frac{7}{9}$ of one roll had not been used.
$\frac{4}{9}$ of the roll was then put on a wall.
How much of the roll remained?

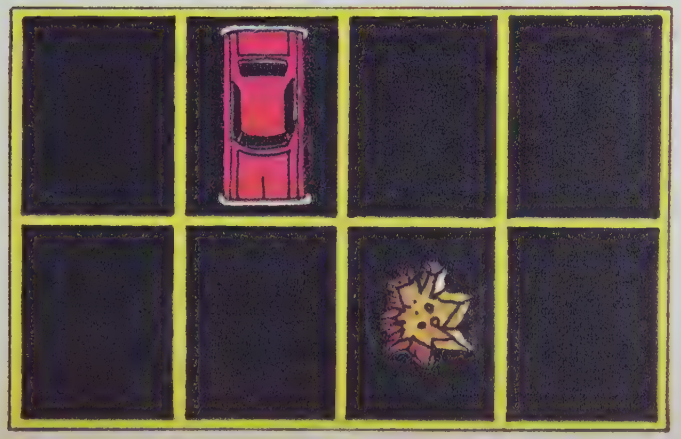


patterns being counted → $\frac{7}{9} - \frac{4}{9} = \frac{3}{9}$ ← fraction of one roll
patterns in all →

You might want to show your work this way: $\frac{7}{9} - \frac{4}{9} = \frac{7-4}{9} = \frac{3}{9}$

SUBTRACT

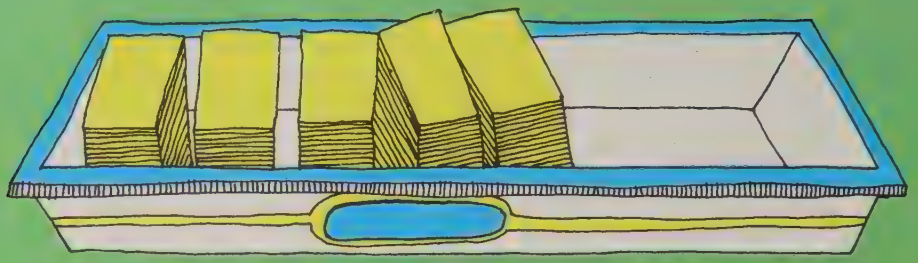
	a	b	c	d
1.	$\frac{4}{5} - \frac{1}{5}$	$\frac{5}{6} - \frac{2}{6}$	$\frac{3}{8} - \frac{1}{8}$	$\frac{6}{9} - \frac{5}{9}$
2.	$\frac{7}{8} - \frac{2}{8}$	$\frac{9}{9} - \frac{2}{9}$	$\frac{7}{10} - \frac{4}{10}$	$\frac{6}{7} - \frac{4}{7}$
3.	$\frac{10}{10} - \frac{5}{10}$	$\frac{4}{5} - \frac{3}{5}$	$\frac{2}{4} - \frac{1}{4}$	$\frac{1}{4} - \frac{1}{4}$
4.	$\frac{8}{9} - \frac{5}{9}$	$\frac{9}{10} - \frac{3}{10}$	$\frac{5}{7} - \frac{3}{7}$	$\frac{1}{10} - \frac{0}{10}$



5. Go back. Ring each answer that can be given a simpler name.

6. Mr. Jones had a parking lot with spaces for 8 cars. He used one space himself, so $\frac{7}{8}$ of the spaces were left. There was a huge hole in $\frac{1}{8}$ of his lot. No cars could park there. How much of his lot was left for parking?

There was $\frac{5}{8}$ of a stick of butter. $\frac{2}{8}$ of a stick was used in cooking. What fraction of a stick of butter was left? $\frac{5}{8} - \frac{2}{8} = \underline{?}$



SUBTRACT

	a	b	c	d	e	f
1.	$\frac{7}{8} - \frac{1}{8}$	$\frac{5}{6} - \frac{1}{6}$	$\frac{4}{5} - \frac{2}{5}$	$\frac{5}{8} - \frac{1}{8}$	$\frac{2}{7} - \frac{1}{7}$	$\frac{3}{10} - \frac{1}{10}$
2.	$\frac{5}{9} - \frac{2}{9}$	$\frac{6}{7} - \frac{5}{7}$	$\frac{7}{10} - \frac{3}{10}$	$\frac{3}{4} - \frac{1}{4}$	$\frac{5}{6} - \frac{1}{6}$	$\frac{3}{5} - \frac{2}{5}$
3.	$\frac{7}{9} - \frac{5}{9}$	$\frac{9}{10} - \frac{3}{10}$	$\frac{4}{5} - \frac{3}{5}$	$\frac{9}{10} - \frac{1}{10}$	$\frac{5}{8} - \frac{3}{8}$	$\frac{3}{7} - \frac{2}{7}$
4.	$\frac{8}{9} - \frac{2}{9}$	$\frac{5}{7} - \frac{2}{7}$	$\frac{1}{8} - \frac{1}{8}$	$\frac{7}{10} - \frac{0}{10}$	$\frac{2}{5} - \frac{1}{5}$	$\frac{3}{8} - \frac{1}{8}$

5. Go back. Ring each answer that can be given a simpler name. Then rename as many as you can.

PROGRESS CHECK

Complete these sentences.

1. $\frac{1}{4} = \frac{?}{8}$
2. $\frac{4}{5} = \frac{?}{10}$
3. $\frac{1}{3} = \frac{?}{9}$
4. $\frac{3}{4} = \frac{?}{12}$
5. $\frac{2}{8} = \frac{?}{4}$
6. $\frac{4}{6} = \frac{?}{3}$
7. $\frac{3}{12} = \frac{?}{4}$
8. $\frac{2}{10} = \frac{?}{?}$

Add.

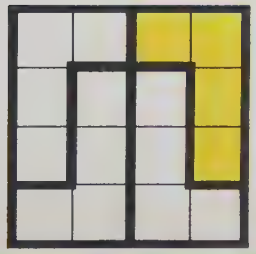
9. $\frac{3}{7} + \frac{3}{7}$
10. $\frac{1}{6} + \frac{5}{6}$
11. $\frac{1}{8} + \frac{3}{8}$
12. $\frac{5}{9} + \frac{4}{9}$

Subtract.

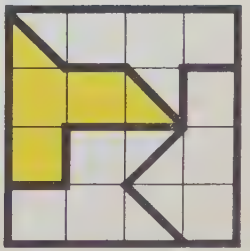
13. $\frac{5}{6} - \frac{1}{6}$
14. $\frac{7}{8} - \frac{5}{8}$
15. $\frac{6}{7} - \frac{3}{7}$
16. $\frac{2}{3} - \frac{2}{3}$

SUPER SLEUTH

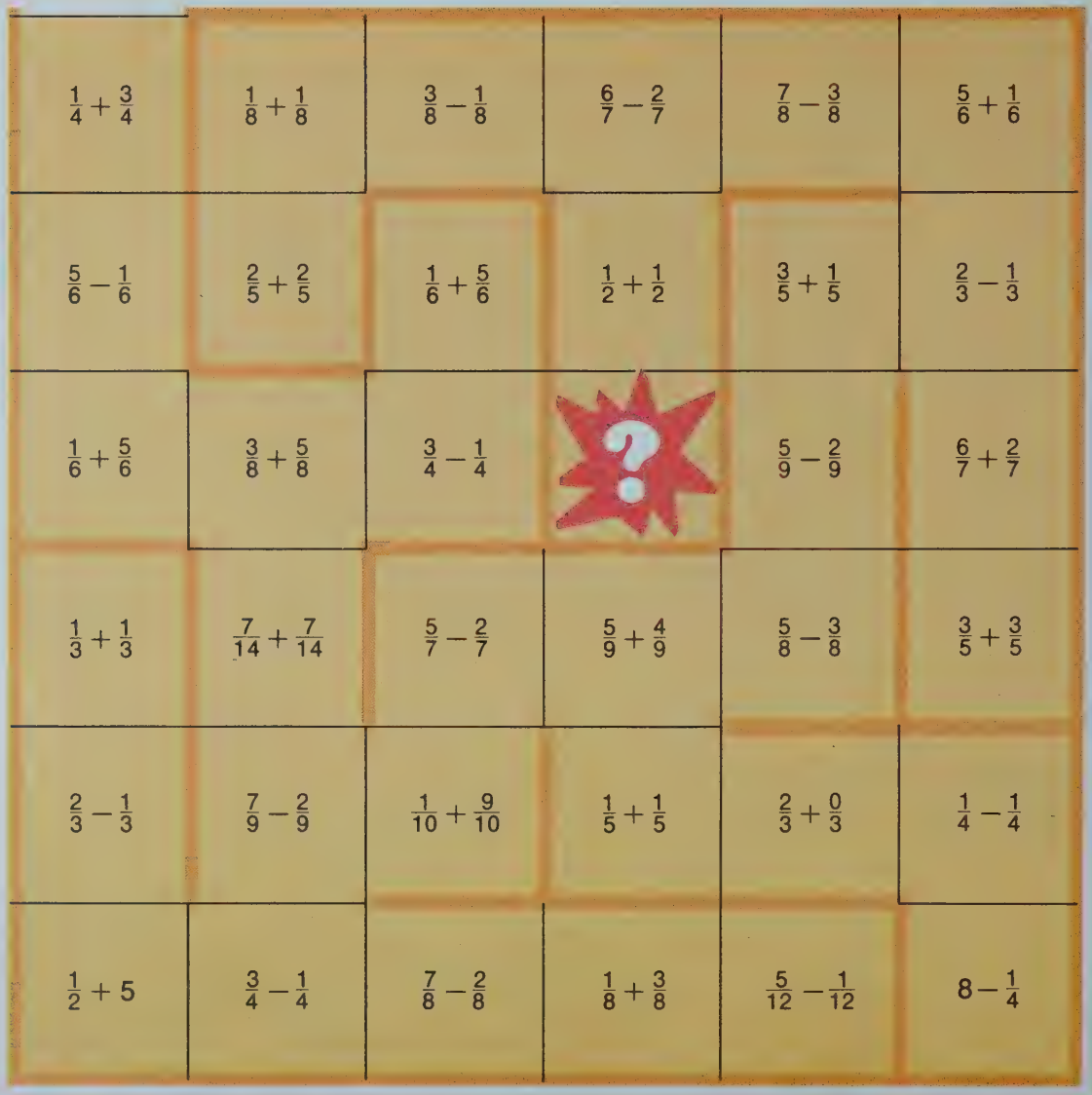
Which regions show $\frac{1}{4}$ shaded?

a 

b

c 

Copy and compute only as many problems as you have to in order to get to the prize. Look out! Look for both addition and subtraction problems.



If you followed the shortest path, you found the answers to each addition problem were alike in some way. How?

This set of problems is put into a 6-by-6 square grid. Then the heavy lines are drawn to form a maze. Draw just the grid. Then make heavy lines so that a different maze is formed.

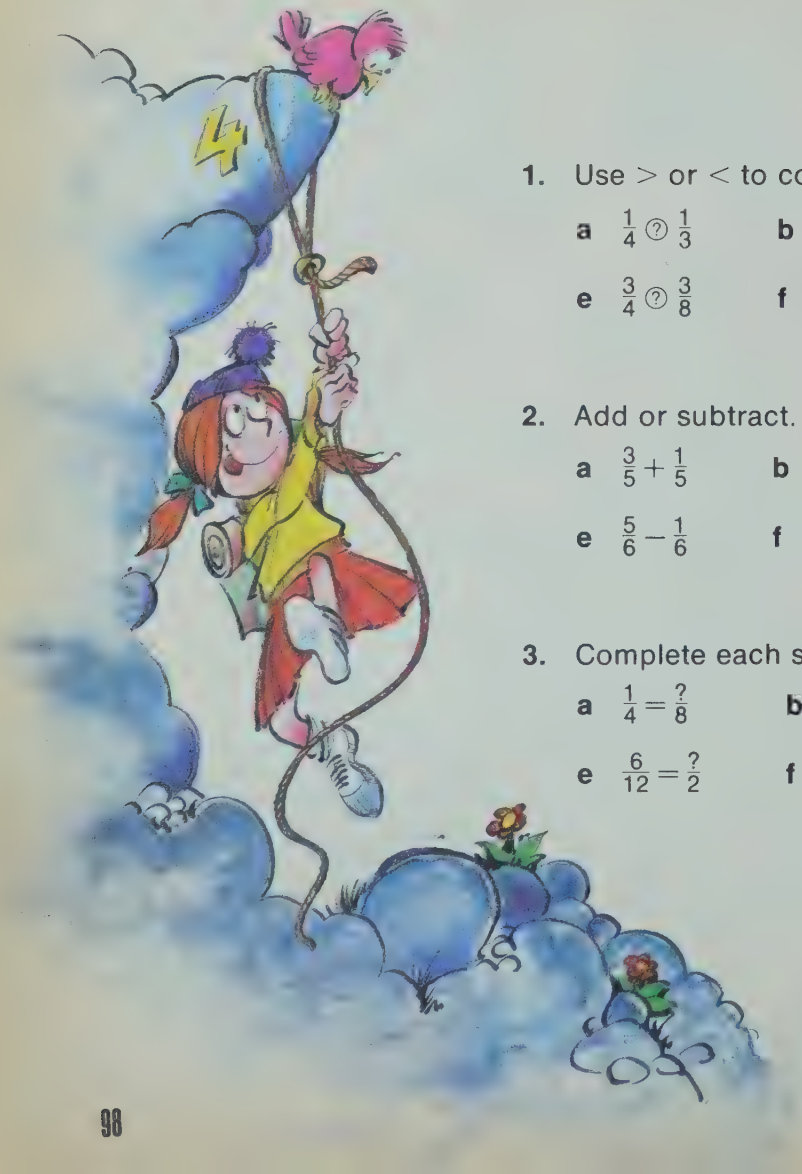
After lunch, the chef asked an assistant to report on the supply of pies. The assistant brought him a list.

1 peach	$\frac{1}{3}$ lemon cream
$\frac{1}{2}$ mince	$\frac{2}{3}$ banana cream
$\frac{5}{6}$ apple	$1\frac{1}{3}$ raspberry

The restaurant's usual serving is $\frac{1}{6}$ of a pie. If the chef did no more baking that day, how many people could have a serving of:

a peach pie?
b mince pie?
c apple pie?
d lemon cream pie?
e banana cream pie?
f raspberry pie?

CHECKOUT



1. Use $>$ or $<$ to compare the following pairs of fractions.

a $\frac{1}{4} \textcircled{?} \frac{1}{3}$ **b** $\frac{2}{5} \textcircled{?} \frac{3}{5}$ **c** $\frac{3}{3} \textcircled{?} \frac{3}{4}$ **d** $\frac{1}{6} \textcircled{?} \frac{1}{8}$

e $\frac{3}{4} \textcircled{?} \frac{3}{8}$ **f** $\frac{2}{3} \textcircled{?} \frac{3}{3}$ **g** $\frac{1}{5} \textcircled{?} \frac{1}{8}$ **h** $\frac{7}{12} \textcircled{?} \frac{7}{2}$

2. Add or subtract. Make sure your answers have their simplest name.

a $\frac{3}{5} + \frac{1}{5}$ **b** $\frac{5}{9} + \frac{2}{9}$ **c** $\frac{4}{9} - \frac{3}{9}$ **d** $\frac{3}{8} + \frac{2}{8}$

e $\frac{5}{6} - \frac{1}{6}$ **f** $\frac{2}{5} + \frac{3}{5}$ **g** $\frac{5}{8} - \frac{1}{8}$ **h** $\frac{3}{12} + \frac{5}{12}$

3. Complete each sentence.

a $\frac{1}{4} = \frac{?}{8}$ **b** $\frac{2}{3} = \frac{?}{6}$ **c** $\frac{3}{5} = \frac{?}{10}$ **d** $\frac{3}{4} = \frac{?}{12}$

e $\frac{6}{12} = \frac{?}{2}$ **f** $\frac{12}{16} = \frac{?}{4}$ **g** $\frac{9}{12} = \frac{?}{4}$ **h** $\frac{8}{10} = \frac{?}{5}$

5

DIVISION

Your turn to be teacher. You have a bunch of papers to correct. There were only four problems assigned.

$7\overline{)918}$ $6\overline{)1844}$

$90\overline{)2005}$ $25\overline{)3030}$

But it's not your day. You can't find the answer key. You look at the first paper. You spot that the first one is wrong. Better get to work.

Find the errors. Show your check to make sure your answer is correct.

Your

GOAL

in this chapter
is to do problems
like this right,
so that you won't
have a paper
like this one.

MISSION POSSIBLE: DIVISION

The word *divide* and the idea of division are part of our world. What does each of the following mean?

1. "United we stand, divided we fall."
2. Divide a chocolate bar into two parts.
3. Divide the class into three groups.
4. Divide the work among all the children.
5. "Divide and conquer."
6. Divide the profits.
7. Mountain School, Junior Division
8. "A house divided against itself cannot stand."
9. Division Street

Here are some situations that do not use the word *divide*. Is the idea of division used?

10. My grandfather split kindling.
11. Let's choose sides.
12. They formed eight teams by counting off.
13. He cut the cake.
14. He shared it with his friends.
15. The dividend cheque came in the mail.
16. Each of us can have three pieces of chicken.

Which ones could actually involve the arithmetic kind of division? What is the arithmetic kind of division? How much do you know about it?

Pretend you and two others worked all summer. You formed a lawn-mowing company called the Sod Squad. You advertised in the local newspaper and got a lot of jobs. The profits were divided at the end of September. Your company had earned a total of $567. This was divided equally among the three of you. How much did each receive?

How many 3s in 567? More than 10? $10 \times 3 = 30$
More than 100? $100 \times 3 = 300$
More than 200? $200 \times 3 = 600$
There are between 100 and 200.
This is your estimate.

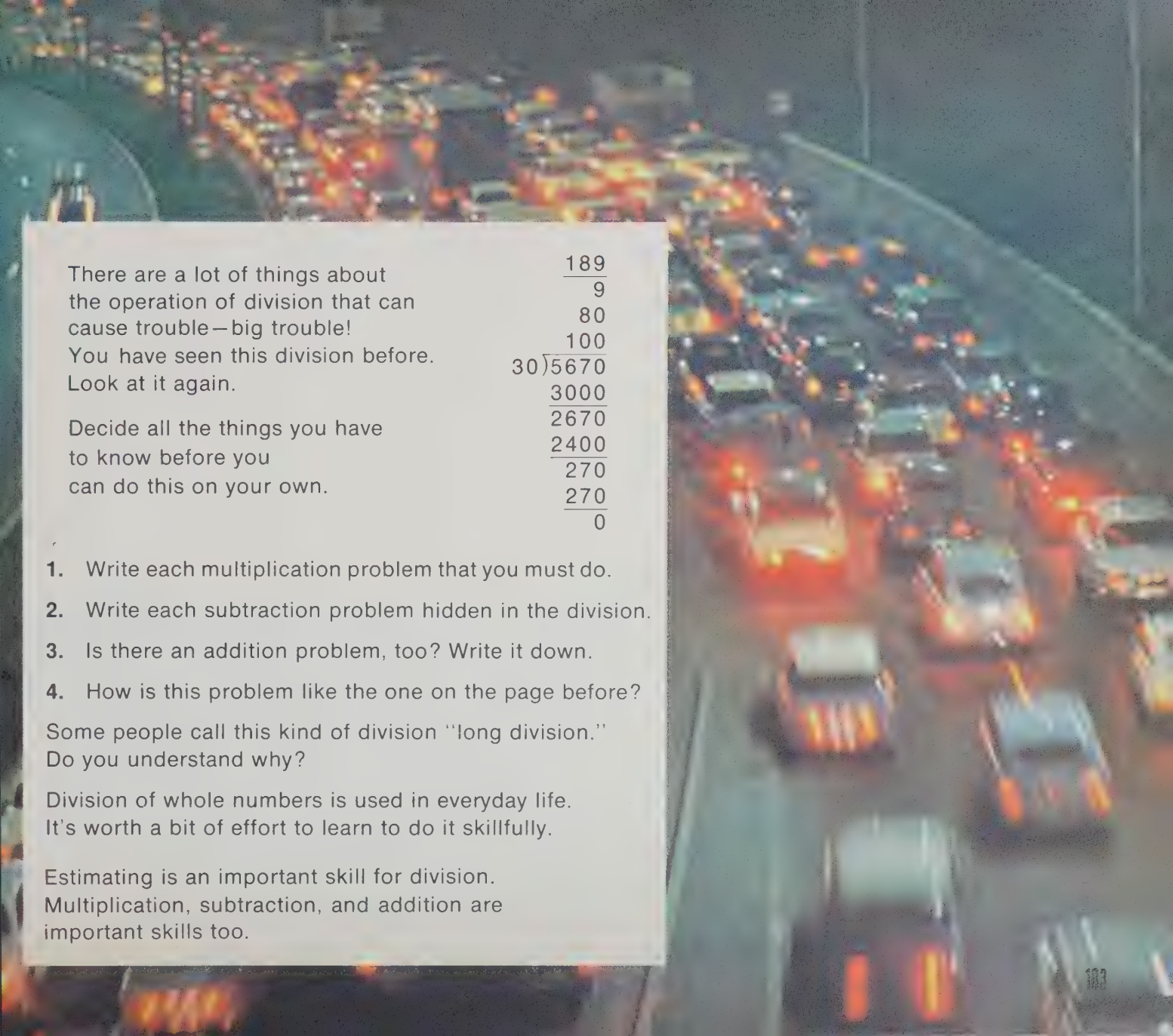
Now divide.

```
  189
    9
   80
  100
3)567    How many 3s in 567?
 300     100 × 3
 267     How many 3s in 267?
 240     80 × 3  Your estimate might have been different.
  27     How many 3s in 27?
  27     9 × 3
   0
```

Is the answer close to the estimate? Is the answer reasonable?

Do you remember examples like this from the last chapter on division? How much do you remember? Find out by doing these problems.

	a	b	c	d	e	f	g
1.	$3\overline{)6}$	$3\overline{)60}$	$3\overline{)600}$	$3\overline{)6000}$	$30\overline{)60}$	$30\overline{)600}$	$30\overline{)6000}$
2.	$6\overline{)300}$	$3\overline{)690}$	$6\overline{)126}$	$7\overline{)714}$	$8\overline{)640}$	$9\overline{)990}$	$4\overline{)368}$



There are a lot of things about the operation of division that can cause trouble—big trouble!
You have seen this division before. Look at it again.

Decide all the things you have to know before you can do this on your own.

$$
\begin{array}{r}
189 \\
\hline
30\overline{)5670} \\
\begin{array}{r}
9 \\
80 \\
100 \\
3000 \\
2670 \\
2400 \\
270 \\
270 \\
0
\end{array}
\end{array}
$$

1. Write each multiplication problem that you must do.
2. Write each subtraction problem hidden in the division.
3. Is there an addition problem, too? Write it down.
4. How is this problem like the one on the page before?

Some people call this kind of division "long division." Do you understand why?

Division of whole numbers is used in everyday life. It's worth a bit of effort to learn to do it skillfully.

Estimating is an important skill for division. Multiplication, subtraction, and addition are important skills too.

Check yourself on the rounding skill.
Round each to the nearest ten.

	a	b	c	d
1.	32	34	35	36
2.	93	94	95	97
3.	11	25	53	79

Check yourself on multiplication.
Don't use paper and pencil.
Do it in your head.

	a	b	c	d
4.	10×1	10×10	100×10	10×2
5.	10×5	100×5	200×5	10×50
6.	20×6	40×6	300×6	50×6
7.	40×8	400×8	70×4	70×40
8.	20×20	30×30	100×20	200×30

PROGRESS CHECK

Check yourself on this skill. *Estimate* your answers.

	a	b	c	d
Group 1	$10\overline{)10}$	$10\overline{)100}$	$10\overline{)1000}$	$10\overline{)10000}$
Group 2	$20\overline{)100}$	$30\overline{)100}$	$40\overline{)100}$	$50\overline{)100}$
Group 3	$60\overline{)100}$	$60\overline{)120}$	$60\overline{)140}$	$60\overline{)160}$

Put your skills to work.
There were 7750 machine parts made. They were to be put into boxes. There were to be 50 in each box. How many boxes would be needed to pack the parts?

$50\overline{)7750}$ How many 50s in 7750?

$100 \times 50 = 5000$ Not enough.
$200 \times 50 = 10\,000$ Too many.
More than 100 but less than 200 boxes will be needed.

```
  155
    5
   50
  100
50)7750    How many 50s in 7750?
 5000    100 × 50
 2750    How many 50s in 2750?
 2500    50 × 50
  250    How many 50s in 250?
  250    5 × 50
    0
```

1. The company packed machine parts according to order. Here is the daily production run. The packing orders are also listed.

Machine parts made each day	**Monday** 8250	**Tuesday** 8500	**Wednesday** 7800	**Thursday** 8750	**Friday** 8550
Number to be packed in each box	50 per carton	20 per carton	40 per carton	70 per carton	10 per carton
Number of boxes	**a**	**b**	**c**	**d**	**e**

***2.** About how many thousands of parts did the company make that week?

1. What do you say to yourself when you see this problem? $78\overline{)8307}$ ("Ugh" is not the correct answer.) You really should round 78 and then ask yourself, About how many 80s in 8307? Complete the following.

a	$67\overline{)825}$	About how many 70s in 825? (Where did the 70 come from?) How many?
b	$58\overline{)2407}$	About how many ■s in 2407? How many?
c	$42\overline{)376}$	About how many ■s in 376? How many?
d	$74\overline{)6890}$	About how many ■s in 6890? How many?

2. Complete this table.

Problem	Rounded division		Estimated Answer
a $8\overline{)121}$	$10\overline{)121}$	$10 \times 10 = 100$ **(Close!)** $20 \times 10 = 200$ **(Too many!)**	10
b $12\overline{)333}$	$10\overline{)333}$	$30 \times 10 = 300$ $40 \times 10 = 400$	30
c $52\overline{)510}$	$50\overline{)510}$	?	?
d $52\overline{)5100}$	?	?	?
e $29\overline{)152}$	?	?	?
f $29\overline{)1520}$	?	?	?
g $29\overline{)15200}$	?	?	?

Estimating is important in doing long-division problems. In fact, sometimes an estimate is all you need. Pretend you are a player on a winning team in the major leagues. You are about to play in the World Series. If you *win* the Series, your team will win about $715 000. This money is to be equally shared by about 35 players, coaches, trainers, and the manager.

Estimate each man's share. **THINK** There are about 40 people that will divide about $700 000. About how many 40s in 700 000?
$40 \times \$10\,000 = \$400\,000$ Too small!
$40 \times \$20\,000 = \$800\,000$ Very close!

$20 000 is a good estimate.

If you *lose* the Series, your team shares only about $450 000. Estimate each man's share. (You could round the $450 000, but let's try leaving it as it is.)

$40 \times \$10\,000 = \$400\,000$ Too small!
$40 \times \$20\,000 = \$800\,000$ Too large!

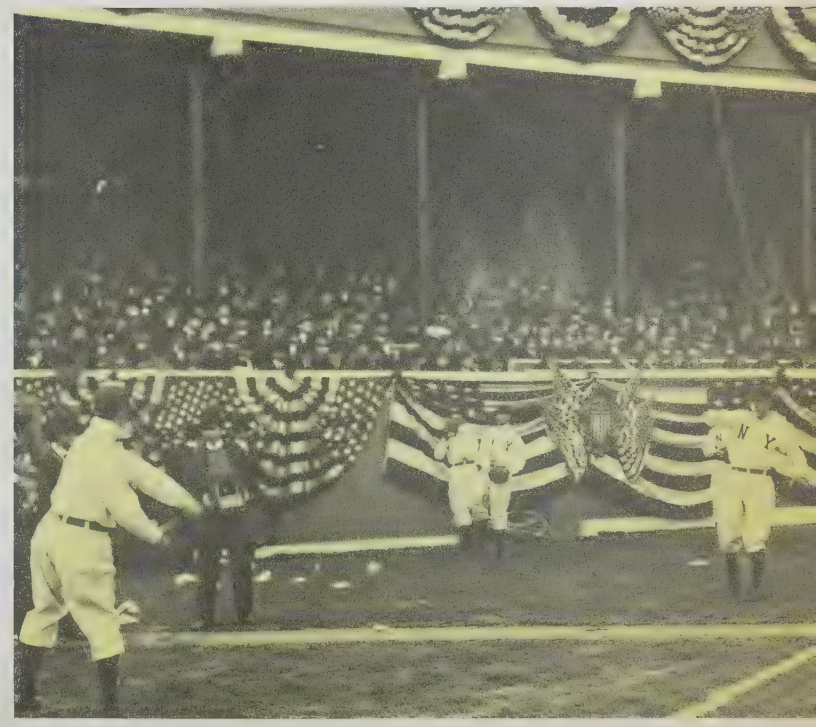
Is your share closer to $10 000 or to $20 000? What is a good estimate for your share?

1. Would it make sense to compute the exact amount?

2. Can you compute the exact amount? Think about these questions before you answer.

   a Will 34, 35, 36, or more men share in the World Series bonus?

   b Are the winner's and loser's bonus dollars known exactly before the Series?



There is no one way to estimate. The purpose of estimating is to get an answer that is reasonable. Get an estimated answer for these problems.

1. Izzy was dizzy from trying to figure out how many hamburgers to order for the scout picnic. He finally ordered 275 hamburgers for 39 people. Estimate the number of hamburgers he ordered for each person. Was Izzy dizzy when he ordered?

2. Ann promised to drive you to camp. Camp is 322 km from home. Ann says she can travel about 75 km/h . Estimate the hours needed for the trip.

3. Sue and Lou have 585 tickets to sell for the tennis tournament. They want to sell them all in 13 days. Estimate how many tickets they need to sell each day.

4. Harry and 30 of his classmates spent Saturday selling tickets for performances of their school play. Each pupil sold about the same number of tickets. Their ticket sales totalled 1001. About how many tickets did each pupil sell?

5. The money Harry and the others collected totalled $1418. About how much did each of the 31 pupils collect for ticket sales?

6. Mr. Nurvis, the teacher, is planning a parade. Every class is to march. 16 children can be in one row. There are 736 children. About how many rows of children can there be?

Estimates can help you find a reasonable answer.
Now it's time to find some exact answers.

Pollution is a health problem almost everywhere. One way to fight pollution is to recycle materials after they have been used. The people at Crystal Springs save and sell glass. They get paid $21 for each tonne (t) of glass. (A tonne is 1000 kg .) Last week they got a cheque for $525. How many tonnes of glass did they sell?

Think How many 21s in 525?

Round 21 to 20.
How many 20s in 525?

$10 \times 20 = 200$ Not enough.
$20 \times 20 = 400$ Not enough.
$30 \times 20 = 600$ Too many.

They sold between 20 and 30 t .

Now you divide.

$$\begin{array}{r} 25 \\ 21\overline{)525} \\ 420 \quad 20 \times 21 \\ \hline 105 \\ 105 \quad 5 \times 21 \\ \hline 0 \end{array}$$

The parts of a division problem have special names.

You already know what a remainder is.
In the problem above, the divisor is → $21\overline{)525}$ ← 25
This is called the dividend.
The answer is the quotient.

Hardly anyone remembers the word *dividend*.
But do try to remember the word *divisor* and the word *quotient*.

Find these quotients.

1. $18\overline{)306}$ **2.** $27\overline{)435}$ **3.** $35\overline{)525}$ **4.** $42\overline{)588}$ **5.** $24\overline{)676}$ **6.** $13\overline{)546}$

The better you get at making an estimate, the fewer steps you will have to do in division. Start looking at your estimates more carefully. Make a good enough estimate to save yourself some work.

Complete this.

THE PROBLEM	DIVISOR SHOULD BE ROUNDED TO	FIRST ESTIMATES	ESTIMATED QUOTIENT
$37\overline{)814}$	40	$20 \times 40 = 800$ $30 \times 40 = 1200$	> 20 < 30
$17\overline{)901}$	20	$40 \times 20 = 800$ $50 \times 20 = 1000$	> 40 < 50
$72\overline{)1872}$	?	?	?
$27\overline{)783}$	?	?	?
$29\overline{)1348}$	?	?	?
$36\overline{)673}$	?	?	
$34\overline{)992}$	?	?	
$86\overline{)5834}$	?		

After you estimate the least possible quotient and the greatest possible quotient, you can think some more. Should the quotient be closer to the least number or to the greatest number?

Pick three problems only. Find the exact quotient.

The students at Greenwood School decided to collect and sell scrap metal for recycling. They began collecting without asking how much they would be paid for each tonne. It took a long time, but they finally had 14 t collected. They sold it all for $2254. How much money per tonne did they get?

Think

How many 14s in 2254? Round 14 to 10.
How many 10s in 2254?

$100 \times 10 = 1000$ Not enough.
$200 \times 10 = 2000$ That's better.
$300 \times 10 = 3000$ Too many.

They got between $100 and $200 per tonne.

They got $161 per tonne ($161/t).
The estimate says this is a reasonable answer.

Find out exactly

$$
\begin{array}{rl}
161 & \\
\hline
1 & \\
60 & \\
100 & \\
14\overline{)2254} & \\
1400 & 100 \times 14 \\
\hline
854 & \\
840 & 60 \times 14 \\
\hline
14 & \\
14 & 1 \times 14 \\
\hline
0 &
\end{array}
$$

Would you have estimated 50? It would have been O.K.

1. Copy and complete these division problems.

a
$$
\begin{array}{rl}
 & ? \\
45\overline{)7650} & \\
4500 & (? \times 45) \\
\hline
3150 & \\
3150 & (? \times 45) \\
\hline
0 &
\end{array}
$$

b
$$
\begin{array}{rl}
 & ? \\
23\overline{)667} & \\
460 & (? \times 23) \\
\hline
207 & \\
207 & (? \times 23) \\
\hline
0 &
\end{array}
$$

c
$$
\begin{array}{rl}
 & ? \\
12\overline{)4560} & \\
3600 & (? \times 12) \\
\hline
960 & \\
960 & (? \times 12) \\
\hline
0 &
\end{array}
$$

2. Answer these.

a How many 19s in 950?

b How many 18s in 5400?

Complete the table

1. $18\overline{)285}$	$20\overline{)300}^{10}$	$10 \times 18 = 180$	10, 20
2. $17\overline{)536}$	$20\overline{)500}^{20}$	?	?
3. $28\overline{)2967}$	?	?	?
4. $73\overline{)12351}$	?	?	
5. $33\overline{)4311}$	?		
6. $27\overline{)663}$			

Something to think about

7. **a** 6 tens + 3 tens = ? , 60 + 30 = ?
   b 6 tens − 3 tens = ? , 60 − 30 = ?
   c 6 tens × 3 tens = ? , 60 × 30 = ?
   d 6 tens ÷ 3 tens = ? , 60 ÷ 30 = ?
   e 9 hundreds + 9 tens = ? , 900 + 90 = ?
   f 9 hundreds − 9 tens = ? , 900 − 90 = ?
   g 9 hundreds × 9 tens = ? , 900 × 90 = ?
   h 9 hundreds ÷ 9 tens = ? , 900 ÷ 90 = ?
   i 8 hundreds + 4 hundreds = ? , 800 + 400 = ?
   j 8 hundreds − 4 hundreds = ? , 800 − 400 = ?
   k 8 hundreds × 4 hundreds = ? , 800 × 400 = ?
   l 8 hundreds ÷ 4 hundreds = ? , 800 ÷ 400 = ?

The man who bought the scrap metal would not pay for part of a tonne.
There are 1000 kg in 1 t .
You can understand why he wouldn't pay for 40 kg .

a Would he pay for 500 kg? Why? Is that fair?
b Would he pay for 990 kg? Why?

1.

How much would you pay for 1 pack of gum if the price were 2 for 15¢? Why?

2.

If you were dividing 3 pieces of cake between 2 people, what could you do with the one piece remaining?

3.

You were supposed to divide your 15 books among 4 visitors. What do you do with the ones remaining?

4.

Suppose you had 5 bags of peaches. Each bag had 4 peaches in it. Four people were to have equal shares. What do you do with the extra bag?

5.

What would you do if you had to share 35¢ with your brother?

As you use division in the real world, you'll find many ways to handle a remainder. When you practise division, write the remainder beside the answer.

PROGRESS CHECK

(1.) $5\overline{)34}$	(2.) $10\overline{)101}$	(3.) $90\overline{)8100}$	(4.) $10\overline{)1000}$
(5.) $70\overline{)950}$	(6.) $2\overline{)0}$	(7.) $33\overline{)7524}$	(8.) $27\overline{)1572}$

One problem below is correct. Finding someone else's mistake is a challenge. Division mistakes are the hardest of all to find. The error may be in multiplication, subtraction, or addition. Find the errors. (There may be more than one in a problem.) Then write the name of the operation done incorrectly.

For example:

```
   185 R17
     5
    80
   100
43)7872
  4300
  3672   7872 − 4300 = 3572, not 3672
  3440   Subtraction error
   232
   215
    17
```

1.

```
   205 R0
     1
     4
   200
35)875
   700
   175
   140
    35
    35
     0
```

2.

```
   101 R35
     2
     9
    90
57)5682
  4130
   552
   413
   149
   114
    35
```

3.

```
    83 R5
     3
    20
    60
34)2827
  2040
   787
   680
   107
   102
     5
```

4.

```
   123 R17
     3
    20
   100
56)7890
  5600
  1290
  1120
   170
   153
    17
```

5.

```
   199 R20
     9
    90
   100
24)4766
  2400
  2366
  2166
   206
   186
    20
```

6.

```
   112 R6
     2
    10
   100
78)8752
  7800
   952
   780
   162
   156
     6
```

7. Which error could have been spotted with a good estimate?

The students who collected scrap metal want to buy $5000 worth of equipment for the school. They need more money — $2760 more. Now how many more tonnes must they collect if the price promised them is $160/t?

Think How many 160s in 2760?

Round 160 to 200.
How many 200s in 2760?

$10 \times 200 = 2000$ Not enough.
$20 \times 200 = 4000$ Too many.

They will need more than 10 but less than 20 t . Find out exactly.

$$
\begin{array}{r}
17\text{ R}40 \\
160\overline{)2760} \\
\underline{1600} \quad 10 \times 160 \\
1160 \\
\underline{1120} \quad 7 \times 160 \\
40
\end{array}
$$

The question was, How many tonnes? What are you going to do with the remainder?

If they sell 17 t, will they get as much money as they need? What is the answer to this problem?

Here are some computation problems. You won't have to worry about what to do with the remainder on these.

1. $250\overline{)7500}$ **2.** $340\overline{)7161}$ **3.** $170\overline{)1192}$ **4.** $110\overline{)5507}$ **5.** $260\overline{)2349}$

The next ones are for very brave people.

6. $620\overline{)84940}$ **7.** $130\overline{)11220}$ **8.** $250\overline{)31250}$ **9.** $305\overline{)15250}$ **10.** $430\overline{)23456}$

When you use division for something important, you have to be right. An estimate tells you if your answer is reasonable. How do you tell if your answer is right? Try multiplication.

1. $4 \times 7 = 28$ True or false?
$7 \times 4 = 28$ True or false?
$28 \div 7 = 4$ True or false?
$28 \div 4 = 7$ True or false?

2. $6 \times 9 = 54$ True multiplication sentence
$9 \times 6 = 54$ True multiplication sentence
$54 \div 6 = 9$ True division sentence
Write another true division sentence using 6, 9, and 54.

3. $14 \times 11 = 154$
Write another true multiplication sentence using 11, 14, and 154.
$154 \div 11 = 14$
Write another true division sentence using 11, 14, and 154.

4. $7018 \div 22 = 319$. This is a true sentence.
Write another true division sentence using 22, 319, and 7018.

5. Divide.

$$\begin{array}{r} 12 \\ 13\overline{)156} \\ \underline{130} \\ 26 \\ \underline{26} \\ 0 \end{array}$$

Check Does $156 \div 13 = 12$?

$$\begin{array}{r} 12 \\ \times\ 13 \\ \hline 36 \\ \underline{120} \\ 156 \end{array}$$

13×12 does equal 156!
Does $156 \div 12 = 13$?

Answer this one QUICKLY! $12\overline{)156}$?

Continue with checking.

6.
$$\begin{array}{r} 7 \\ 9\overline{)65} \\ 63 \\ \hline 2 \end{array}$$

$65 \div 9 = 7$ R2 Check it!

$9 \times 7 = 63$ and $63 + 2 = 65$
or $(9 \times 7) + 2 = 65$ is another way of writing it.

7.
$$\begin{array}{r} ? \\ 8\overline{)49} \end{array}$$

$49 \div 8 = 6$ R1 Check it. Use multiplication.

DIVIDE AND CHECK

8. $301 \div 17$ or $17\overline{)301}$

9. $625 \div 15$ or $15\overline{)625}$

10. $532 \div 25$ or $25\overline{)532}$

PROGRESS CHECK

(1.) Divide and check. $1565 \div 95$ or $95\overline{)1565}$

(2.) Divide and check. $2345 \div 35$ or $35\overline{)2345}$

(3.) Divide and check. $3486 \div 42$ or $42\overline{)3486}$

*(4.) Divide and check. $1111 \div 100$ or $100\overline{)1111}$

The form that you have been using to write a division problem is a good one. You just about can't get lost while you are dividing, subtracting, multiplying, and adding. You can always use that form if you want to.

There is another form that saves some writing. If you understand it and like it, you can use it.

750 people must get from the train station to the stadium. One bus holds 50 people. How many busloads will there be?

There is no change in the thinking steps.

THINK How many 50s in 750?

The estimate is more important than ever.

$10 \times 50 = 500$

$20 \times 50 = 1000$

More than 10 buses but less than 20 buses will be needed.

You start to write the problem in the same way.

Write only the 1 in the tens place.

```
   15
50)750
   50   1 ten × 50 = 50 tens   Write 50.
  250   Now you are ready for the ones.
  250   5 × 50 = 250   Write the 5 in the quotient.
    0
```

It does look a bit like magic.

But don't be afraid of something new. There is no rush. Take time out to do the problem the old way. Did you get the same quotient?

Study another example.

$19\overline{)874}$ Estimate first. Round 19 to 20.
How many 20s in 874?
$40 \times 20 = 800$ That's close.
$50 \times 20 = 1000$ Too many.
There are more than 40 but
less than 50 20s in 874.

Now compute.

$$\begin{array}{r} 4 \\ 19\overline{)874} \\ 76 \\ \hline 11 \end{array}$$

4 tens × 19 = 76 tens O.K. so far?

You are ready for the ones in the next step.

$$\begin{array}{r} 46 \\ 19\overline{)874} \\ 76\downarrow \\ \hline 114 \\ 114 \\ \hline 0 \end{array}$$

$(6 \times 19 = 114)$ Write the 6 in the quotient.

When you use this method, finish the multiplication before you write the digit in the quotient. You will want to make sure you are right.

The only place where you can have trouble is sloppy writing.
For example:

$$\begin{array}{r} 4 \\ 19\overline{)874} \\ 76 \\ \hline \end{array}$$

WHERE IS THE TENS PLACE?

WHAT IS TO BE SUBTRACTED FROM WHAT?

If you have problems like this, you'd better stick with the other form.

Look at the two ways of writing a division problem. Look to see how they are alike. How are they different?

The problem is $27\overline{)1836}$.

The first step for both is an estimate.

Think Round 27 to 30.
How many 30s in 1836?
$60 \times 30 = 1800$
$70 \times 30 = 2100$
It's less than 70 but more than 60.

First step

$$\begin{array}{r} 60 \\ 27\overline{)1836} \\ \underline{1620} \\ \end{array} \quad 60 \times 27$$

First step

$$\begin{array}{r} 6 \\ 27\overline{)1836} \\ \underline{162} \\ \end{array} \quad 6 \text{ tens} \times 27 = 162 \text{ tens}$$

How are the first steps alike? How are they different?

Next step

$$\begin{array}{r} 68 \\ 8 \\ 27\overline{)1836} \\ \underline{1620} \\ 216 \\ \underline{216} \\ 0 \end{array} \quad \begin{array}{l} \\ \\ \\ 60 \times 27 \\ \\ 8 \times 27 \\ \end{array}$$

Next step

$$\begin{array}{r} 68 \\ 27\overline{)1836} \\ \underline{162} \\ 216 \\ \underline{216} \\ 0 \end{array} \quad \begin{array}{l} \\ \\ 6 \text{ tens} \times 27 = 162 \text{ tens} \\ \\ 8 \times 27 = 216 \\ \end{array}$$

How are the last steps alike? How are they different?

Try just two problems, using the shorter method. If you don't like it, you can use the other method for the rest of these problems.

1. How many 63s in 882?
2. How many 39s in 702?
3. How many 47s in 799?
4. How many 78s in 858?
5. How many 23s in 437?
6. How many 36s in 396?
7. How many 57s in 855?
8. How many 68s in 884?
9. How many 82s in 984?

EXTRA PRACTICE

1. Round each number to the nearest ten. Be careful.

a 587	**b** 681	**c** 43	**d** 9007
e 11 111	**f** 209	**g** 330	**h** 1006

2. Round each number to the nearest hundred.

a 1567	**b** 1199	**c** 956	**d** 1096

3. Estimate answers.

a $8\overline{)16}$	**b** $8\overline{)170}$	**c** $8\overline{)161}$	**d** $80\overline{)161}$
e $70\overline{)79}$	**f** $70\overline{)790}$	**g** $70\overline{)7900}$	**h** $7\overline{)790}$
i $82\overline{)4107}$	**j** $82\overline{)417}$	**k** $53\overline{)395}$	**l** $350\overline{)3905}$

4. Estimate. Divide. Use either form. Check.

a $5\overline{)17}$	**b** $5\overline{)170}$	**c** $50\overline{)170}$	**d** $50\overline{)1700}$
e $40\overline{)181}$	**f** $40\overline{)1810}$	**g** $39\overline{)181}$	**h** $39\overline{)1810}$
i $27\overline{)550}$	**j** $27\overline{)555}$	**k** $270\overline{)5550}$	**l** $27\overline{)5550}$
m $69\overline{)5400}$	**n** $69\overline{)5420}$	**o** $690\overline{)5400}$	**p** $690\overline{)5420}$

CHECKOUT

Your goal in this chapter was to gain skill in division. Did you? Prove it. Do the following problems.

1. Round each number to the nearest ten.

a 49 **b** 125 **c** 764 **d** 5777

2. Round each number to the nearest hundred.

a 549 **b** 279 **c** 1487 **d** 5550

3. There are 856 students at Greenbank School. About how many hundreds of students are there? (Round to the nearest hundred.)

4. A total of 964 people paid for tickets to the game. About how many hundreds of people bought tickets? (Round to the nearest hundred.)

5. Estimate answers. Show what you did to get an estimate.

a 5×97 **b** 18×27

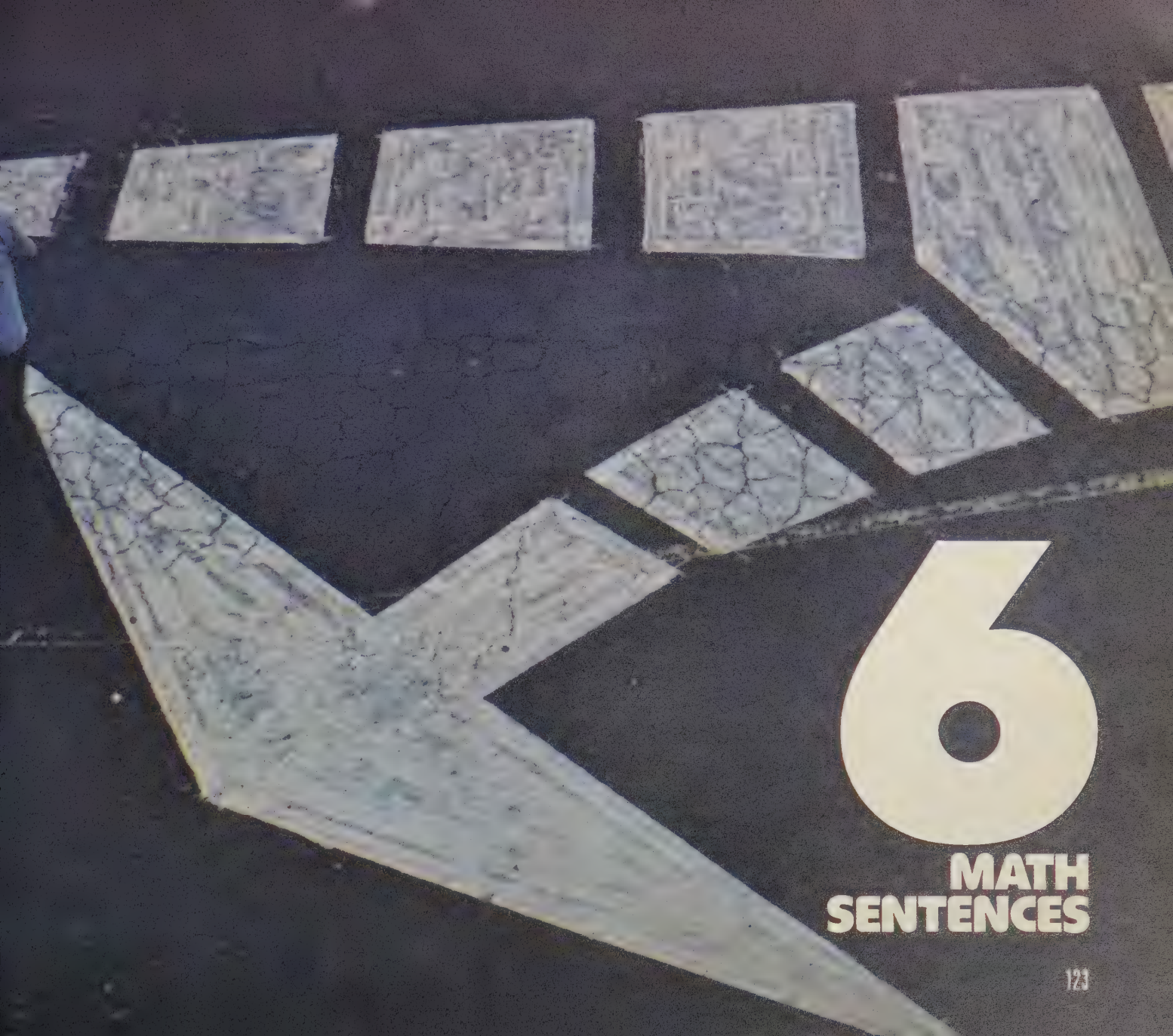
c $6\overline{)691}$ **d** $19\overline{)99}$

e $67\overline{)131}$ **f** $57\overline{)2988}$

6. Estimate. Divide. Check.

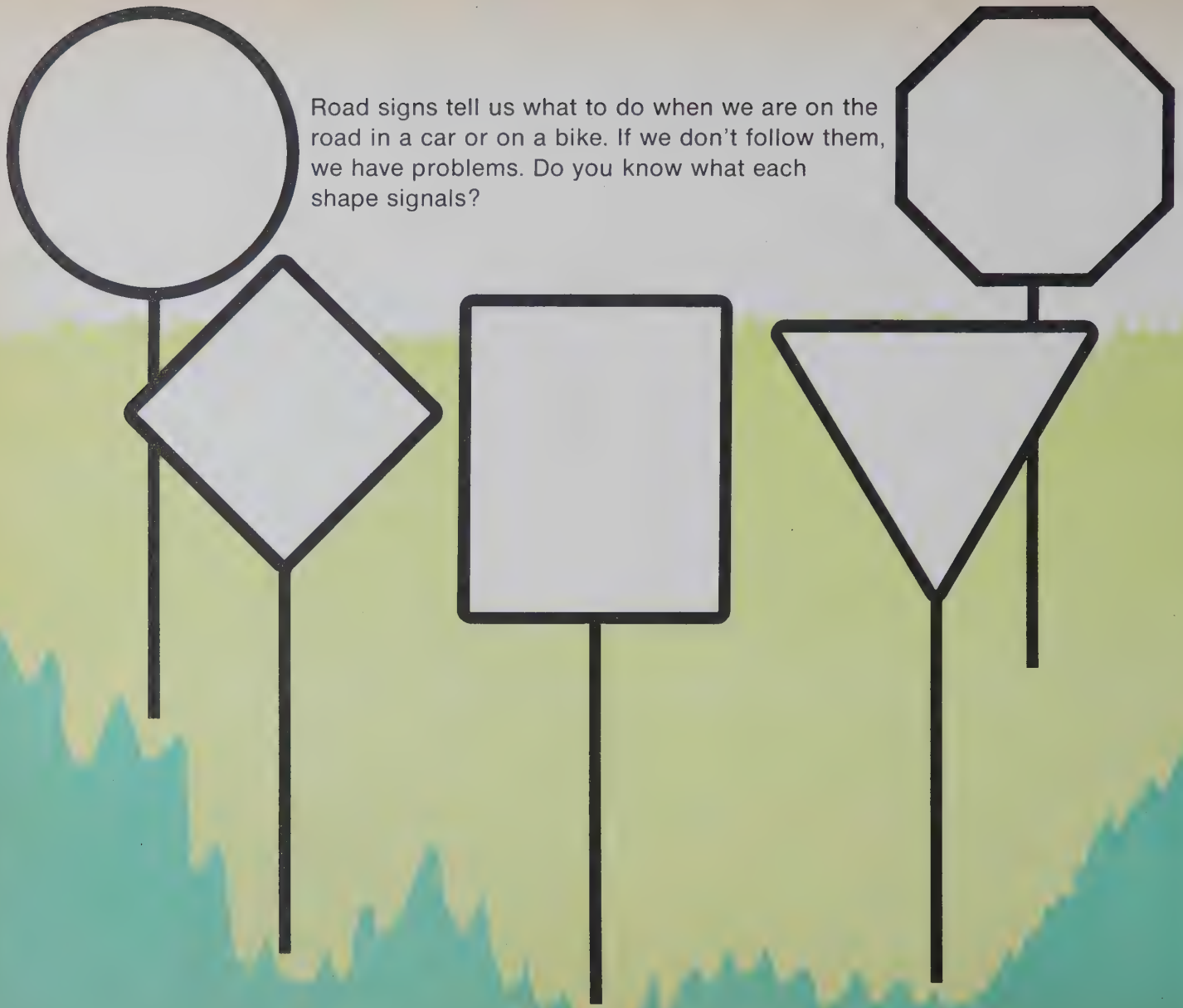
a $30\overline{)210}$ **b** $31\overline{)217}$ **c** $78\overline{)9123}$

d $83\overline{)6247}$ **e** $44\overline{)3210}$ **f** $12\overline{)4560}$

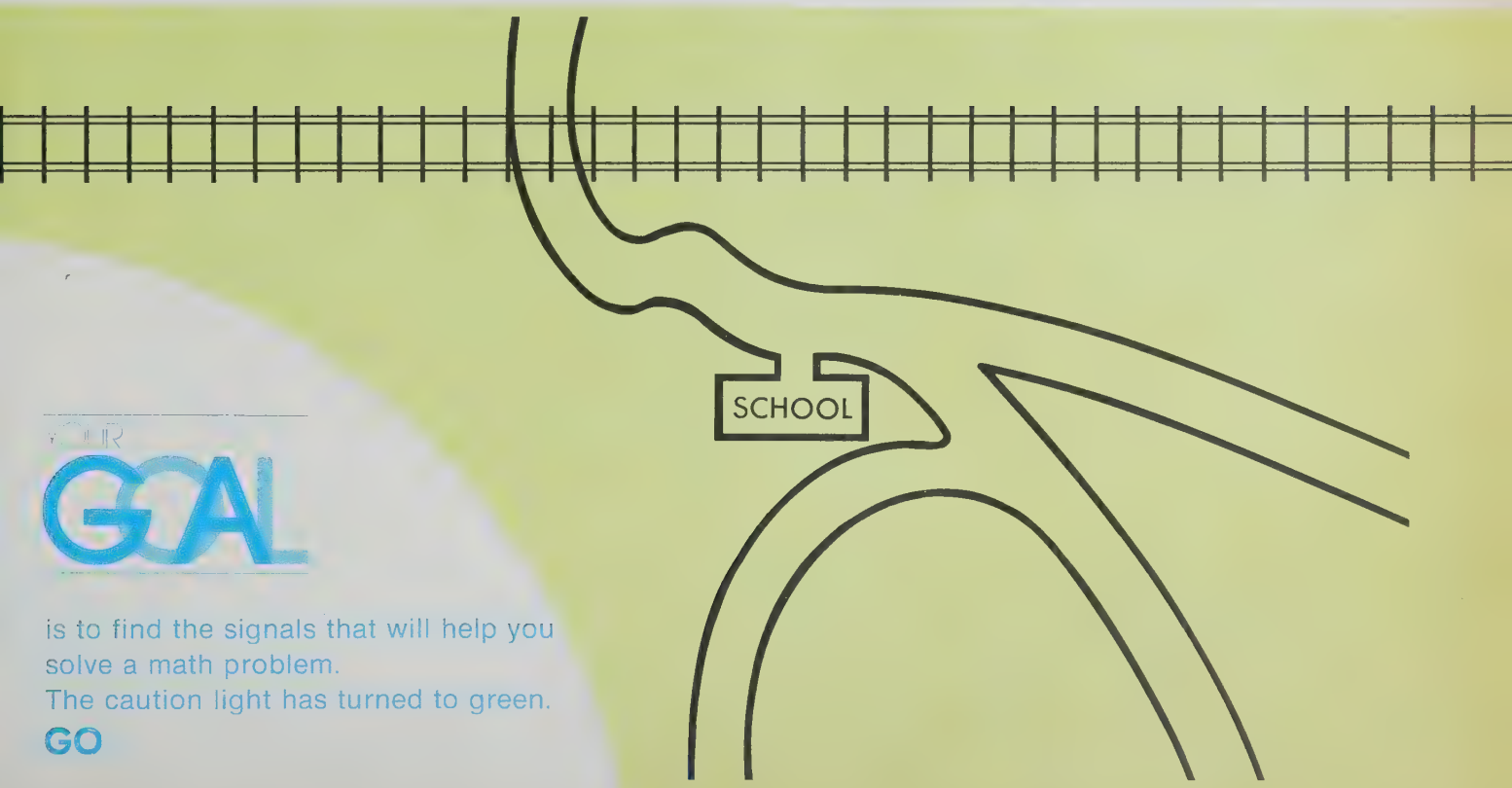


6 MATH SENTENCES

Road signs tell us what to do when we are on the road in a car or on a bike. If we don't follow them, we have problems. Do you know what each shape signals?



Pretend the picture below shows a new road. You are in charge of placing traffic signs. Where should they go? What should the sign signal?



YOUR GOAL

is to find the signals that will help you solve a math problem.
The caution light has turned to green.
GO

Math has symbols too. If we don't follow the directions they give, we have a problem. Without the symbols, we don't have a problem at all—at least not a math problem.

1. Pretend you are writing a math book. You are in charge of placing the operation and relation symbols to make true sentences. Where should they go in each set of three digits? What should the symbols signal?

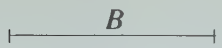
a 3 5 7	**b** 17 8 9	**c** 4 2 3
d 7 10 17	**e** 2 4 8	**f** 2 2 4
g 9 3 3	**h** 21 7 3	**i** 7 0 0
j 4 12 3	**k** 8 0 8	**l** 1 5 6

2. You have written math sentences with three numbers and one operation symbol and one relation symbol. Write more true sentences using only the relation symbols $>$, $<$, and $=$.

a $0 \,?\, 1$	**b** $10 \,?\, 9$	**c** $\frac{2}{2} \,?\, 1$
d $\frac{1}{3} \,?\, \frac{2}{6}$	**e** $1\frac{1}{2} \,?\, 1$	**f** $8 \,?\, 7$
g $\frac{2}{8} \,?\, \frac{1}{8}$	**h** $99 \,?\, 97$	**i** $\frac{3}{2} \,?\, 1\frac{1}{2}$
j $\frac{6}{8} \,?\, \frac{3}{4}$	**k** $\frac{9}{9} \,?\, \frac{7}{7}$	**l** $\frac{0}{4} \,?\, \frac{0}{5}$

1. **a** How long is segment A?

b How long is segment B?



c How long are A and B together?

d Write a math sentence.

2. **a** How long is segment C?

b How long is segment D?

c How much longer is segment C than segment D?

d Write a math sentence.

3. **a** What is the length of this book to the nearest centimetre?

b What is the width of this book to the nearest centimetre?

c How much longer is it than wide?

d Write a math sentence.

4. You must be 16 years old to drive a car.

a How old are you?

b How many years must you wait to drive a car?

c Write a math sentence.

5. Sometimes simple little problems can fool you. Look out for this one.

a How many pages in this chapter?

b What page are you doing?

c How many pages more to do in this chapter?

d Write a math sentence.

A math sentence may be

TRUE

or

FALSE

1.

Is each of these sentences **TRUE?**

a $10 + 5 = 15$

b $14 \times 2 > 25$

c $18 \div 3 = 6$

d $10 < 16$

Is each of these sentences **FALSE?**

e $10 + 5 = 16$

f $3 \times 9 > 30$

g $14 \div 7 < 1$

h $15 < 10$

2.

What about these sentences? True or false?

a $15 + 10 = n$ **b** $14 \times n > 32$ **c** $n < 5$

Do you know whether they are true? false? Your answer depends on what n stands for. Right? These are called open sentences.

3.

Do all math sentences have to have an operation symbol such as $+$, $-$, $\times$, or $\div$?

4.

Do all math sentences have to have a relation symbol such as $=$, $\neq$, $>$, or $<$?

5.

What do all open sentences have that true sentences and false sentences do not have?

6.

Mark gets a weekly allowance of 25 cents. Mark says that if he saves all of his allowance each week, he will have $4.00 in 16 weeks.
Is his statement true? false?

A math sentence can help you decide.

$16 \times 25 = 400$ Is that a true sentence?
Is Mark's statement true or false?

True or False

1 Jan practised 30 minutes before school and 20 minutes after school. She said she practised for an hour.

a Was her statement true or false?

b Write a math sentence to back up your answer.

2 Sam delivers papers every day of the week. This takes 20 minutes each morning. He said he spends more than 100 minutes every week just delivering papers.

a Was his statement true or false?

b Write a math sentence to back up your answer.

3 Ellen reads comics. Last week she read 28 comics in 7 days. She said she read 5 a day.

a Is her statement true or false?

b Write a math sentence to back up your answer.

4 Simon had begun collecting shells. His uncle gave him 4 more. "Great!" said Simon. "Now I have 13 shells altogether." How many shells did he have to start with? Does an open sentence tell the number he began with?

$$n + 4 = 13$$

a Can you make that open sentence true?

b What numbers will make it true?

c What numbers will make it false?

5 Find a number that will make each open sentence true.

a $24 \div n = 3$ **b** $8 + n = 20$

c $2 \times n = 2$ **d** $14 - n = 14$

e $6 + n < 8$ **f** $5 - n > 4$

A replacement for a letter, a box, or a question mark can make a sentence true or false. A replacement that makes the sentence true is a solution to the sentence.

1. Find solutions for each open sentence.

a $27 \div n = 3$

b $6 \times 8 = r$

c $y - 12 = 7$

d $24 + 40 = s$

e $24 \div 4 = q$

f $\frac{1}{2} + b = 1\frac{1}{2}$

g $\frac{1}{4} + b = \frac{3}{4}$

h $37 - n = 15$

i $n + \frac{1}{4} = \frac{3}{4}$

2. Which of these sentences are true? Which are false? Which are open?

a $\frac{4}{5} + \frac{1}{10} > 1$

b $32 \div a = 8$

c $7 \times 6 \neq 40$

d $9 \times 5 < 40$

e $\frac{1}{3} > \frac{4}{12}$

f $96 \div 32 = 3$

g $18 \div b = 6$

h $100 \times 17 = n$

i $154 - n = 67$

j $36 \div 6 = 5$

k $4 \times b = 28$

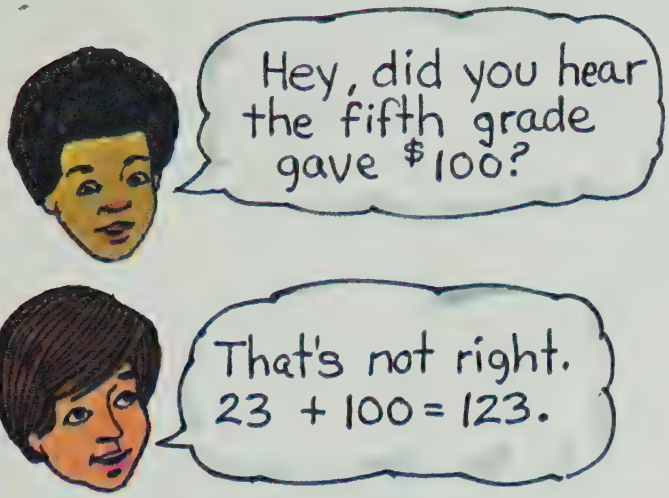
l $c \times 7 = 49$

3. Find solutions for the open sentences in problem 2.

4. Are all equations math sentences? Are all math sentences equations? Can an equation be an open sentence? What relation symbol must every equation have?

Jason read in the school newspaper that the fifth grade had made a contribution to the activity fund. This raised the total to $140.

Everyone knew there was only $23 in the fund before the fifth grade's contribution.



Simon guessed again. "Maybe it was $120." Jason hated to argue, but he had to say, "No. $23 + 120 = 143$." Simon guessed again. "How about $117?" Was this guess more accurate? Was it right?

Let's use math sentences to recap the discussion.

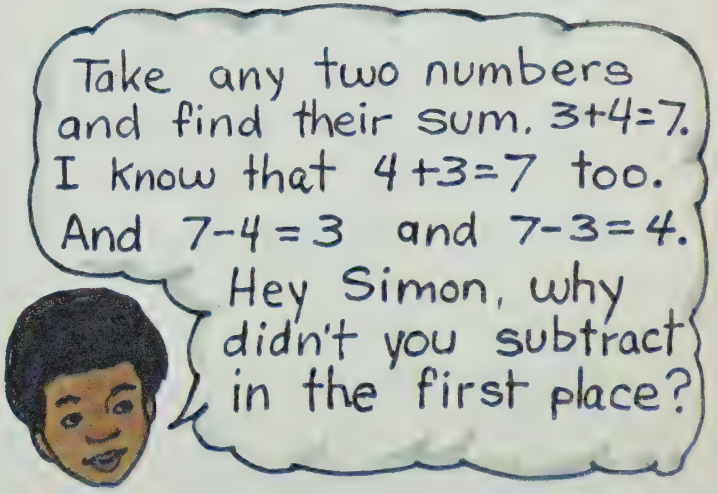
$23 + n = 140$

Can n be 100? Nope! $23 + 100 = 123$

Can n be 120? Wrong again! $23 + 120 = 143$

Can n be 117? Right! $23 + 117 = 140$

Simon was probably good at estimating. Jason was good at computing. He remembered something special about addition and subtraction.


Take any two numbers and find their sum. 3+4=7. I know that 4+3=7 too. And 7-4=3 and 7-3=4.
Hey Simon, why didn't you subtract in the first place?

$23 + n = 140$ can be thought of as $n = 140 - 23$.

Is that magic, or does it work all the time?

Simon said he wasn't guessing to find an answer; he was estimating. What's the difference between a good guess and an estimate? Sometimes an estimate isn't good enough. An accurate answer must be computed.

Jason saw an addition equation with one number to be added missing, so he subtracted to find the answer.

$$23 + n = 140$$
$$n = 140 - 23$$

1. Use Jason's method to solve the following.

a $621 + n = 896$ **b** $256 + n = 450$ **c** $127 + n = 216$

2. Find solutions for these sentences.

a $163 + n > 200$ **b** $256 + n < 257$ **c** $525 + n > 532$

3. Find a solution for each sentence.

a $128 + n > 212$ **b** $128 + n < 212$ **c** $n = 212 - 128$

d $89 + n > 265$ **e** $89 + n < 265$ **f** $n = 265 - 89$

g $248 + n > 285$ **h** $248 + n < 285$ **i** $n = 285 - 248$

4. Which is needed—an accurate answer or an estimated answer?

a How much money you should take to the store

b How much money you can spend at the store

c How long it takes to get to school

d How long it takes to get home

e How many people in your city

f How many people are registered to vote

g How many people do vote

You will use this information on the next page.

Every math sentence must have what mathematicians call a relation symbol. There are lots of these symbols. The relation symbols you use the most are $=$, $>$, and $<$. In a sentence they tell how one number relates to another.

Look back at the situations in problem 4 on the page before.

4. **a** and **b** Let t stand for the money you take.
Let s stand for the money you spend.
Can t equal s? Can t be greater than s? Can t be less than s? What would happen?

4. **c** and **d** Let t stand for the time it takes to go to school.
Let h stand for the time it takes to go home.
Can t equal h? Can t be greater than h? Can t be less than h? Why?

4. **e, f,** and **g** Let p stand for people.
Let r stand for those registered to vote.
Let v stand for those that do vote.
Can p equal r? Can p be greater than r? Can r be greater than v?

Look out! Can p equal v? Can v be greater than r? Can p be less than r?

This is another one of those times when we have to think carefully. Can the math sentences on this page be called true or false? Why?

Go back over the situations. Assign a number to each letter. (Be reasonable when you put the numbers into the story situation.) Find out if the sentence can be true or false when you are working with numbers.

Not every math sentence needs the operation symbol $+$, $-$, $\times$, or $\div$.
Lots of math sentences have them, though.

1. Find a solution for each sentence.

a $500 + 250 = n$ **b** $200 + 175 = n$ **c** $125 + 175 = n$ **d** $395 + 195 = n$

No trouble, right? All you had to do was add. Keep on.

e $400 + n = 650$ **f** $600 + n = 867$ **g** $135 + n = 241$ **h** $275 + n = 474$

Not quite so easy, right? Did you really *add* to find the value of n?

If both Simon and Jason would get to work, it would help.

$300 + n = 475$ Simon says the solution is less than 475.
Jason says you should subtract to find how much it is.
$n = 475 - 300$ Now that's not so bad. What's the solution?

PROGRESS CHECK

Now it's your turn. Find the solution.

(1.) $536 + n = 860$ (2.) $728 + n = 887$ (3.) $467 + n = 996$ (4.) $143 + n = 971$

(5.) $752 + n = 980$ (6.) $685 + n = 900$ (7.) $245 + n = 570$ (8.) $489 + n = 600$

Which of the following are true? Which are false? Which are open?

(9.) $3 \times 9 > 28$ (10.) $365 - 214 = 151$ (11.) $9 \times 8 = 54$ (12.) $9 + 6 > 15$

(13.) $72 \div n = 9$ (14.) $3 + 9 > 15 - 3$ (15.) $35 < 9 \times 4$ (16.) $25 + 25 = n$

If there are 33 rows and there are 6 trees in each row, how many trees in all?

Jason wrote an open sentence to find the answer.

$$33 \times 6 = t$$

What is the answer to the problem?

$$33 \times 6 = 198$$

Is this answer close to Simon's estimate?

Solve these. You may want to start by writing an open sentence. Maybe you won't. But you are asked to write a true math sentence to show the right answer.

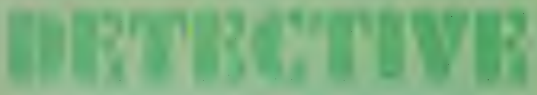
1. There were 6 trucks. Each truck hauled 100 bundles of towels. How many bundles in the 6 trucks in all?

2. Ma Dell made 60. She put the same number into each of 5 boxes. How many in each box?

3. There were 24 in one room. There were 36 in another room. How many in all?

4. Sam had 36. He gave 15 away. How many did he have left?

Sometimes when you solve word problems, you have to be a



You have to find word clues to know what arithmetic to do.

1. What operation *might* these words signal?

a How many in all?
b How much faster?
c How many were left?
d He added more.
e What is the total?
f How many in each?

2. See if your clues work. Find the answers.

a 25 boxes. 5 in each box. How many in all?

b 42 came early. 50 more got there. How many in all?

c He ran the distance in 3 minutes. She ran the distance in $2\frac{1}{2}$ minutes. How much faster was she?

d There were 20. 5 went home. How many were left?

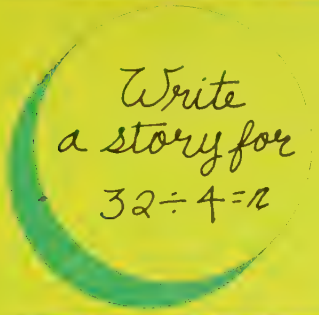
e Each bag would hold 5. There were 6 bags. There were 35 to be packed. How many were left?

f There were 50. He added some more. Now there were 60. How many did he add?

g There are 25 in one book, 30 in the second book, and 10 in the third book. What is the total number in the three books?

3. You make up a simple story that has this question at the end: How many in each?

Lots of times word clues do help you decide how to compute. But always read the whole problem, or you might get fooled.



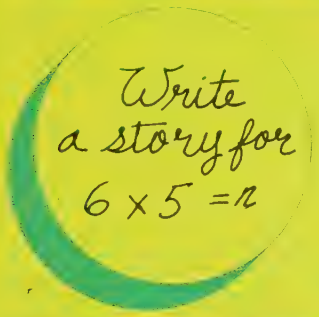
Joe's thought:

If 32 pennies are divided equally among 4 boys, how much money will each have?

Jackie's idea:

How many girls will get packages of gum if each girl gets 4 packages of gum and there are 32 packages in all?

Two stories. One math sentence. Same answer?



Hank writes:

There are 6 teams and 5 players on each team. How many players?

Pete says:

Last summer I worked for 6 days and was paid $5 a day. How much money should I have received?

Two stories. One math sentence. Same answer?

1. You write a story to fit $32 \div 4 = n$.

2. Write a story for any two of these. Then solve every sentence.

a $66 \div 11 = n$ **b** $15 \times 4 = n$ **c** $35 + a = 92$

d $133 \div n = 19$ **e** $190 \times n = 950$ **f** $n - 42 = 77$

You can make up a story for just about any math sentence. But the important skill is to complete a true math sentence. A true math sentence is one good way to find out if your solution to the problem is correct.

Here's a copy of Jimmy's homework. He was supposed to write open sentences to fit the stories and then solve the sentences. He made some mistakes. Some answers are not even reasonable. Find the **mistakes** he made.

1. Of the 24 students in the class, 16 were never absent. How many were sometimes absent?

   $24-16=a$
   $8=a$

2. Mark bought some model airplane kits. Each kit had 37 pieces. There were 185 pieces all together. How many kits did Mark buy?

   $185 \times 37 = K$
   $6845 = K$

3. 17 of the airplanes that arrived today at Swegis Airport were jets. 43 of the airplanes that arrived today were not jets. How many airplanes arrived today?

   $a = 43 - 17$
   $a = 26$

4. If a gram of prevention is worth 16 g of cure, how much is 32 g of prevention worth?

   $C \times 16 = 32$
   $C = 32 \div 16$
   $C = 2$

5. Each of the 27 booths at the carnival had 17 prizes. How many prizes in all?

   $27 \times 17 = p$
   $459 = p$

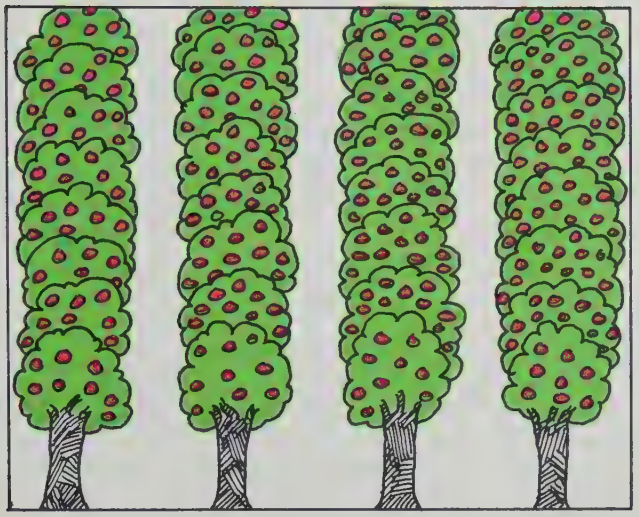
6. 144 avocados were packed in a box with 12 on each layer of the box. How many layers?

   $144 \times 12 = l$
   $1728 = l$

How could Jimmy straighten out his thinking on problems 2, 3, 4, and 6?

Solve

Write a true math sentence that recaps the story.



1. The 29 students of Miss Brown's class were planning a field trip. 17 in the class did not want to go to the local museum.
   Did the remaining students have enough votes for a majority?

2. Jerry owns a large terrarium in which there are 16 brown, warty toads. He says he has twice as many soft, slimy snails as he has brown, warty toads. How many soft, slimy snails does he have?

3. Rita gets 25¢ pocket money each week. If she saves all of her money, how many weeks will it take her to save $3.00?

4. Long-playing records are made in a press. The press can make 1 record a minute. How many records could 3 presses make in 11 minutes?

5. An orchard about 30 m long and 25 m wide has 8 trees in each of its 4 rows. One day each row yielded a total of 24 baskets of fruit.
   This is part of a story. Write four versions of the story so that you have an addition problem in the first story, a subtraction problem in the second story, a multiplication problem in the third story, and a division problem in the fourth story.

Sometimes you get into a situation where you have to find the solution to two sentences to get the problem solved. The solution to the first sentence is part of the second sentence. You might think of these as two-step problems.

---

1 Jane went to the store for her mother. She bought bread for 32 cents and a can of soup for 17 cents. How much change should she get if she gave the clerk a dollar?

First you figure out what the total bill was. Then you can find how much change.

---

2 Ping-Pong balls were packed for shipment with 36 balls in a tray. There were 4 trays in each layer and 8 layers in the carton. How many balls are there in each carton?

First you figure out how many balls in each layer. Then you can find how many in the 8 layers.

---

3 Robbie wants to buy 2 records that cost $3.67 each. He has only $7.14. Can he buy 2? If he can't, how much more does he need?

First you find out how much 2 records cost. Then you can find out how much money he needs.

---

4 Bill earned money babysitting. He charged 50 cents an hour. He took care of Junior Jones for 3 hours Tuesday. He watched Sissy Smith for 2 hours Wednesday. How much money did he earn these two days?

There certainly is more than one way you can organize the information to find the answer to this one. Write true sentences to show what your two steps were.

Would you believe that sometimes you can have too much information? You don't need all the facts that are available.

Some of the following problems have information that you don't need. Write a true math sentence to show the information that you used to get the solution.

1 The fuel tank in Bob's car holds 40 ℓ. If Bob drives under 88 kilometres per hour (km/h), he gets 9 kilometres per litre (km/ℓ). If he drives over 88 km/h, he gets only 7 km/ℓ .

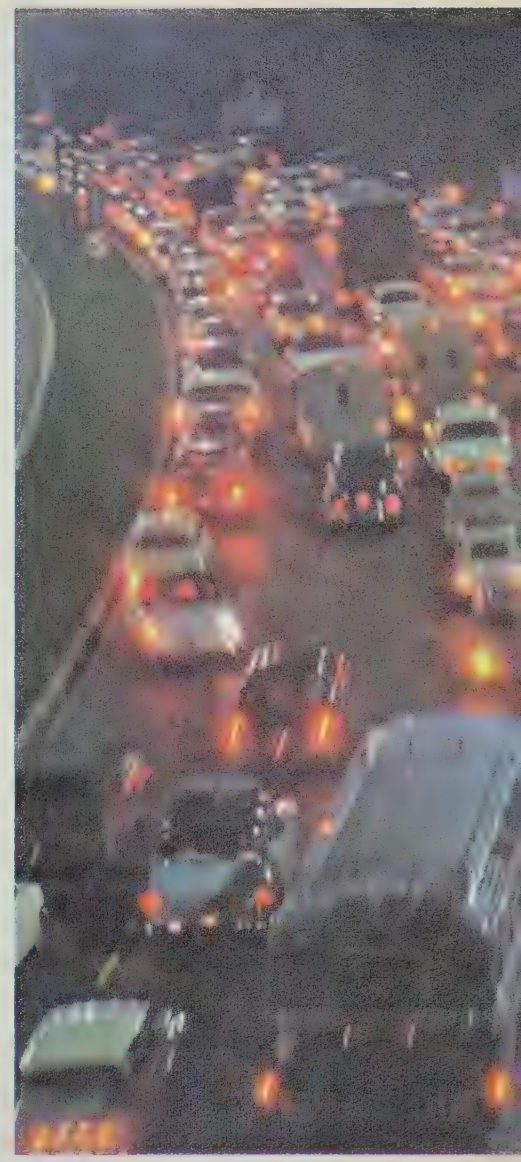
a How many kilometres could he drive with a full tank if he kept under 88 km/h all the time?

b How far could he travel with a full tank if he drove over 88 km/h the entire time?

c If he drives under 88 km/h the entire time, how many more kilometres can he travel on a full tank than if he drives over 88 km/h the entire time?

2 The fuel tank in Susan's motorcycle holds 9ℓ, but she gets almost 160 km/ℓ . If Susan rode 155 km Monday and 347 km Tuesday, what distance did she ride altogether?

3 The company pays Mr. Slick 8¢ per kilometre to cover expenses for his car. Oil costs 60¢ per litre. His car used 1ℓ last week. Fuel is 14¢/ℓ . He bought 450ℓ last week. He drove 1900 km last week. How much did the company pay him for car expenses last week?



For Experts Only

Write a true math sentence for the stories that have enough information. Make up information for those stories that need more. Then write a true math sentence for them, too.

1 There are 330 000 000 trees growing in Leafland. 320 100 000 of these are oaks. 6 600 000 of the trees are elms. The rest of the trees are willows. How many willows are there in Leafland?

2 One column of newsprint is 33 cm long. There are 3 lines to a centimetre. About how many words would fit in the column?

3 Herbie the humpback whale ate 5000 herring for lunch. Each herring had just eaten 7000 little shellfish. Each little shellfish had just eaten 130 000 teeny, tiny plants. How many teeny, tiny plants did Herbie eat?

4 There are 340 children in Marsha's school. 130 buy lunch at the school. Some eat lunch at home. How many bring their lunch to school?

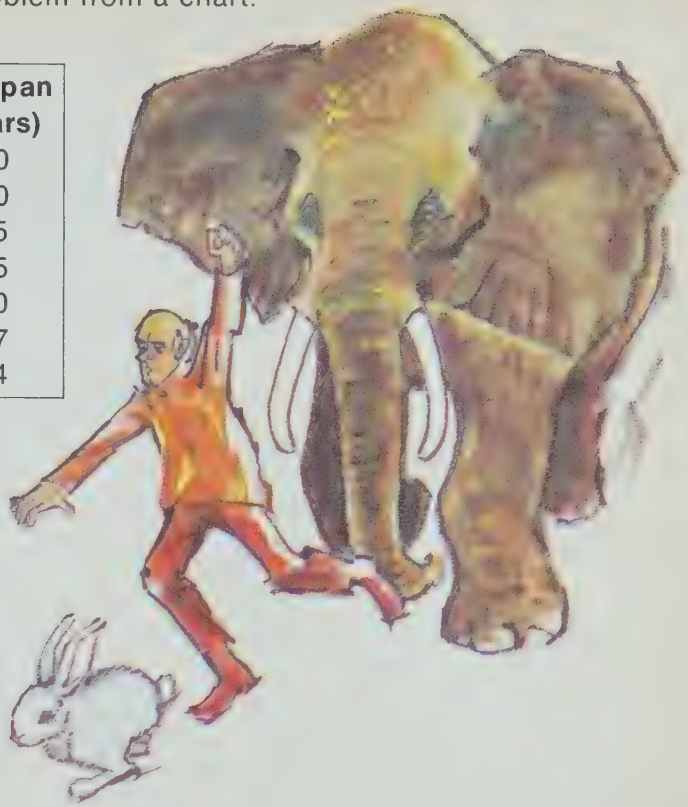
5 Tim put 3 problems on the board. Mary put on 7. Sam put on twice as many as Tim and Mary together. How many problems are on the board?

Sometimes we have to get information to solve a problem from a chart.

Here is an example.

The chart lists seven kinds of animals, including man. It tells two things about each kind: how fast it can run and how long it can live.

Animal	Speed (km/h)	Lifespan (years)
Lion	80	20
Zebra	64	20
Rabbit (tame)	56	5
Cat (tame)	48	15
Human being	44	70
Elephant	40	47
Pig (tame)	18	14



Answer the following questions.
You may want to use math sentences to help you.

1. How much faster is a lion than a zebra?
2. According to this chart, would you be able to catch a pet rabbit running away from you?
3. An angry elephant is chasing you. Who can run faster, you or the elephant? How much faster?
4. What is the difference between the speeds of the fastest and the slowest animals listed?
5. What is the difference between the longest lifespan and the shortest lifespan for animals listed on the chart?
6. Anita has two pets — a rabbit and a cat. Which pet is likely to live longer? How many years longer?
7. The zebra at Zenith Zoo has lived $\frac{7}{20}$ of its expected lifespan. How old is the zebra?

CHECKOUT

1. Which are true sentences?

a $14 \times 4 = 42$ **b** $63 > 8 \times 8$ **c** $63 < 7 \times 9$

d $\frac{3}{5} > \frac{1}{3}$ **e** $\frac{3}{5} - \frac{1}{3} > 1$ **f** $74 - 27 = n$

2. Copy each set of three digits. Write an operation and a relation symbol to complete a true sentence.

a 3 4 12 **b** 0 8 8 **c** 5 4 11

d 4 2 2 **e** 5 3 15 **f** $\frac{1}{4}$ $\frac{3}{4}$ 1

3. Use $>$, $<$, or $=$ to make true sentences.

a 16 15 **b** $\frac{6}{8}$ $\frac{3}{4}$ **c** $\frac{3}{8}$ $\frac{1}{2}$

4. Solve the following problems. Then write a true sentence to show that your answer is correct.

a 16 cars.
8 in each row.
How many rows?

b 12 meals.
3 meals each day.
How many days?

c 9 boats.
5 people in each.
How many people?

d 10 packages.
10 kg each.
How many kg?

7 FRACTIONS

Renaming a fraction can mean a lot of things.
Look at the different kinds of renaming.

THINK about *what* you do as you rename these.

1. What is the simplest name for each of these?

a $\frac{2}{4}$ **b** $\frac{14}{12}$ **c** $\frac{6}{9}$ **d** $\frac{5}{5}$ **e** $\frac{10}{15}$

2. Rename each whole number as a fraction.

a $3 = \frac{?}{2}$ **b** $1 = \frac{?}{8}$ **c** $1 = \frac{?}{3}$

d $1 = \frac{?}{10}$ **e** $2 = \frac{?}{4}$ **f** $4 = \frac{?}{4}$

3. Try to rename each of these mixed numbers as a fraction.

a $1\frac{1}{10}$ **b** $2\frac{1}{5}$ **c** $2\frac{2}{7}$ **d** $10\frac{11}{12}$

That's enough.

4. And would you believe there is still one more type of renaming? Rename each pair of fractions so that they have a common denominator.

a $\frac{1}{3}$ and $\frac{1}{6}$ **b** $\frac{3}{4}$ and $\frac{5}{8}$ **c** $\frac{3}{6}$ and $\frac{1}{2}$

That's enough.

5. If you put it all together, you can do problems like these. Try them.

a $1\frac{3}{4} + 1\frac{1}{8}$ **b** $8\frac{5}{6} - 2\frac{1}{3}$ **c** $2\frac{2}{5} + 1\frac{1}{10}$

d $3\frac{1}{2} - 1\frac{1}{6}$ **e** $4\frac{1}{2} + 2\frac{1}{4}$ **f** $2\frac{7}{8} - 1\frac{3}{4}$

Your goal is to become really good at renaming fractions.

Did you make any mistakes on problem 1 on the page before?
Maybe this will help.

That's a common factor.

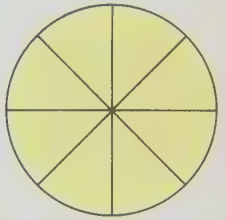
$\frac{2}{4}$ — Is there a common factor? $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

$\frac{8}{12}$ — Is there a common factor?
2 is a common factor, but so is 4.
Use the greatest common factor and save time. $\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Practise on these. Write the simplest name for each.

	a	b	c	d	e	f	g	h	i	j
1.	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{6}{12}$	$\frac{5}{10}$	$\frac{7}{14}$	$\frac{3}{9}$	$\frac{2}{10}$	$\frac{6}{8}$	$\frac{10}{12}$	$\frac{8}{8}$

The last problem looks easy, BUT $\frac{8}{8} = 1$.
Maybe you had to draw a picture for yourself.
But drawing pictures takes a long time.
The denominator tells how many parts in one.
If the numerator is the same number as the denominator, then the fraction says you are considering the whole thing.



Renaming 2 is not a problem.
THINK One whole thing is divided into $\frac{8}{8}$.

Then 2 whole things are the same as $\frac{8}{8} + \frac{8}{8} = \frac{8+8}{8} = \frac{16}{8}$

Practise on these.
Rename each whole number as a fraction.

	a	b	c	d	e	f
2.	$1 = \frac{?}{6}$	$2 = \frac{?}{2}$	$3 = \frac{?}{3}$	$5 = \frac{?}{8}$	$4 = \frac{?}{3}$	$2 = \frac{?}{3}$
3.	$1 = \frac{?}{9}$	$3 = \frac{?}{4}$	$2 = \frac{?}{6}$	$4 = \frac{?}{4}$	$3 = \frac{?}{5}$	$5 = \frac{?}{10}$

Renaming a mixed number as a fraction is *not* a very big step.
$2\frac{1}{10}$ says that you have 2 whole things and $\frac{1}{10}$ more.
2 is the same as $\frac{10}{10} + \frac{10}{10}$, or $\frac{20}{10}$. You have $\frac{1}{10}$ more.
So $\frac{20}{10} + \frac{1}{10} = \frac{21}{10}$. AND $\frac{21}{10}$ is the fraction name for $2\frac{1}{10}$.

Time for practice.
Rename each mixed number as a fraction.

	a	b	c	d	e	f	g
1.	$1\frac{5}{6}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{6}$	$2\frac{3}{10}$	$3\frac{1}{3}$	$2\frac{3}{5}$
2.	$3\frac{3}{8}$	$2\frac{2}{3}$	$4\frac{1}{4}$	$2\frac{2}{5}$	$2\frac{5}{6}$	$4\frac{3}{4}$	$3\frac{1}{6}$

Renaming a fraction as a mixed number isn't hard either. Start slowly.

Which of these fractions can be renamed as mixed numbers?

	a	b	c	d	e	f	g
3.	$\frac{5}{3}$	$\frac{3}{5}$	$\frac{7}{5}$	$\frac{5}{7}$	$\frac{7}{3}$	$\frac{3}{7}$	$\frac{0}{4}$
4.	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{6}{9}$	$\frac{9}{6}$	$\frac{12}{5}$	$\frac{5}{12}$	$\frac{9}{8}$

Use $\frac{7}{3}$ as an example. **Think** How many $\frac{3}{3}$s in $\frac{7}{3}$?

There are 2 and $\frac{1}{3}$ more.
$\frac{7}{3}$ can be renamed $2\frac{1}{3}$.

Time for more practice.
Rename each fraction as a mixed number.

	a	b	c	d	e	f	g
5.	$\frac{3}{2}$	$\frac{7}{5}$	$\frac{9}{4}$	$\frac{13}{4}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{7}{6}$
6.	$\frac{7}{3}$	$\frac{9}{2}$	$\frac{16}{5}$	$\frac{19}{8}$	$\frac{11}{6}$	$\frac{15}{4}$	$\frac{21}{10}$

1 Add. Rename the sums as mixed numbers.

Example: $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ THINK $\frac{3}{3} = 1$. How many $\frac{3}{3}$s in $\frac{4}{3}$?
There's 1 and $\frac{1}{3}$ more. $\frac{4}{3}$ can be renamed $1\frac{1}{3}$.

a $\frac{5}{6}+\frac{2}{6}$ **b** $\frac{3}{9}+\frac{7}{9}$ **c** $\frac{4}{5}+\frac{3}{5}$ **d** $\frac{5}{7}+\frac{6}{7}$ **e** $\frac{7}{8}+\frac{4}{8}$

2 The next set of problems requires renaming—lots of it.

Example: $\frac{3}{8} + \frac{7}{8}$ First you add. Your answer is $\frac{10}{8}$. Now you rename $\frac{10}{8}$ as $1\frac{2}{8}$.
And you can rename again, because $\frac{2}{8} = \frac{1}{4}$.
Your final answer is $1\frac{1}{4}$.

Try these. Your challenge is to find the simplest name for each sum.

a $\frac{3}{4}+\frac{3}{4}$ **b** $\frac{5}{6}+\frac{5}{6}$ **c** $\frac{7}{8}+\frac{5}{8}$ **d** $\frac{2}{3}+\frac{2}{3}$ **e** $\frac{1}{5}+\frac{4}{5}$

f $\frac{5}{6}+\frac{3}{6}$ **g** $\frac{4}{5}+\frac{2}{5}$ **h** $\frac{5}{8}+\frac{5}{8}$ **i** $\frac{9}{10}+\frac{3}{10}$ **j** $\frac{5}{12}+\frac{11}{12}$

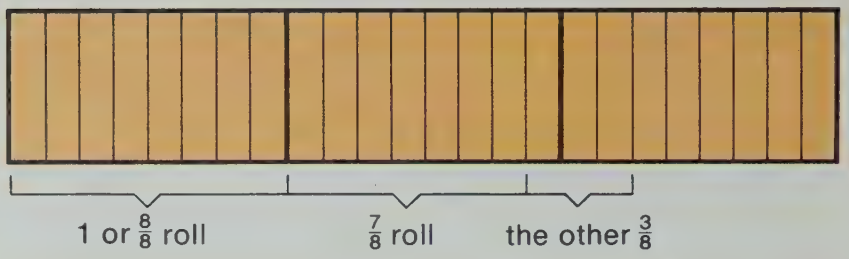
Time to look at some situations.

3 He had to spend $\frac{3}{4}$ of an hour on his music.
Homework would take another $\frac{3}{4}$ hour.
How long before he would be free?

4 She used $1\frac{7}{8}$ rolls of wallpaper for one wall.
She used $\frac{3}{8}$ of a roll more around the windows.
How many rolls did she use in all?

There is something new in that last problem. What is it?
Make sure you know how to add with mixed numbers.

Let the picture help you get an answer for $1\frac{7}{8} + \frac{3}{8}$.



Let's look at that last problem. The paper for the one wall can be thought of as $\frac{15}{8}$. Another $\frac{3}{8}$ is needed. $\frac{15}{8} + \frac{3}{8} = \frac{18}{8}$ AND that answer has to be renamed. $\frac{18}{8} = 2\frac{2}{8}$. You must rename AGAIN. What is the answer?

That is a real-world problem for you. Sometimes it seems that you never find the answer. Be patient. You'll make it.

Try another problem.

They wanted to make fudge and a chocolate cake.
The fudge recipe called for $2\frac{1}{4}$ squares of baking chocolate.
The cake needed $1\frac{3}{4}$ squares.
How much chocolate do they need to make both?

Skip the picture this time. Save even more time by writing the problem this way:

$$\begin{array}{r} 2\frac{1}{4} \\ +\ 1\frac{3}{4} \\ \hline 3\frac{4}{4} \end{array}$$

First add the fractions.
Then add the whole numbers.
And finally rename the answer.
How many squares of chocolate?

Your turn. Add these. Name each in simplest form.

1. $1\frac{1}{2} + 1\frac{1}{2}$ **2.** $2\frac{2}{3} + 1\frac{1}{3}$ **3.** $2\frac{5}{6} + 4\frac{1}{6}$ **4.** $1\frac{3}{5} + 2\frac{2}{5}$ **5.** $1\frac{3}{8} + 1\frac{5}{8}$

Do you think that set of problems was rigged? Why?

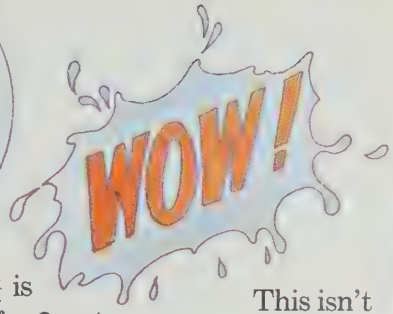
You practised only one type of renaming. Look at all types.

$$2\tfrac{3}{8} + 1\tfrac{1}{8} = 3\tfrac{4}{8}, \text{ or } 3\tfrac{1}{2}$$

There is nothing hard here.

$$1\tfrac{3}{4} + 1\tfrac{1}{4} = 2\tfrac{4}{4}, \text{ or } 3$$

The answer $2\frac{4}{4}$ is another name for $2 + 1$. That's not hard.



This isn't as bad as it looks, but take your time with this one.

$$1\tfrac{5}{6} + 1\tfrac{2}{6} = 2\tfrac{7}{6}, \text{ or } 3\tfrac{1}{6}$$

Study this example:

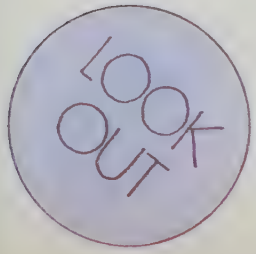
$$2\tfrac{2}{3} + 1\tfrac{2}{3} = 3\tfrac{4}{3}$$

What is the simplest name?

$\frac{4}{3} = 1\frac{1}{3}$

$3 + 1\frac{1}{3} = 4\frac{1}{3}$ That's your answer in simplest form.

Take time to practise only this type of renaming.
Name each in simplest form.



	a	b	c	d	e	f	g
1.	$3\frac{5}{4}$	$2\frac{5}{3}$	$6\frac{8}{5}$	$9\frac{4}{3}$	$5\frac{13}{10}$	$4\frac{11}{6}$	$1\frac{17}{9}$
2.	$2\frac{6}{4}$	$4\frac{12}{10}$	$6\frac{9}{6}$	$3\frac{12}{8}$	$3\frac{14}{10}$	$1\frac{8}{6}$	$2\frac{10}{4}$

Add. Write the simplest name for each sum.

	a	b	c	d	e
1.	$1\frac{3}{5}$	$4\frac{2}{3}$	$3\frac{5}{6}$	$7\frac{3}{8}$	$3\frac{4}{7}$
	$+\ 2\frac{2}{5}$	$+\ 1\frac{1}{3}$	$+\ 1\frac{1}{6}$	$+\ 2\frac{5}{8}$	$+\ 4\frac{3}{7}$

The next ones may look different, but compute the same way.

2.	$5\frac{4}{5}$	$4\frac{5}{9}$	$2\frac{7}{8}$	$3\frac{3}{5}$	$1\frac{5}{7}$
	$+\ \frac{3}{5}$	$+\ \frac{7}{9}$	$+\ \frac{5}{8}$	$+\ \frac{4}{5}$	$+\ \frac{3}{7}$

Back to word problems again.

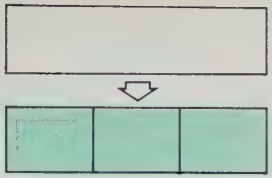
3. Sara bought $2\frac{1}{3}$ dozen plain doughnuts and $1\frac{1}{3}$ dozen jelly doughnuts for the party. How many dozen doughnuts did she buy?

4. Jamie took $10\frac{1}{2}$ minutes to do question 1. He took only 8 minutes for question 2.

a How much time did he take?

b He had 20 minutes to do the whole page. How much time was left?

That's one way to get you to think about subtraction. Believe it or not, subtraction comes next!

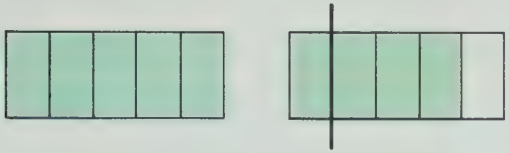


$1 - \frac{2}{3} = \underline{?}$ Can this be done? Rename 1 as $\frac{3}{3}$.

Why rename? Now can you subtract? $\frac{3}{3} - \frac{2}{3} = \underline{?}$

1. Subtract. (You will have to rename the whole number.)

a $1 - \frac{3}{5}$ **b** $1 - \frac{7}{9}$ **c** $2 - \frac{1}{2}$ **d** $3 - \frac{3}{4}$ **e** $1 - \frac{7}{12}$ **f** $4 - \frac{7}{9}$



$1\frac{4}{5} - \frac{3}{5}$ can also be pictured.
How many fifths shaded?
Take away $\frac{3}{5}$. How many fifths remain? $1\frac{4}{5} - \frac{3}{5} = \underline{?}$

This problem could have been written →

$$\begin{array}{r} 1\frac{4}{5} \\ -\ \frac{3}{5} \\ \hline ? \end{array}$$

Use this vertical form whenever you want to.

2. Subtract.

a $1\frac{2}{3} - \frac{1}{3}$ **b** $1\frac{4}{5} - \frac{3}{5}$ **c** $2\frac{1}{3} - \frac{1}{3}$ **d** $2\frac{3}{10} - \frac{2}{10}$ **e** $1\frac{5}{9} - \frac{4}{9}$ **f** $1\frac{5}{6} - \frac{4}{6}$

You might get out of practice if you don't do some renaming. Don't let that happen. Subtract some more. Write the simplest name for each answer.

g $1\frac{3}{4} - \frac{1}{4}$ **h** $1\frac{9}{10} - \frac{7}{10}$ **i** $2\frac{5}{6} - \frac{1}{6}$ **j** $3\frac{5}{8} - \frac{3}{8}$ **k** $3\frac{11}{12} - \frac{1}{12}$ **l** $4\frac{9}{10} - \frac{1}{10}$

The vertical form is best when you subtract mixed numbers. Study the example first.

$$\begin{array}{r} 4\frac{4}{5} \\ -\,1\frac{1}{5} \\ \hline 3\frac{3}{5} \end{array}$$

← First subtract the fractions.

← Then subtract the whole numbers.

You know you might have to rename in subtraction, too.

$$\begin{array}{r} 3\frac{3}{4} \\ -\,2\frac{1}{4} \\ \hline 1\frac{2}{4} \text{ or } 1\frac{1}{2} \end{array}$$

Try this one:

$$\begin{array}{r} 4\frac{2}{5} \\ -\,2\frac{2}{5} \\ \hline ? \end{array}$$

Is your answer a mixed number?

Show how good you are at subtraction. Do these. Give the simplest name for your answers.

	a	b	c	d	e	f	g
1.	$5\frac{7}{8} - 1\frac{1}{8}$	$1\frac{7}{9} - 1\frac{1}{9}$	$2\frac{5}{6} - 1\frac{1}{6}$	$4\frac{3}{5} - 1\frac{3}{5}$	$2\frac{5}{6} - 2\frac{5}{6}$	$6\frac{5}{9} - 3\frac{2}{9}$	$3\frac{5}{8} - 1\frac{1}{8}$
2.	$10\frac{9}{10} - 8\frac{3}{10}$	$6\frac{5}{8} - 4\frac{3}{8}$	$7\frac{2}{5} - 4\frac{1}{5}$	$8\frac{4}{9} - 2\frac{2}{9}$	$10\frac{4}{5} - 2\frac{1}{5}$	$6\frac{3}{10} - 5\frac{1}{10}$	$8\frac{1}{8} - 6\frac{1}{8}$

3. The dog chewed up $\frac{2}{5}$ of the rope. How much of the rope is left?

4. Bill had $\frac{1}{4}$ of the pizza. Jan had $\frac{1}{4}$. Dan had $\frac{1}{4}$. Is there any left for you? How much?

SUBTRACT

1. Rename when necessary.

a $2\frac{2}{3} - 1\frac{1}{3}$ **b** $2\frac{1}{2} - 1\frac{1}{2}$ **c** $4\frac{3}{4} - 2\frac{1}{4}$ **d** $3\frac{5}{6} - 1\frac{1}{6}$ **e** $4\frac{11}{12} - 2\frac{5}{12}$

2. Recopy these in vertical form if you want to. Don't forget to rename.

a $3\frac{4}{5} - 1\frac{2}{5}$ **b** $5\frac{7}{10} - 3\frac{3}{10}$ **c** $1\frac{7}{12} - 1\frac{5}{12}$ **d** $2\frac{7}{8} - 1\frac{3}{8}$

PROGRESS CHECK

Compute. Watch out! You are to do both addition and subtraction. Rename when necessary.

(1.) $1\frac{1}{2} + \frac{1}{2}$ (2.) $2\frac{1}{2} + \frac{1}{2}$ (3.) $3\frac{1}{2} + \frac{1}{2}$ (4.) $1\frac{1}{2} - \frac{1}{2}$

(5.) $2\frac{1}{2} - \frac{1}{2}$ (6.) $3\frac{1}{2} - \frac{1}{2}$ (7.) $2\frac{1}{4} + \frac{3}{4}$ (8.) $2\frac{3}{4} - \frac{1}{4}$

(9.) $1\frac{3}{4} - \frac{1}{4}$ (10.) $\frac{5}{6} + \frac{5}{6}$ (11.) $\frac{5}{6} - \frac{5}{6}$ (12.) $\frac{5}{6} - \frac{0}{6}$

Are you ready?

Try this one.

$$4\frac{1}{5} - 2\frac{2}{5}$$

Can you subtract the fractions?

There is a curve ball in the subtraction game.

When this problem comes up in whole numbers, you rename—a ten as ten ones, for example. Can $4\frac{1}{5}$ be renamed? Rename one of the whole numbers as $\frac{5}{5}$. How many fifths then?

$4\frac{1}{5}$ *Rename* → $3\frac{1}{5} + \frac{5}{5}$ *Rename* → $3\frac{6}{5}$

$-\ 2\frac{2}{5}$ *Rewrite* → $-\ 2\frac{2}{5}$ $-\ 2\frac{2}{5}$

O.K. now?

Try these. You'll have to rename.

1. LOOK

$5\frac{1}{3}$ → $4\frac{4}{3}$

$-\ 2\frac{2}{3}$ $-\ 2\frac{2}{3}$

NO! **O.K. now.**

2.

$3\frac{2}{7}$ → $2\frac{9}{7}$

$-\ 1\frac{5}{7}$ $-\ 1\frac{5}{7}$

3. $6\frac{1}{6} - 3\frac{5}{6}$ **4.** $4\frac{1}{5} - 2\frac{2}{5}$ **5.** $7\frac{2}{5} - 2\frac{3}{5}$ **6.** $3\frac{1}{4} - 1\frac{3}{4}$ **7.** $5\frac{1}{3} - 2\frac{2}{3}$

If you're not sure, ask about these.

8. $4 - 1\frac{3}{4}$ **9.** $6 - 4\frac{3}{5}$ **10.** $5 - 3\frac{1}{2}$ **11.** $3 - 2\frac{1}{2}$

12. $3 - 1\frac{2}{3}$ **13.** $4 - 2\frac{5}{6}$ **14.** $8 - 7\frac{7}{8}$ **15.** $10 - 1\frac{1}{10}$

PROGRESS CHECK

Subtract. Watch out for renaming.

1. $7\frac{2}{5} - 2\frac{1}{5}$

2. $3\frac{1}{4} - 1\frac{3}{4}$

3. $4\frac{7}{9} - 2\frac{4}{9}$

4. $5\frac{5}{6} - 3\frac{1}{6}$

5. $2\frac{1}{5} - 1\frac{2}{5}$

6. $2\frac{3}{8} - 1\frac{5}{8}$

7. $9\frac{1}{4} - 3\frac{3}{4}$

LOOK OUT!

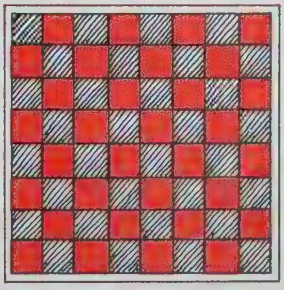
8. $5 - 1\frac{1}{2}$

9. $4\frac{2}{3} - 1$

*10. $2\frac{3}{4} - 1\frac{1}{2}$

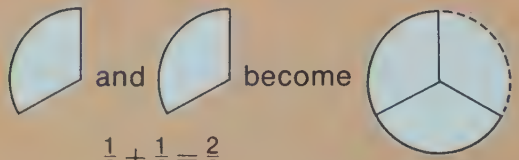
*11. $3\frac{5}{8} - 1\frac{1}{4}$

SUPER SLEUTH



What fraction of the board is black?

When we add or subtract fractions less than one, the pie model works too.



$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ or 1

All you have to do is put the pieces together. Right?

$\frac{1}{2}+\frac{1}{3}=$ **?** How do we name it?

$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ Look at the denominators. They are the same.

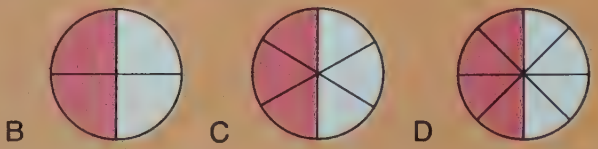
$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ Look at the denominators. They are the same.

$\frac{1}{2}+\frac{1}{3}=\underline{?}$ The denominators are the troublemakers.

All we have to do is get the denominators of both fractions to show the same number of parts.

Look at the unshaded part. What fraction names it?

What fraction names these unshaded parts?



You know that $\frac{1}{2}$ can be named $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$ or $\frac{5}{10}$ or $\frac{6}{12}$ or $\frac{7}{14}$ or $\frac{8}{16}$ or $\frac{9}{18}$ or $\frac{10}{20}$ or . . .

You know that $\frac{1}{3}$ can be named $\frac{2}{6}$ or $\frac{3}{9}$ or $\frac{4}{12}$ or $\frac{5}{15}$ or . . . Hold everything!
There is no need to go on.

$\frac{1}{2}+\frac{1}{3}$ can be renamed as $\frac{3}{6}+\frac{2}{6}$ You know how to name that answer.

$\frac{3}{8} + \frac{1}{4}$ ← Look. The denominators do not show the same number of parts.

Can $\frac{3}{8}$ be renamed as $\frac{?}{4}$? NO!
But can $\frac{1}{4}$ be renamed as $\frac{?}{8}$?
So rename. Get common denominators.

Rename $\frac{3}{8} + \frac{1}{4}$ → $\frac{2}{8}$

Rewrite $\frac{3}{8} + \frac{2}{8} = \underline{?}$

$\frac{1}{2} + \frac{1}{4}$ ← Get common denominators.

Can $\frac{1}{2}$ be renamed as $\frac{?}{4}$?

Rename $\frac{1}{2} + \frac{1}{4}$ → $\frac{2}{4}$

Rewrite $\frac{2}{4} + \frac{1}{4} = \underline{?}$

You will be finding a lot of common denominators.
Practise. Find a common denominator for each pair of fractions.

1. $\frac{1}{2}, \frac{1}{8}$ **THINK** Can $\frac{1}{2}$ be renamed as $\frac{?}{8}$? Do it.

2. $\frac{1}{6}, \frac{1}{3}$ **THINK** Can $\frac{1}{6}$ be renamed as $\frac{?}{3}$? NO!
Try again. Can $\frac{1}{3}$ be renamed as $\frac{?}{6}$? Do it.

3. $\frac{2}{3}, \frac{1}{9}$ **THINK** Can $\frac{2}{3}$ be renamed as $\frac{?}{9}$? Do it.

4. $\frac{3}{4}, \frac{1}{8}$ **THINK** Can $\frac{3}{4}$ be renamed as $\frac{?}{8}$? Do it.

5. $\frac{1}{12}, \frac{2}{3}$ **THINK** Can $\frac{1}{12}$ be renamed as $\frac{?}{3}$? NO!
Try again. Can $\frac{2}{3}$ be renamed as $\frac{?}{12}$? Do it.

6. $\frac{1}{2}, \frac{1}{6}$ **THINK** Can $\frac{1}{2}$ be renamed as $\frac{?}{6}$? Do it.

Fractions must have *common denominators* before you can add.

$\frac{1}{2} + \frac{3}{4} = \frac{?}{?}$ Find the common denominator.

Rename, using the common denominator. $\frac{2}{4}$

Rewrite the problem. $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$ Easy, isn't it?

1. Here is the problem. $\frac{2}{3} + \frac{1}{6} = \frac{?}{?}$ Find the common denominator.

Rename, using the common denominator. $\frac{?}{6}$

Rewrite the problem. $\frac{?}{6} + \frac{1}{6} = \frac{?}{6}$ Now you're ready to add!

2. Try another problem. $\frac{1}{8} + \frac{1}{4} = \frac{?}{?}$ Find the common denominator.

Rename, using the common denominator. $\frac{?}{8}$

Rewrite the problem. $\frac{1}{8} + \frac{?}{8} = \frac{?}{8}$ Now you can add.

3. Try one more. $\frac{2}{3} + \frac{1}{9} = \frac{?}{?}$ Find the common denominator.

Rename, using the common denominator. $\frac{?}{?}$

Rewrite the problem. $\frac{?}{?} + \frac{?}{?} = \frac{?}{?}$

Now you're ready to practise addition on your own.
Remember –

Step 1 Write the problem; find the common denominator.

Step 2 Rename, using the common denominator.

Step 3 Rewrite the problem; then add.

Step 4 Write the simplest name for your answer.

1. $\frac{1}{2}+\frac{1}{4}$ **2.** $\frac{1}{3}+\frac{7}{6}$ **3.** $\frac{3}{8}+\frac{1}{2}$

4. $\frac{4}{9}+\frac{1}{3}$ **5.** $\frac{3}{4}+\frac{5}{8}$ **6.** $\frac{5}{10}+\frac{2}{5}$

7. $\frac{1}{3}+\frac{1}{6}$ **8.** $\frac{1}{2}+\frac{7}{10}$ **9.** $\frac{5}{6}+\frac{1}{2}$

Subtraction is done the same way. Try these.
Don't forget the four easy steps.

10. $\frac{1}{2}-\frac{1}{6}$ **11.** $\frac{7}{12}-\frac{1}{4}$ **12.** $\frac{4}{3}-\frac{11}{12}$

13. $\frac{3}{5}-\frac{3}{10}$ **14.** $\frac{7}{6}-\frac{2}{3}$ **15.** $\frac{5}{12}-\frac{1}{6}$

16. Compute. Watch for the operation signs.

a $\frac{1}{2}+\frac{3}{10}$ **b** $\frac{3}{8}+\frac{1}{4}$ **c** $\frac{2}{3}-\frac{4}{9}$ **d** $\frac{5}{6}+\frac{2}{3}$

e $\frac{11}{12}-\frac{3}{4}$ **f** $\frac{4}{3}-\frac{5}{9}$ **g** $\frac{1}{12}+\frac{5}{6}$ **h** $\frac{4}{7}-\frac{3}{14}$

PROGRESS CHECK

Practise your renaming skills as you add these fractions.

(1.) $\frac{2}{3} + \frac{2}{3}$ Common denominator? You're ready to add.

$$\frac{2}{3} + \frac{2}{3} = \frac{2+2}{3} = \frac{4}{3}$$ Now what?

That was too easy. That's no fun.

(2.) $\frac{2}{3} + \frac{5}{6}$ Common denominator? Find one. *Rename* $\frac{2}{3} = \frac{?}{6}$

Write the problem with a common denominator. $\frac{4}{6} + \frac{5}{6} = \frac{?}{6}$

NOW WHAT?

Your turn. Find the sums, renaming the fractions when you need to. Rename your answers.

(3.) $\frac{1}{5} + \frac{3}{10} = \underline{?}$ (4.) $\frac{1}{3} + \frac{1}{9} = \underline{?}$ (5.) $\frac{1}{4} + \frac{3}{8} = \underline{?}$ (6.) $\frac{2}{3} + \frac{1}{6} = \underline{?}$

(7.) $\frac{1}{3} + \frac{5}{12} = \underline{?}$ (8.) $\frac{1}{6} + \frac{5}{12} = \underline{?}$ (9.) $\frac{1}{4} + \frac{1}{12} = \underline{?}$ (10.) $\frac{5}{9} + \frac{2}{3} = \underline{?}$

(11.) $\frac{4}{5} + \frac{1}{10} = \underline{?}$ (12.) $\frac{1}{2} + \frac{3}{8} = \underline{?}$ *(13.) $\frac{7}{10} + \frac{3}{100} = \underline{?}$ *(14.) $\frac{8}{10} + \frac{19}{100} = \underline{?}$

Just for fun

Lucky Fractions

This lucky card contains all the answers to the addition and subtraction problems on this page. After you do a problem, look for your answer on the lucky card and put a marker on it. The first one has been done for you. Do only as many problems as you need to win the game.

LUCKY				
$\frac{35}{10}$	$\frac{5}{8}$	$\frac{84}{100}$	$\frac{3}{20}$	$\frac{9}{12}$
$\frac{5}{18}$	$\frac{5}{15}$	$\frac{7}{8}$	$\frac{3}{10}$	$\frac{3}{8}$
$\frac{6}{5}$	$\frac{8}{12}$	$\frac{7}{10}$	$\frac{15}{6}$	$\frac{11}{12}$
$\frac{1}{9}$	$\frac{7}{9}$	$\frac{11}{12}$	$\frac{17}{9}$	$\frac{5}{6}$
$\frac{3}{100}$	$\frac{2}{4}$	$\frac{10}{6}$	$\frac{11}{14}$	$\frac{3}{10}$

1. $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$
2. $\frac{5}{3} + \frac{2}{9} = \underline{?}$
3. $\frac{3}{4} - \frac{1}{4} = \underline{?}$
4. $\frac{7}{8} - \frac{1}{2} = \underline{?}$
5. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \underline{?}$
6. $\frac{7}{10} + \frac{14}{100} = \underline{?}$
7. $\frac{6}{5} - \frac{13}{15} = \underline{?}$
8. $\frac{4}{9} - \frac{1}{3} = \underline{?}$
9. $\frac{7}{10} - \frac{2}{5} = \underline{?}$
10. $\frac{43}{100} - \frac{4}{10} = \underline{?}$
11. $\frac{5}{6} + \frac{1}{12} = \underline{?}$
12. $\frac{1}{3} + \frac{1}{6} + \frac{2}{6} = \underline{?}$
13. $\frac{5}{6} + \frac{5}{3} = \underline{?}$
14. $\frac{5}{2} - \frac{5}{6} = \underline{?}$
15. $\frac{7}{10} - \frac{2}{5} = \underline{?}$
16. $\frac{1}{3} + \frac{5}{12} = \underline{?}$
17. $\frac{3}{4} - \frac{1}{12} = \underline{?}$
18. $\frac{4}{7} + \frac{3}{14} = \underline{?}$
19. $\frac{3}{8} + \frac{1}{4} = \underline{?}$
20. $\frac{6}{2} + \frac{5}{10} = \underline{?}$
21. $\frac{1}{9} + \frac{2}{3} = \underline{?}$
22. $\frac{4}{3} - \frac{5}{12} = \underline{?}$
23. $\frac{11}{18} - \frac{1}{3} = \underline{?}$
24. $\frac{13}{20} - \frac{5}{10} = \underline{?}$
25. $\frac{1}{8} + \frac{6}{8} = \underline{?}$

For Experts Only

A $2\frac{1}{2}$ units

B $1\frac{1}{4}$ units

How much more is A than B?
What is the difference between A and B?

$2\frac{1}{2}$ ——— *Rename.* ——→ $2\frac{2}{4}$

You need a common denominator.

$-\ 1\frac{1}{4}$ ——— *Rewrite.* ——→ $-\ 1\frac{1}{4}$

■■ units

Rename before you subtract. You might have to rename the answer, too.

C $2\frac{1}{4}$ units

D $1\frac{1}{8}$ units

1. What is the difference between C and D?

E $2\frac{7}{8}$ units

F $1\frac{3}{4}$ units

2. What is the difference between E and F?

G $1\frac{3}{4}$ units

H $1\frac{3}{8}$ units

3. What is the difference between G and H?

I 3 units

J $1\frac{7}{8}$ units

4. What is the difference between I and J?

CAN YOU THINK ABOUT MULTIPLICATION AS ADDITION?

1. $4 \times 6 = \underline{?}$ You know the answer is 24. If you didn't know the answer, you could find it by adding.

$4 \times 6 = 24$ could be written

$6 + 6 + 6 + 6 = 24$

2. You can always find the product of two whole numbers by adding. But if you used addition to complete $232 \times 17 = \underline{?}$ adding 232 seventeens would take a long time. So you multiply.

3. Lester likes pie. His mother served each of the 4 people at dinner $\frac{1}{5}$ of a pie. How much of the pie did they eat?

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

This can be rewritten as $4 \times \frac{1}{5} = \frac{?}{5}$. How much pie?

4. Every day Tiny Thirston drinks $\frac{3}{8}$ of a bottle of ZIP ZAP juice.

a How much does he drink in 2 days?

$$\frac{3}{8} + \frac{3}{8}$$

or $2 \times \frac{3}{8} = \frac{?}{8}$ bottle. How much?

b How much in a week?

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

or $7 \times \frac{3}{8} = \frac{?}{8}$ bottles. How many bottles is that?

Can the idea of repeated addition be used in multiplying fractions?

$\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=\frac{12}{5}$ — This is the sum of the numerators.

$3\times\frac{4}{5}=\frac{12}{5}$ — Where does this number come from?

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{8}{3}$ — You know where this comes from.

$4\times\frac{2}{3}=\frac{8}{3}$ — What about this?

$3\times\frac{3}{4}=\frac{?}{4}$ — What is the correct numerator?

1. Use multiplication or addition to solve these problems.

a $2\times\frac{1}{3}=\underline{?}$ **b** $3\times\frac{2}{3}=\underline{?}$ **c** $4\times\frac{1}{2}=\underline{?}$

d $5\times\frac{1}{6}=\underline{?}$ **e** $3\times\frac{3}{8}=\underline{?}$ **f** $2\times\frac{2}{5}=\underline{?}$

2. Look at the first problem again. $2\times\frac{1}{3}=\frac{2}{3}$
Is the numerator in the answer the same as the product of $2\times\frac{1}{3}=\frac{2}{3}$?
Could it be rewritten as $\frac{2\times1}{3}=\frac{2}{3}$?

3. Could other problems be rewritten this way?
Complete $4\times\frac{2}{9}=\underline{?}$ Now complete $\frac{4\times2}{9}=\underline{?}$
Are both the answers the same? They should be.

4. Complete.

a $3\times\frac{2}{5}=\frac{3\times2}{5}=\underline{?}$ **b** $2\times\frac{2}{7}=\frac{2\times2}{7}=\underline{?}$ **c** $6\times\frac{4}{9}=\frac{6\times4}{?}=\underline{?}$

d $5\times\frac{2}{3}=\frac{?\times?}{3}=\underline{?}$ **e** $9\times\frac{5}{6}=\frac{?\times?}{?}=\underline{?}$ **f** $7\times\frac{3}{10}=\frac{?\times?}{?}=\underline{?}$

1. Find these products.

a $3 \times \frac{1}{2} = \underline{?}$ That's easy, isn't it?

b $\frac{1}{2} \times 3 = \underline{?}$ That's a new one.
How do you do it?

Try this $\frac{1}{2} \times 3$ means $\frac{1}{2}$ of 3. Suppose you want to find $\frac{1}{2}$ of 3 apples. One way to do it is to take $\frac{1}{2}$ of each apple. Then add the $\frac{1}{2}$s.

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \times \frac{1}{2}$$

Hey—we know that product is $\frac{3}{2}$.
$3 \times \frac{1}{2} = \frac{3}{2}$ and $\frac{1}{2} \times 3 = \frac{3}{2}$

You knew that you could change the order of two whole-number factors and get the same product. Now you know you can change the order of two fraction factors, too.

$$3 \times \frac{1}{2} = \frac{3 \times 1}{2} = \frac{3}{2} \quad \text{and} \quad \frac{1}{2} \times 3 = \frac{1 \times 3}{2} = \frac{3}{2}$$

2. Find these products.

a $2 \times \frac{1}{5} = \frac{2 \times 1}{5} = \underline{?}$

b $\frac{1}{5} \times 2 = \frac{1 \times 2}{5} = \underline{?}$

c $\frac{4}{5} \times 3 = \frac{4 \times 3}{5} = \underline{?}$

d $\frac{3}{8} \times 4 = \frac{3 \times ?}{8} = \underline{?}$

e $\frac{5}{6} \times 7 = \frac{? \times ?}{?} = \underline{?}$

f $\frac{2}{7} \times 5 = \underline{?}$

g $\frac{5}{9} \times 7 = \underline{?}$

h $\frac{2}{3} \times 4 = \underline{?}$

i $\frac{2}{3} \times 6 = \underline{?}$

PROGRESS CHECK

Compute.

1. $\frac{1}{2} \times 6 = \underline{?}$
2. $\frac{3}{10} \times 8 = \underline{?}$
3. $\frac{5}{11} \times 2 = \underline{?}$
4. $3 \times \frac{7}{9} = \underline{?}$
5. $4 \times \frac{2}{7} = \underline{?}$
6. $\frac{2}{3} \times 10 = \underline{?}$
7. $5 \times \frac{3}{2} = \underline{?}$
8. $\frac{7}{8} \times 5 = \underline{?}$
9. $\frac{2}{11} \times 7 = \underline{?}$
10. $10 \times \frac{9}{10} = \underline{?}$

Have you ever heard of situations like these?

1. You plan to study half of the next half hour.
   How much time would that be? $\frac{1}{2}$ of what?

2. Look—there's a fourth of the pie left.
   You're going to eat half of it.
   How much of the pie will you eat? $\frac{1}{2}$ of what?

3. You and your sister are painting a fence. $\frac{1}{2}$ of it is your share. You have painted $\frac{1}{2}$ of your share.
   How much have you painted? $\frac{1}{2}$ of what?

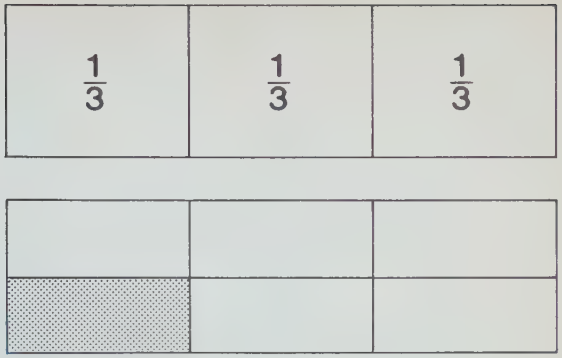
4. You have $3 to spend. You have spent $\frac{1}{2}$ of it.
   How much of the money have you spent? $\frac{1}{2}$ of what?

5. You have $\frac{2}{3}$ left. You give your best friend $\frac{1}{2}$ of that.
   How much did you give your friend? $\frac{1}{2}$ of what?

1. Greedy shared his candy bar with his two friends. First he marked it into thirds. He could have shared it then, but he didn't.

He marked each $\frac{1}{3}$ into 2 equal parts.
He gave each friend one of these parts.

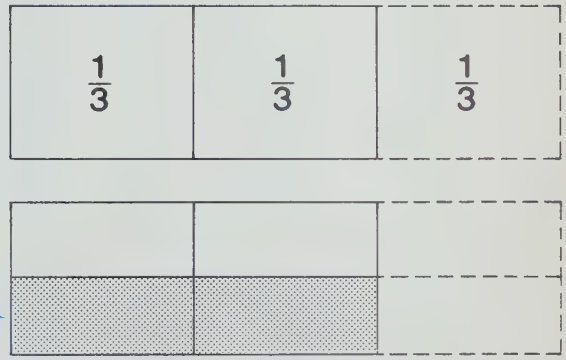
Greedy gave $\frac{1}{2}$ of $\frac{1}{3}$ candy bar to each friend.
What fractional part did each friend get?
The picture shows $\frac{1}{6}$. Does $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$?



2. Another day Greedy wasn't so greedy. He decided to give $\frac{1}{2}$ of $\frac{2}{3}$ of his candy bar to each of his two friends.

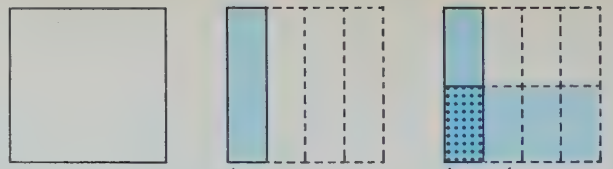
He marked $\frac{1}{2}$ of each $\frac{1}{3}$.
Then he gave each friend $\frac{1}{2}$ of $\frac{2}{3}$ of his candy bar.

Does $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$?
He gave each of his friends $\frac{2}{6}$ of his candy bar.
How much did he have left for himself?
Did he do it the hard way?



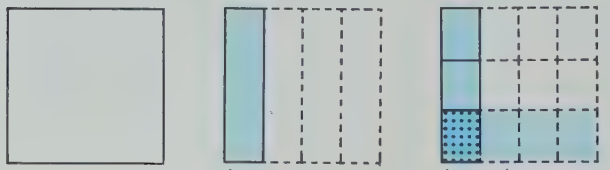
3. You know that $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ and that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.
What is $\frac{1}{2} \times \frac{3}{3}$?
Draw a model to show your answer.
(First show $\frac{3}{3}$. Then show $\frac{1}{2}$ of the $\frac{3}{3}$.)

1.



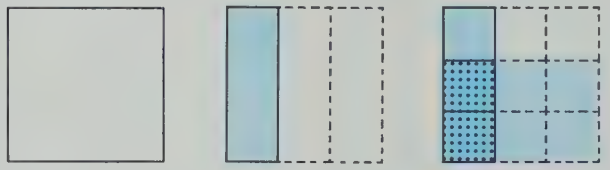
1 square | $\frac{1}{4}$ square | $\frac{1}{2}$ of $\frac{1}{4}$ square | $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

2.



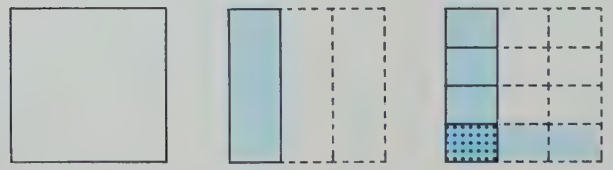
1 square | $\frac{1}{4}$ square | $\frac{1}{3}$ of $\frac{1}{4}$ square | $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ | What's $\frac{2}{3} \times \frac{1}{4}$?

3.



1 square | $\frac{1}{3}$ square | $\frac{2}{3}$ of $\frac{1}{3}$ square | $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ | What's $\frac{1}{3} \times \frac{1}{3}$?

4.



1 square | $\frac{1}{3}$ square | $\frac{1}{4}$ of $\frac{1}{3}$ square | $\frac{1}{4} \times \frac{1}{3} = \underline{?}$ | What's $\frac{3}{4} \times \frac{1}{3}$?

5. Look at the *denominators* of the first problem. Does $2 \times 4 = 8$?
Look at the *numerators* of the first problem. Does $1 \times 1 = 1$?

6. Look at the denominators of each of the other problems.
Then look at the numerators. Have you made a discovery?

1. Study this problem and its model. Watch the numerators and denominators.

This model shows $\frac{2}{3}$.

The problem is $\frac{1}{4}$ of $\frac{2}{3}$.

Now the model has been divided into 4 equal parts. You're interested in one of the parts.
How many parts in all now? How many shaded? $\frac{1}{4} \times \frac{2}{3} = \underline{?}$

2. Draw a model to show $\frac{1}{4}$ of $\frac{1}{2}$ $\left(\frac{1}{4} \times \frac{1}{2}\right)$.

3. Try finding these products without drawing a model.

a $\frac{1}{2} \times \frac{1}{4}$ **b** $\frac{1}{2} \times \frac{1}{2}$ **c** $\frac{1}{2} \times \frac{3}{4}$

d $\frac{2}{4} \times \frac{1}{4}$ **e** $\frac{2}{4} \times \frac{2}{4}$ **f** $\frac{2}{4} \times \frac{3}{4}$

4. If you could compute the products in the last problem, you can answer these questions.

a How much is $\frac{1}{3}$ of $\frac{1}{2}$ of a day?

b $\frac{1}{2}$ of $\frac{3}{4}$ piece of cloth is how much?

c How much is $\frac{1}{2}$ of $\frac{1}{2}$ of an hour?

Are you sure of your work when you multiply two fractions? If not, show your work like this:

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Multiply.

	a	b	c	d
1.	$\frac{1}{2} \times \frac{2}{5}$	$\frac{2}{5} \times \frac{1}{3}$	$\frac{1}{4} \times \frac{1}{4}$	$\frac{2}{9} \times \frac{2}{3}$
2.	$\frac{2}{3} \times \frac{3}{4}$	$\frac{7}{8} \times \frac{3}{4}$	$\frac{2}{3} \times \frac{2}{3}$	Look out! $\frac{4}{9} \times \frac{0}{2}$

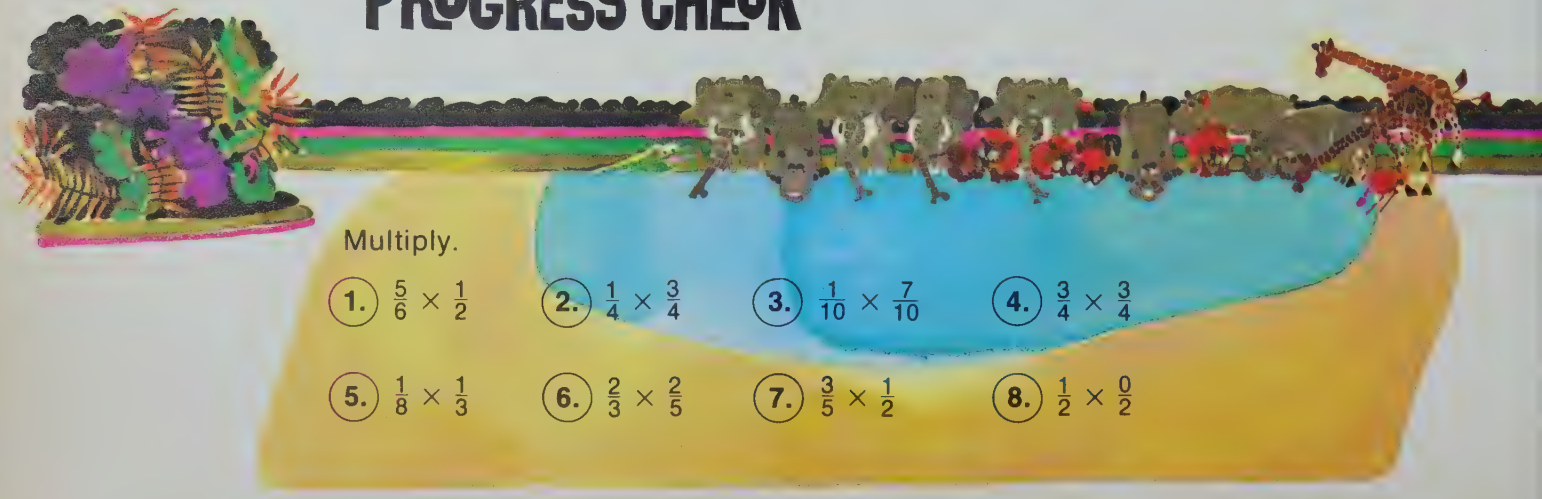
3. He bought $\frac{1}{2}$ loaf of bread. He gave $\frac{1}{4}$ of it to his brother. How much did he give away?

4. She had $\frac{1}{4}$ of a dollar. She gave $\frac{1}{5}$ of it to her sister. What part of a dollar did she give to her sister? How many cents was that?

5. There was $\frac{2}{3}$ jar of jam. She used $\frac{1}{2}$ of it. How much did she use?

6. He had $\frac{1}{2}$ of a dollar. He put $\frac{1}{10}$ of it in his bank. What part of a dollar did he save? How many cents was that?

PROGRESS CHECK



Multiply.

1. $\frac{5}{6} \times \frac{1}{2}$	**2.** $\frac{1}{4} \times \frac{3}{4}$	**3.** $\frac{1}{10} \times \frac{7}{10}$	**4.** $\frac{3}{4} \times \frac{3}{4}$
5. $\frac{1}{8} \times \frac{1}{3}$	**6.** $\frac{2}{3} \times \frac{2}{5}$	**7.** $\frac{3}{5} \times \frac{1}{2}$	**8.** $\frac{1}{2} \times \frac{0}{2}$

1. Compute. *Make sure every product has its simplest name.*
This set of problems has some interesting patterns.
Watch for them.

a $\frac{1}{5} \times \frac{1}{4}$ **b** $\frac{1}{5} \times \frac{2}{4}$ **c** $\frac{1}{5} \times \frac{3}{4}$ **d** $\frac{1}{5} \times \frac{4}{4}$

e $\frac{2}{5} \times \frac{1}{4}$ **f** $\frac{2}{5} \times \frac{2}{4}$ **g** $\frac{2}{5} \times \frac{3}{4}$ **h** $\frac{2}{5} \times \frac{4}{4}$

i Did you find any patterns? Which ones?

Each of these problems is different. Compute.
Products should be named with their simplest names.

	a	**b**	**c**	**d**
2.	$\frac{1}{4} \times \frac{4}{5}$	$\frac{1}{6} \times \frac{2}{3}$	$\frac{2}{5} \times \frac{3}{4}$	$\frac{9}{9} \times \frac{3}{4}$
3.	$\frac{3}{5} \times \frac{1}{3}$	$\frac{2}{3} \times \frac{5}{6}$	$\frac{1}{2} \times \frac{2}{7}$	$\frac{2}{5} \times \frac{1}{4}$
4.	$\frac{5}{6} \times \frac{2}{3}$	$\frac{1}{4} \times \frac{2}{5}$	$\frac{1}{3} \times \frac{3}{5}$	$\frac{2}{7} \times \frac{1}{2}$

5. If you were hungry, would you rather have—

a $\frac{1}{4}$ of $\frac{1}{2}$ hamburger or $\frac{1}{2}$ of $\frac{1}{4}$?

b $\frac{1}{3}$ of $\frac{2}{3}$ milkshake or $\frac{2}{3}$ of $\frac{1}{3}$?

c $\frac{4}{5}$ of $\frac{1}{2}$ order of french fries or $\frac{1}{2}$ of $\frac{4}{5}$?

d $\frac{3}{4}$ of $\frac{1}{6}$ piece of cake or $\frac{1}{6}$ of $\frac{3}{4}$?

e $\frac{1}{8}$ of $\frac{1}{2}$ pizza or $\frac{1}{2}$ of $\frac{1}{8}$?

EXTRA PRACTICE

Compute. Products should be named with their simplest names.

	a	b	c	d
1.	$\frac{1}{3} \times \frac{2}{5}$	$\frac{2}{3} \times \frac{1}{7}$	$\frac{3}{7} \times \frac{2}{5}$	$\frac{5}{9} \times \frac{1}{3}$
2.	$\frac{1}{3} \times \frac{4}{5}$	$\frac{3}{5} \times \frac{2}{3}$	$\frac{2}{7} \times \frac{1}{4}$	$\frac{8}{9} \times \frac{1}{2}$
3.	$\frac{5}{6} \times \frac{1}{2}$	$\frac{5}{7} \times \frac{2}{3}$	$\frac{5}{8} \times \frac{3}{4}$	$\frac{5}{9} \times \frac{0}{5}$

Try these word problems. Each problem needs two answers.

4. George had $\frac{3}{4}$ dozen eggs.
He used $\frac{1}{3}$ of the eggs for an omelette.
What part of a dozen did George use?
How many eggs was that?

5. Monday, Bob's homework took $\frac{3}{4}$ of an hour.
Tuesday it took $\frac{2}{3}$ of that time.
What part of an hour did it take Tuesday?
How many minutes was that?

6. Lynn had $\frac{3}{5}$ of a dollar.
She spent $\frac{1}{2}$ of her money for a pen.
What part of a dollar did she spend?
How many cents was that?

7. Frank has to paint $\frac{2}{5}$ of a 100-board fence.
Alice has to paint only $\frac{1}{2}$ as much as Frank.
What fraction of the job is Alice's?
How many boards is that?

Think about these problems. Write Yes or No for each answer.

8.	Is $\frac{1}{2}$ of $\frac{1}{3}$ the same as	**a** $\frac{1}{3}$ of $\frac{1}{2}$?	**b** $\frac{1}{6}$ of 6?
9.	Is $\frac{1}{4}$ of $\frac{2}{3}$ the same as	**a** $\frac{1}{4}$ of $\frac{5}{9}$?	**b** $\frac{2}{3}$ of $\frac{1}{4}$?
10.	Is $\frac{1}{5}$ of $\frac{1}{7}$ the same as	**a** $\frac{1}{5}$ of 7?	**b** $\frac{1}{7}$ of $\frac{0}{5}$?

CHECKOUT

Add or subtract. Write the simplest name for your answer.

	a	b	c	d
1.	$4\frac{3}{8}$	$2\frac{1}{3}$	$7\frac{4}{5}$	$3\frac{7}{8}$
	$+ 2\frac{7}{8}$	$+ 1\frac{4}{9}$	$- 3\frac{3}{5}$	$- 1\frac{3}{4}$

Multiply. Write the simplest name for your answer.

2. $2 \times \frac{1}{4}$ $\quad \frac{2}{3} \times 4$ $\quad \frac{5}{6} \times \frac{1}{2}$ $\quad \frac{3}{5} \times \frac{1}{3}$

SUPER SLEUTH

Look at this set of problems. The answer to the first problem became the top number of the next problem.

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ $\quad \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$ $\quad \frac{5}{8} + 1\frac{1}{4} = ?$ $\quad ? - \frac{7}{8} = 1$

If you complete this series correctly, your last answer will match the one printed.

$5\frac{7}{12} + \frac{2}{3} = ?$ $\quad ? - 1\frac{5}{12} = ?$ $\quad ? - 2\frac{1}{3} = ?$ $\quad ? - \frac{5}{8} = ?$ $\quad ? + 4\frac{1}{2} = 5\frac{11}{8}$ or $6\frac{3}{8}$

8
DECIMALS

A fraction can be written in many ways. For example, $\frac{3}{4}$ of a dollar can be expressed as \$0.75.

Take time to look at decimal notation.

Money coins are something everyone knows about. Their value is written with decimal notation. It is a fractional part of one dollar.

$\frac{1}{10}$ of a dollar is the same as \$0.10.

$\frac{1}{100}$ of a dollar is the same as \$0.01.

Is there a difference between \$10 and \$0.10? Of course there is! \$10 means ten dollars and \$0.10 means ten cents. For safety's sake it's always a good idea to put a 0 to the left of the decimal point if there is no other digit there.

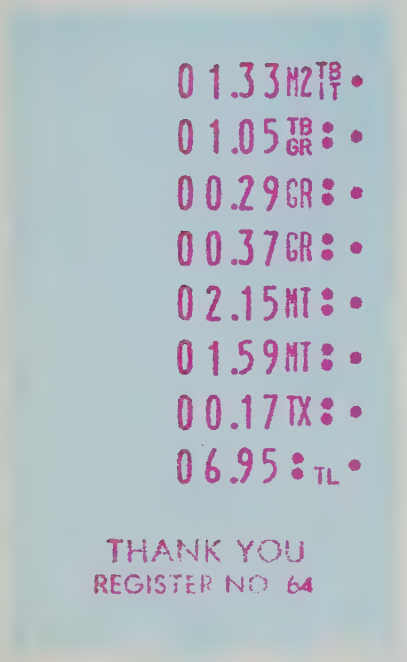
Write each amount of money with a \$ and a decimal point.

1. If $\frac{1}{10}$ of a dollar is \$0.10, what is $\frac{2}{10}$ of a dollar? What is $\frac{1}{5}$ of a dollar?
2. If $\frac{1}{10}$ of a dollar is \$0.10, what is $\frac{5}{10}$ of a dollar? What is $\frac{1}{2}$ of a dollar?
3. If $\frac{1}{10}$ of a dollar is \$0.10, what is $\frac{10}{10}$ of a dollar?
4. How would you write:
   a $1\frac{2}{10}$ dollars?
   b $2\frac{5}{10}$ dollars?
   c $3\frac{15}{100}$ dollars?
   d $4\frac{6}{10}$ dollars?
   e $\frac{12}{10}$ dollars?
5. What are some slang phrases you have heard used for coins? Try to find out how some phrases started.

GOAL

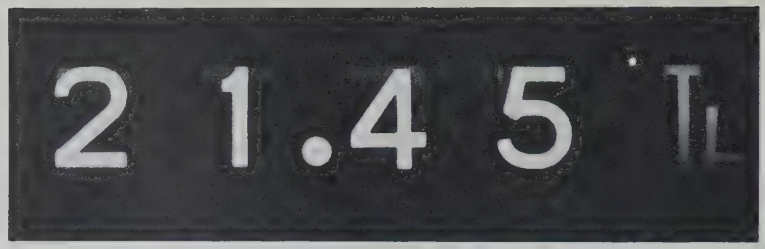
If you know a lot about money, this chapter will be easy. Your goal is to operate with decimal fractions.

You go shopping.
You get a piece of paper like this:



What is it?
What does it say?

Look at this figure from a cash register.



The 2 tells you how many ten-dollar units.
The 1 tells you how many one-dollar units.
The 4 tells how many ten-cent units.
What does the 5 tell?

1. Now consider $34.68. How many ten-dollar units? How many one-dollar units? How many ten-cent units? How many one-cent units?

2. A ten-dollar unit is worth ? dollars. A one-dollar unit is worth ? 10¢ coins. A 10¢ coin is worth ? cents.

3. A dollar is worth ? cents. Ten dollars are worth ? 10¢ coins.

We use decimals for things other than money.
But they work the same way. Look at this decimal.

21.45

The 2 means 2 tens.
The 1 means 1 one.
The dot is called a decimal point.
The 4 means 4 *tenths*.
The 5 means 5 *hundredths*.

Compare this with the cash-register figure.

Decimal notation for a fraction is not limited to money.

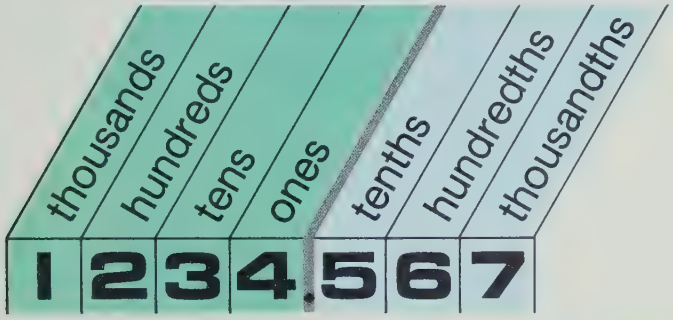
Time may be reported in tenths of a second.

Mass may be reported in tenths or hundredths or thousandths of a kilogram.

Length may be reported in hundredths or thousandths of a metre.

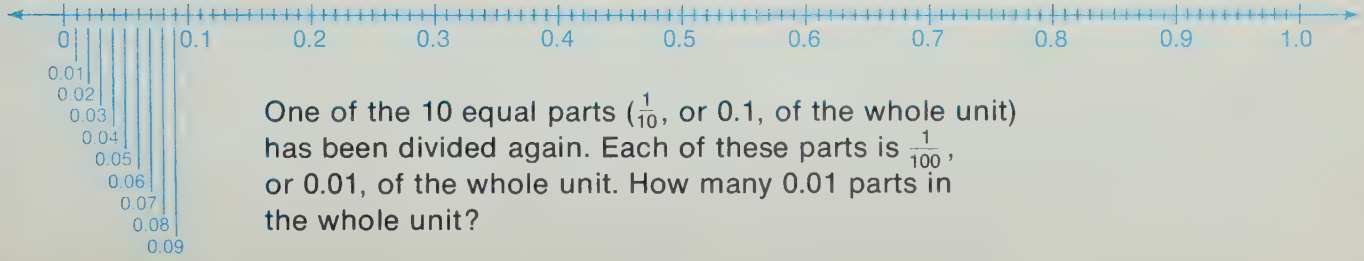
Capacity may be reported in tenths or thousandths of a litre.

The place-value chart you have worked with can be expanded.

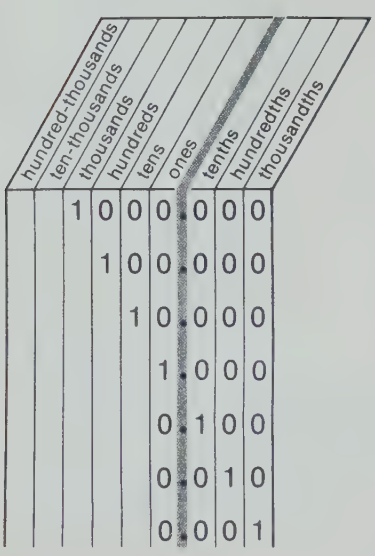


Decimals name fractional parts of 1.
0.5 is the same as $\frac{5}{10}$.
0.56 names the same number as $\frac{56}{100}$.
0.567 names the same number as $\frac{567}{1000}$.

These fractions are not new to you. The number line below shows one unit divided into 10 equal parts. Each part is $\frac{1}{10}$, or 0.1, of the whole unit.



One of the 10 equal parts ($\frac{1}{10}$, or 0.1, of the whole unit) has been divided again. Each of these parts is $\frac{1}{100}$, or 0.01, of the whole unit. How many 0.01 parts in the whole unit?



1. Read each of the numbers on the chart.

2. Is 1000.000 the same number as 1000?

3. Is 10.000 the same number as 10?

4. Is 0.1 the same number as 0.100?

5. Is 0.01 the same number as 0.010?

6. Is 100 the same number as 0.001?

7. Name the larger number.
Use the chart for help if you need it.

a 10 or 1 **b** 0.1 or 1
c 10 or 0.01 **d** 100 or 10
e 0.1 or 0.01 **f** 1 or 0.1
g 1000 or 100 **h** 0.01 or 0.001

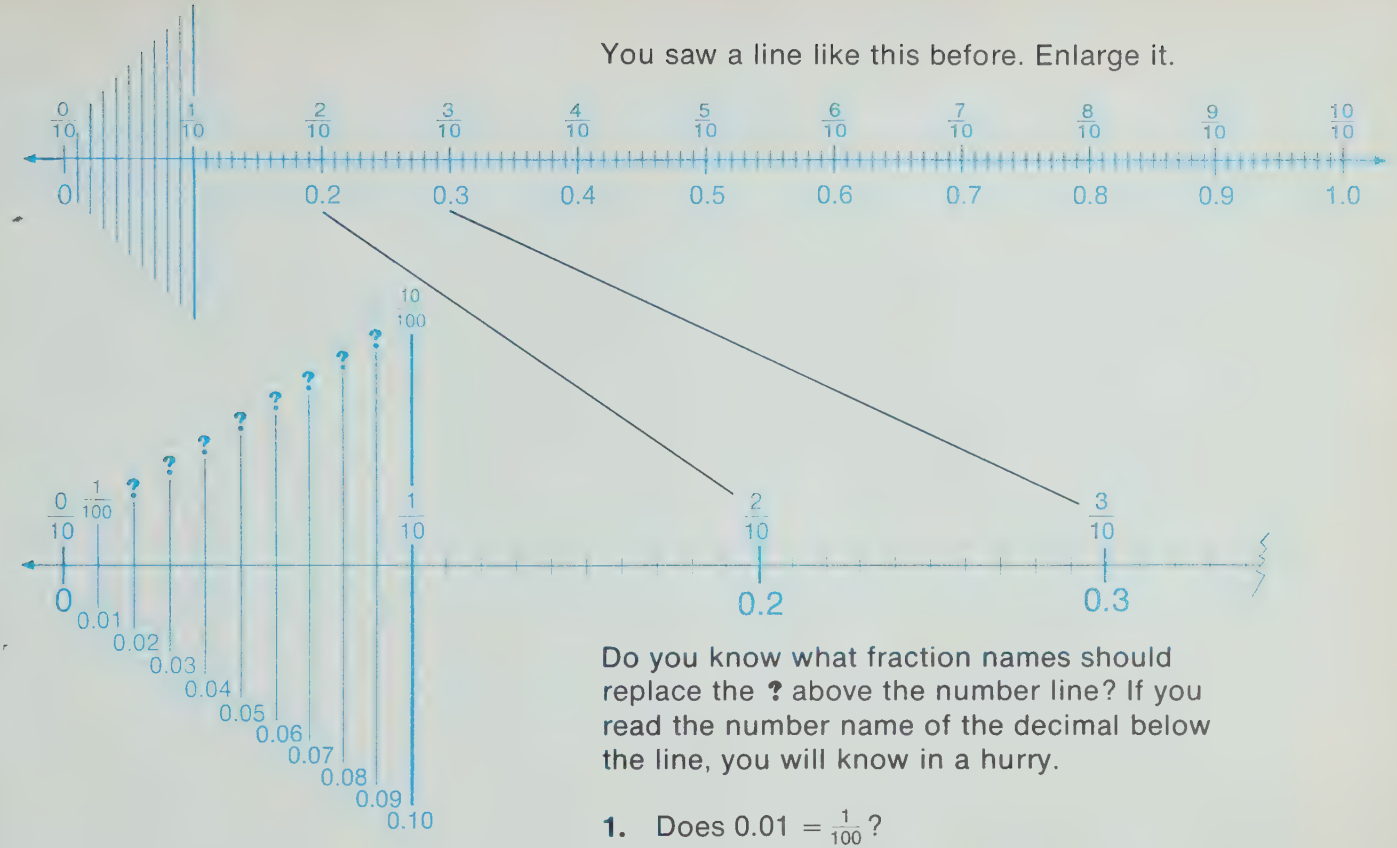
8. If you had 0.1 dollar, would you have the same amount as someone else who had 0.10 dollar?

9. If you had 0.80 dollar, would you have the same amount as someone else who had 0.8 dollar?

10. Answer yes or no to these.

a Does 0.10 = 0.100? **b** Does 0.1 = 0.100?
c Does 1.0 = 1.00? **d** Does 0.400 = 0.40?
e Does 0.1 = 0.11? **f** Does 0.11 = 0.111?

You saw a line like this before. Enlarge it.



Do you know what fraction names should replace the **?** above the number line? If you read the number name of the decimal below the line, you will know in a hurry.

1. Does $0.01 = \frac{1}{100}$?

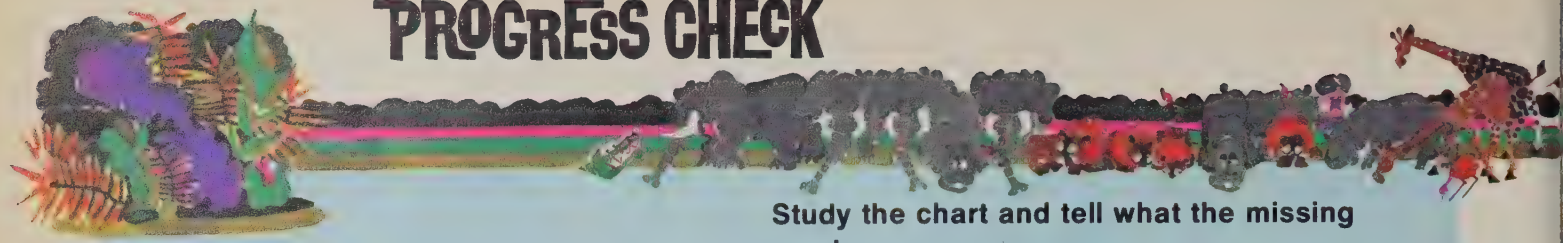
a Does $0.10 = \frac{10}{100}$? **b** Does $\frac{1}{10} = 0.1$?

2. Do 0.1, 0.10, $\frac{1}{10}$, and $\frac{10}{100}$ all name the same number?

3. What is the place value of each digit in the following numbers?

a 12.56 **b** 100.01 **c** 21.10

PROGRESS CHECK



Decimals are really just another kind of fraction. In this chart the decimals in the first column are equal to the fractions in the second column. The third column tells the meaning of each.

Study the chart and tell what the missing numbers are.

	Decimal	Fraction	What it means
1.	0.1	$\frac{1}{10}$	1 tenth
2.	0.01	$\frac{1}{100}$	1 hundredth
3.	0.001	$\frac{1}{1000}$	1 thousandth
4.	0.6	$\frac{?}{10}$	6 tenths
5.	0.02	$\frac{2}{?}$	2 hundredths
6.	0.005	$\frac{5}{1000}$	? thousandths
7.	0.48	?	48 hundredths
8.	0.039	$\frac{39}{1000}$	? thousandths
9.	0.125	$\frac{125}{1000}$	? thousandths
10.	0.708	$\frac{708}{1000}$	? thousandths

11. **What is the largest number in the chart?**

Now is the time to start adding with decimals.

Suppose you walk 0.3 km to a friend's house.
Then you walk 0.4 km to the park.
How far do you walk in all?

$$\begin{array}{r} 0.3 \\ +\ 0.4 \\ \hline 0.7 \end{array} \qquad \left(\text{or } \frac{3}{10} + \frac{4}{10} = \frac{7}{10}\right)$$

You have already walked 0.7 km . Now you walk another 0.5 km going home. How far have you walked in all?

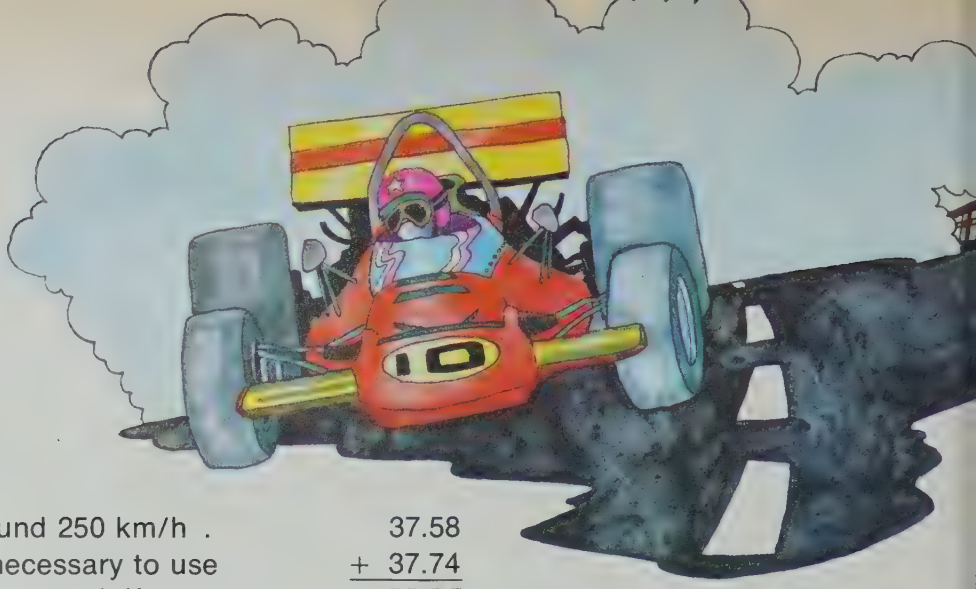
$$\begin{array}{r} 0.7 \\ +\ 0.5 \\ \hline ? \end{array} \qquad \left(\text{or } \frac{7}{10} + \frac{5}{10} = \frac{12}{10}\right)$$

What is the answer this time?

If you remember the place value of the numbers you are adding, there are no new rules. It is the same as adding whole numbers.

Here is some practice. Add.

1. 0.6 + 0.3	**2.** 0.4 + 0.5	**3.** 0.8 + 0.3	**4.** 4.5 + 0.2	**5.** 0.4 + 7.3
6. 6.1 + 3.7	**7.** 5.8 + 0.6	**8.** 0.9 + 4.4	**9.** 3.56 + 2.73	**10.** 9.81 + 7.47



You should be able to add and subtract decimals with hundredths because it is just like using money. Here is an example.

Formula I racing cars hit speeds around 250 km/h . With such speeds, officials found it necessary to use clocks that measure hundredths of a second. If a car turned one lap in 37.58 seconds (s) and another lap in 37.74 s, how many seconds did the car take to make the two laps?

$$\begin{array}{r} 37.58 \\ +\ 37.74 \\ \hline \blacksquare\blacksquare.\blacksquare\blacksquare \end{array}$$

Jim ran the 100 m race in 12.4 s .
Betty ran it in 11.9 s .
How much faster than Jim did Betty run the 100 m?

Here are some practice exercises. Subtract.

1. $\begin{array}{r} 0.9 \\ -\ 0.7 \\ \hline \end{array}$	**2.** $\begin{array}{r} 0.8 \\ -\ 0.3 \\ \hline \end{array}$	**3.** $\begin{array}{r} 1.8 \\ -\ 0.4 \\ \hline \end{array}$	**4.** $\begin{array}{r} 2.6 \\ -\ 0.5 \\ \hline \end{array}$	**5.** $\begin{array}{r} 3.6 \\ -\ 0.7 \\ \hline \end{array}$
6. $\begin{array}{r} 5.5 \\ -\ 0.9 \\ \hline \end{array}$	**7.** $\begin{array}{r} 8.2 \\ -\ 3.5 \\ \hline \end{array}$	**8.** $\begin{array}{r} 6.3 \\ -\ 5.4 \\ \hline \end{array}$	**9.** $\begin{array}{r} 9.74 \\ -\ 4.63 \\ \hline \end{array}$	**10.** $\begin{array}{r} 8.19 \\ -\ 7.41 \\ \hline \end{array}$

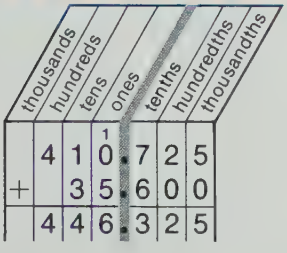
You know how to add and subtract money.
Cents represent hundredths.

June earned $1.25 on Monday.
She earned $2.50 on Tuesday.
How much did she earn in the two days?

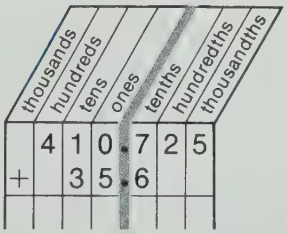
She went shopping. She took $2.50 with her.
She bought only one thing. It cost $0.57.
How much money did she have left?

When you did those problems, you probably didn't even think about place value.
But start thinking about it now. You grouped ones, tens, hundreds, and thousands when you computed with whole numbers. You group tenths, hundredths, and thousandths when you compute with decimal fractions.

Addition

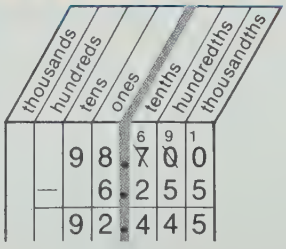


Could the addition problem have been written in this way?

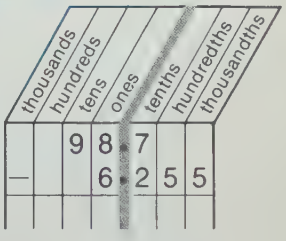


Would the answer be different? Why?

Subtraction



Could the subtraction problem have been written this way?



Could you still subtract? How?

You have used estimation to help you before. It is especially useful when you are working with decimals. If you put a decimal point in the wrong place—***WOW!*** Here is an example.

A department store bills you $5.87 for a shirt and $4.13 for a tie. The total comes to $100. Is this total reasonable?

Using estimation, you can see quickly that a mistake has been made. The total should be about $10, not $100.

$$\begin{array}{r} 5.87 \\ +\ 4.13 \end{array} \longrightarrow \begin{array}{r} 6 \\ +\ 4 \\ \hline 10 \end{array}$$

Probably you already have good estimation sense from working with money. But you should develop this same sense working with other decimals, too.

PROGRESS CHECK

Here is some mixed practice. Estimate. Then find exact answers. Make sure your answers are reasonable.

1. $\begin{array}{r} 3.5 \\ +\ 6.0 \end{array}$

2. $\begin{array}{r} 45.7 \\ -\ 8.9 \end{array}$

3. $\begin{array}{r} 5.0 \\ -\ 3.5 \end{array}$

4. $\begin{array}{r} 14.76 \\ +\ 23.14 \end{array}$

5. $5.2 + 3.8 = \underline{?}$

6. $17.3 - 9.5 = \underline{?}$

7. $\begin{array}{r} 54.71 \\ +\ 6.09 \end{array}$

8. $31.50 + 32.76 = \underline{?}$

9. $20.09 - 7.77 = \underline{?}$

10. $\begin{array}{r} 10.05 \\ -\ 7.99 \end{array}$

11. $\begin{array}{r} 15.00 \\ -\ 1.11 \end{array}$

12. $\begin{array}{r} 5.70 \\ +\ 3.58 \end{array}$

13. $\begin{array}{r} 15.76 \\ +\ 14.83 \end{array}$

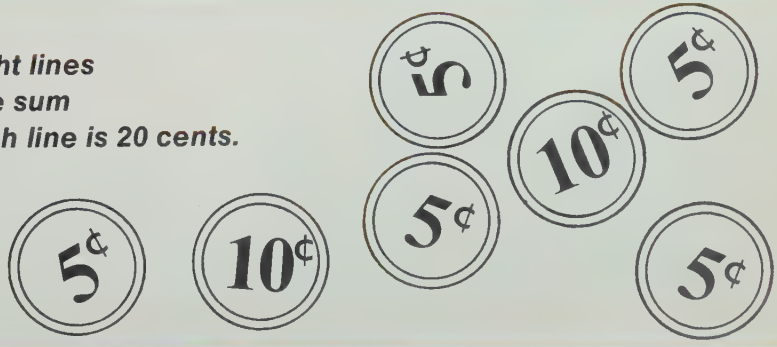
Roundy Wheeler borrowed $37 from his dad to buy a bicycle. He paid him back at the rate of $1.75 a week for the first 5 weeks and $2.25 a week from the sixth week on until he paid it all back. Copy and complete the payment table. It's for the first ten weeks.

How many weeks did it take to pay off the loan?
How much was the last payment?

End of week	Amount paid	Balance owed after payment
1	$1.75	$35.25
2	$1.75	?
3	?	?
4	?	?
5	?	?
6	$2.25	?
7	?	?
8	?	?
9	?	?
10	?	?

SUPER SLEUTH

Place these coins in three straight lines so that the sum in each line is 20 cents.



For Experts Only

Bob delivers newspapers every morning. If he delivers a paper every day, the customer pays $0.80 a week. The cost for weekdays only is $0.50 a week, and for weekend delivery only it is $0.35 a week. Customers usually pay on Saturdays. The payments of 15 customers last Saturday are recorded below. Complete this table.

	Customer name	Weekly charge	Paid for this many weeks	Total paid	Paid with	Given this change
1.	Mrs. Seaver	$0.80	2	a	$2	b
2.	Mr. Lacher	$0.35	4	a	$2	b
3.	Miss Liere	$0.35	6	a	$5	b
4.	Miss Smith	$0.50	2	a	$1	b
5.	Mrs. Erickson	$0.80	2	a	$5	b
6.	Mrs. Humphrey	$0.80	3	a	$3	b
7.	Mrs. Dormal	$0.50	5	a	$3	b
8.	Mr. Stone	$0.50	3	a	$10	b
9.	Miss Snyder	$0.35	4	a	$5	b
10.	Mrs. Campbell	$0.35	5	a	$2	b
11.	Mr. Koslowski	$0.35	3	a	$2	b
12.	Mr. Jameson	$0.80	2	a	$5	b
13.	Mrs. Randall	$0.80	4	a	$5	b
14.	Mrs. Clark	$0.35	5	a	$10	b
15.	Miss Adams	$0.50	4	a	$2	b
16.	TOTAL			a	$62	b

This had better equal this plus this

or you are in trouble!

CHECKOUT

1. Do 0.1, 0.10, and 0.100 mark the same point on the number line?

2. Do 0.3, 0.03, and 0.003 mark the same point on the number line?

3. Do 4, 4.0, and 4.00 mark the same point on the number line?

4. Do 0.006, 0.06, and 0.6 mark the same point on the number line?

5. Order each set from smallest to largest.

   a 1.0, 1.1, 10.1 **b** 0.01, 0.10, 1.1 **c** 0.11, 0.10, 0.01

6. Add.

   a 0.65 and 0.34 **b** 0.50 and 0.50 **c** 0.49 and 0.52

   d 0.82 and 0.88 **e** 3.75 and 2.25 **f** 1.37 and 2.72

   g 3.5 and 6.75 **h** 10.1 and 9.9 **i** 0.99 and 0.01

   j 50 and 0.05 **k** 0.02 and 20 **l** 9.2 and 0.8

7. Subtract.

   a 0.75 – 0.23 **b** 0.94 – 0.58 **c** 0.80 – 0.37

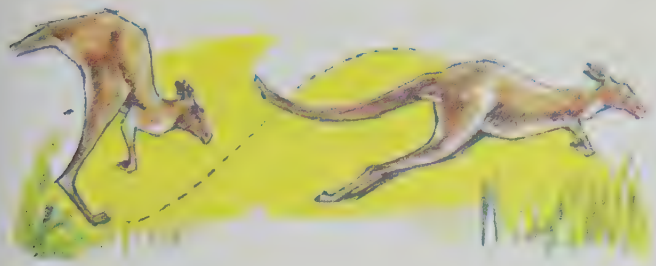
   d 1.52 – 0.61 **e** 10.30 – 0.71 **f** 1.00 – 0.31

   g 1.05 – 0.37 **h** 1.5 – 0.7 **i** 1.8 – 0.95

   j 40 – 0.09 **k** 0.98 – 0.09 **l** 3.70 – 3.69

9 MEASUREMENT

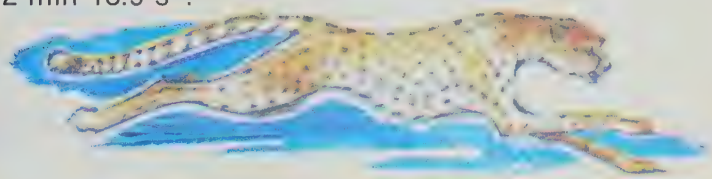
DID YOU KNOW?



A kangaroo can jump as far as 12 m in a single leap. The human record for the long jump, set in 1968, is 8.90 m .

The smallest mammal in the world, the shrew, has a mass of only a few grams. One kind of shrew has an average adult mass of only 2.5 g .

The fastest land animal in the world, the cheetah, can cover a kilometre in about half a minute. The human record for the 1000 m run, established in 1974, is 2 min 13.9 s .



A camel can go for days without water, but when it does drink, it gulps more than 120 ℓ in a few minutes. The average adult human being needs a minimum of 2500 ml of water daily.

YOUR GOAL is to become more skilful at measuring.

Learn to estimate accurately and measure precisely.

Use measurements to suit your needs.

SYMBOLS

Like our number system, our measurement system is international.
Like number symbols, our measurement symbols are understood everywhere.
Measurement symbols should always be used with numerals.

Here are some examples.

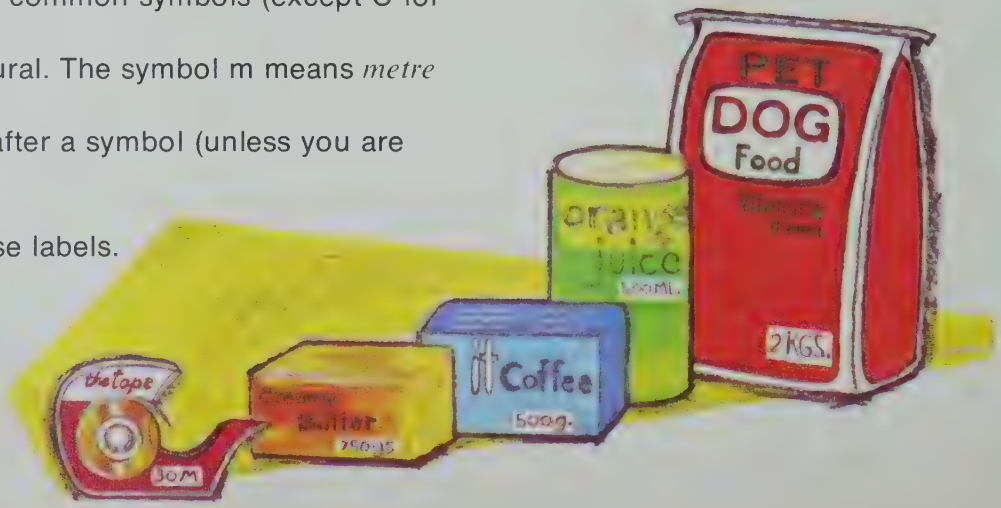
0.75 kg 24.06 m 3525 ℓ 8 500 000 km 30°C

You know the rules about writing numbers and about using decimal notation for a fractional part of a measurement.

There are rules for writing measurement symbols too.
Here are four rules to remember.

1. You leave a space between a numeral and a symbol (except in temperature readings).
2. You use small letters for common symbols (except C for Celsius).
3. You don't add *s* for a plural. The symbol m means *metre* or *metres*.
4. You don't use a period after a symbol (unless you are ending a sentence).

There are some errors on these labels.
Find the errors.
Write correct measurements.



YOU'VE COME A LONG WAY!

You know about place value and decimals.
You know about common units of measurement.
You know about symbols.

Do you know about PREFIXES?

Prefixes are word parts added before other words.
Prefixes are used in names for our measurements.

A metric prefix always has the same meaning.
It always uses the same letter (or letters) as part of a symbol.
For example:

The prefix *kilo-* means *one thousand*, and uses the letter *k* when it forms part of a symbol.
A *kilo*metre is 1000 m, or 1 km; a *kilo*gram is 1000 g, or 1 kg .

The prefix *milli-* means *one thousandth*, and uses the letter *m* when it forms part of a symbol.
A *milli*metre is 0.001 m, or 1 mm; a *milli*litre is 0.001 ℓ or 1 ml .

The chart below looks like a place-value chart — and for a good reason. Our system of measurement is related to the decimal system. The headings give the prefixes with their symbols and meanings. The words to which the prefixes are added take the place of the ones column in a place-value chart.

Some words and symbols are missing. Copy the chart and fill in the missing items.

Prefix Symbol Meaning	kilo- k thousand (1000)	hecto- h hundred (100)	deca- da ten (10)		deci- d tenth (0.1)	centi- c hundredth (0.01)	milli- m thousandth (0.001)
Units of Length	kilometre km	hectometre hm	. . . . . dam	metre m	decimetre . .	. . . . . cm	. . . . . mm
Units of Mass	. . . . . kg	hectogram . .	. . . . . dag	gram g	. . . . . dg	centigram . .	. . . . . mg
Units of Capacity	kilolitre . .	. . . . . hl	decalitre . .	litre ℓ	. . . . . dl	centilitre . .	. . . . . ml

Most people don't need to use all the units of measure.
Here are the common units that are used most often.

LENGTH	kilometre	metre	centimetre	millimetre
CAPACITY	kilolitre	litre	millilitre	
MASS	tonne	kilogram	gram	milligram

Remember *tonne* (t) is the common unit used for a mass of 1000 kg .

Choose your answers from the list of common units.

1. **a** Which is the largest unit of length? The smallest?
b Which is the largest unit of capacity? The smallest?
c Which is the largest unit of mass? The smallest?

2. Write the measurement that would be commonly used to describe each item.

a The capacity of a large pail: 0.1 hl 1 dal 10 ℓ
b The distance of an Olympic sprint: 1 hm 1 dam 100 m
c The length of a long walk: 5 km 50 hm 500 dam
d The mass of a thick slice of bread: 4 dag 40 g 400 dg
e The capacity of an oil drum: 2 hl 200 ℓ 2000 dl
f The mass of a postage stamp: 0.2 dg 2 cg 20 mg

Try these without looking at any list.

3. Give the missing numbers.

a ? m = 1 km	**b** ? m = 1 hm	**c** 1 m = ? cm	**d** ? mm = 1 m
e ? ℓ = 1 dal	**f** ? ℓ = 1 kl	**g** 1 ℓ = ? ml	**h** 100 ℓ = ? hl
i ? g = 1 kg	**j** 1 g = ? mg	**k** 1 t = ? kg	**l** 10 dg = ? g

WHEN YOU'RE TALKING ABOUT MEASUREMENT . . .

You may need to give only an *estimate*.
Nan drives about 8 km to her office.
The game will last about two hours.

You may need a *precise* or *close measurement*.
The window pane is 480 mm by 312 mm .
The winner's time was 11.8 s .

1. Write Yes for each item for which an estimate is likely to be close enough.
Write No if an estimate will not do.

a Length of a kite string
b Clearance for vehicles under a bridge
c Safety limit for a boatload
d Time you will take to read 100 pages
e Correct dosage of a medicine
f Amount of water you drink daily
g Dimensions of a lawn you cut
h Dimensions of an Olympic-sized pool
i Government report on rainfall
j Mass of your desk

2. If you want to give a precise or close measurement, you need to choose a suitable unit.
For each item, name the unit that seems most suitable for the measurement asked for.

a Distance to the moon
b Length of a pin
c Mass of a cent
d Mass of a railway car
e Capacity of a tank car
f Capacity of an eye-dropper

3. Each item (**a** to **f**) gives three possible measurements for the same thing. Some measurements have been made to the nearest whole number. Some are made to the nearest tenth. Some are to the nearest hundredth, and some to the nearest thousandth.
For each item, write the measurement that comes closest to a precise measurement.

a	2 km	1.9 km	1.87 km	**b**	3.86 kg	4 kg	3.9 kg
c	3.1 ℓ	3.09 ℓ	3.088 ℓ	**d**	21.5 s	22 s	21.49 s
e	1.8 m	1.76 m	1.758 m	**f**	9.486 kl	9.5 kl	9.49 kl

How well do you know millimetres?

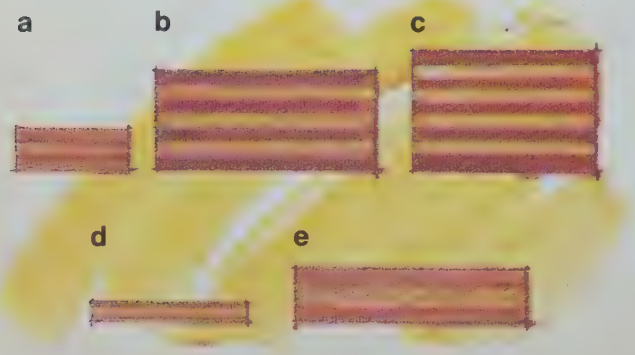
You can find out by estimating and then measuring.
If you really know millimetres, your estimate will be very close to the measurement.

First make a copy of this table for your records.

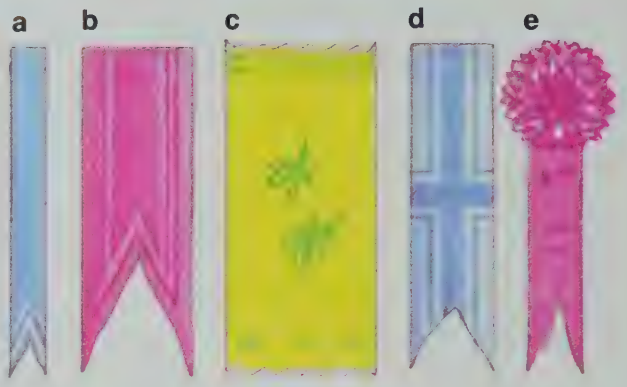
Exercise		Estimate	Measure	Difference
1.	a	mm	mm	mm
	b	mm	mm	mm
	c	mm	mm	mm

1. Estimate in millimetres, and then use a rule to check. Record your findings in the table.
   - **a** How wide is your little finger?
   - **b** How thick is the cover of this book?
   - **c** How thick is the whole book?

2. Add to your table for records on the thickness of the plywood pieces shown below. Estimate first, and then use your rule.



3. Add to the table your estimate and then the measurement of the width of each ribbon shown below.



If you really know millimetres, you should know how to express them as centimetres. Remember, a millimetre is a *tenth* of a centimetre. Our measurement system is a decimal system.
So 54 mm = 5.4 cm .

4. Use your table, and write in centimetres the measurement you recorded for each part of exercise 2.

5. Write in centimetres the measurement you recorded for each part of exercise 3.

How well do you know centimetres?

Many small things must be measured in millimetres. Centimetre measurement is less exact, but it may be all that you need. You can find out how well you know centimetres — again by estimating and measuring.

You have two "handy rules" for making estimates in centimetres: your fingers and your handspan.

Use the rule at the side of this page to find which of your fingers is about 1 cm wide. Then find how many fingers measure 5 cm.

Now use the rule to measure your handspan. It is the number of centimetres you can reach with your fingers spread as far apart as possible.

Draw a 4-column table like that on page 196. Change the symbol to cm, and give estimates and measurements to the nearest centimetre.

1. Use your fingers for estimating. Complete the records on your table.
How long is **a** your hand? **b** your foot? **c** your pencil?
How wide is **d** this book? **e** your workbook?

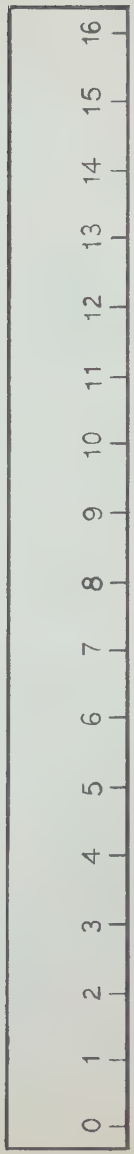
2. Use your handspan for estimating. Complete the records on your table.
How long is **a** your arm? **b** your leg? **c** the longer side of your desk?
What is **d** your height? **e** the height of your desk?

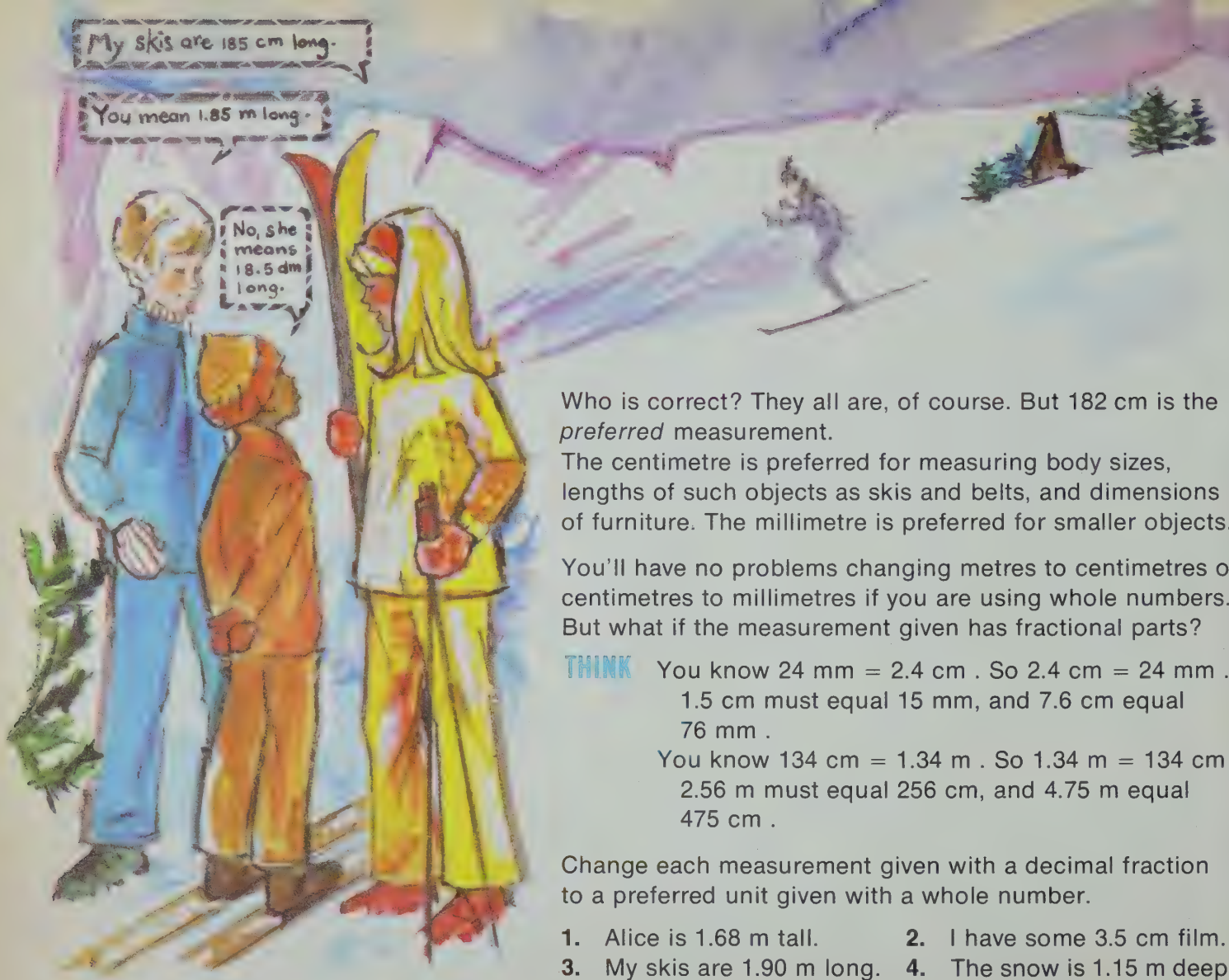
Centimetre measurements may be expressed as metres. A *centi*metre is a *hundredth* of a metre. You must have two digits to the right of the decimal point when you express centimetres as metres.

9 cm = 0.09 m 19 cm = 0.19 m 109 cm = 1.09 m

3. Write in metres the measurement you recorded for each part of exercise 1.

4. Write in metres the measurement you recorded for each part of exercise 2.





Who is correct? They all are, of course. But 182 cm is the *preferred* measurement.
The centimetre is preferred for measuring body sizes, lengths of such objects as skis and belts, and dimensions of furniture. The millimetre is preferred for smaller objects.

You'll have no problems changing metres to centimetres or centimetres to millimetres if you are using whole numbers. But what if the measurement given has fractional parts?

THINK You know 24 mm = 2.4 cm . So 2.4 cm = 24 mm . 1.5 cm must equal 15 mm, and 7.6 cm equal 76 mm .
You know 134 cm = 1.34 m . So 1.34 m = 134 cm . 2.56 m must equal 256 cm, and 4.75 m equal 475 cm .

Change each measurement given with a decimal fraction to a preferred unit given with a whole number.

1. Alice is 1.68 m tall.
2. I have some 3.5 cm film.
3. My skis are 1.90 m long.
4. The snow is 1.15 m deep.
5. The ski is 5.5 cm wide.
6. That belt is 2.8 cm wide.

How important is the metre? It's *very* important!

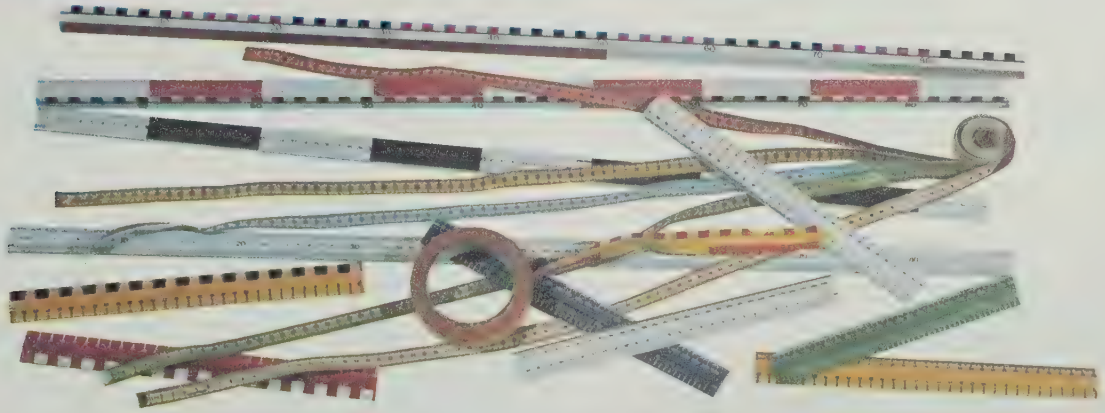
Its invention was the beginning of the **METRIC SYSTEM** of measurement.

About two hundred years ago, some French scientists decided to invent a system of measurement that would suit our decimal (base-ten) number system. The metre was the first standard unit they chose. It gave the new system its name.

The metre is the **BASE UNIT** of length in the metric system.

All other units for measuring length are based on the metre. Larger units measure the length of a metre multiplied by 10, 100, 1000, and so on. Smaller units measure the length of a metre divided by 10, 100, 1000, and so on. You already know the prefixes that show the numerical relationships of these other units to the metre.

The metre is the **PREFERRED UNIT** for many measurements of length.



Measurements given in metres include room dimensions, distances on golf courses, depths of lakes and rivers, heights of buildings and mountains, distances in many Olympic events, and lengths of carpeting and other materials.

How about the gram?

The scientists who invented the metre had to choose a new standard unit for measuring *mass* — the amount of matter in an object. They decided to relate it to their new units for measuring length.

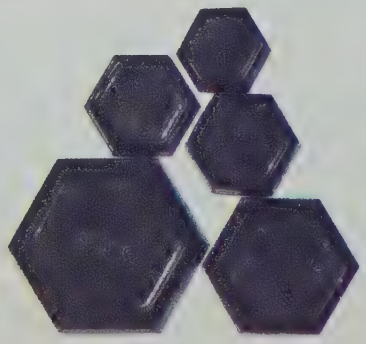
Imagine a cube of water 1 cm long, 1 cm wide, and 1 cm high. The mass of such a cube became the new standard unit for mass — the **gram.**

All other units for measuring mass are related to the gram. The metric prefixes used for units of length are used for the units of mass. The prefixes show the relationships of the other units of mass to the gram.

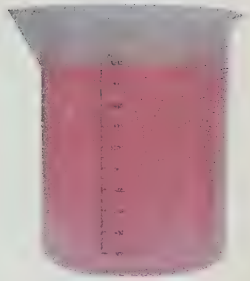
Because the gram itself is a small measurement, it is not the base unit for mass. The base unit is the mass of 1000 cm cubes of water — the **kilogram.**

Imagine one million centimetre cubes of water: the mass of water that would fill a tank 1 m long, 1 m wide, and 1 m high. This mass of 1000 kg has a special name — **tonne.**

A small car has a mass of about 1 t . What are some other things you might want to measure in tonnes?



How about the litre?



Another measurement the scientists had to consider was *capacity* — the amount a container will hold. Again they decided to relate their new standard for capacity to the new units for measuring length.

Imagine a container that holds 1000 cm cubes of water. The capacity of that container became the new standard unit for capacity — the **litre.**

The litre is not called a base unit in the metric system, but all other measures of capacity are related to it. The metric prefixes show the relationships of other units of capacity to the litre.

Metres, grams, and litres are all important units in our system of measurement. Our measures of length, mass, and capacity are related to each other.

A 1 cm cube holds 1 ml of water that has a mass of 1 g .
1000 cm cubes hold 1 ℓ of water that has a mass of 1 kg .
1 000 000 cm cubes hold 1 kl of water that has a mass of 1 t .

The official name for the system is the International System of Units — SI for short. SI symbols mean the same in all languages. Most countries today are either using SI or changing to it.

Why is an international system like SI useful?

Must a litre container always be in the form shown on this page?
What other shapes can you think of?

TEMPERATURE MEASUREMENTS — SI

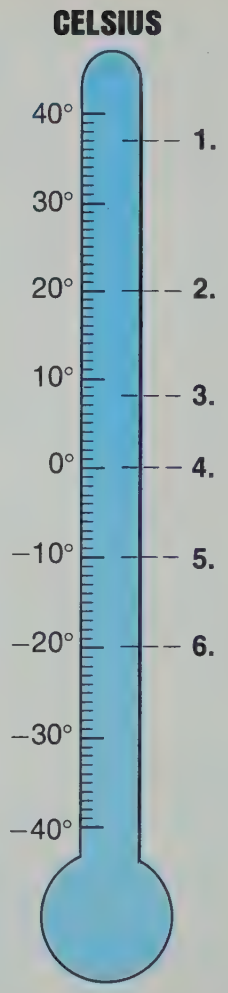
The common SI units for temperature measurements were invented about 200 years ago by a Swedish astronomer named Anders Celsius. He set his scale by fixing the freezing point of water at 0°, and the boiling point at 100°. The hundred equal divisions between these two points and other equal divisions above and below them are known as degrees Celsius (symbol °C).

Temperatures below 0°C are called minus temperatures and written with a minus sign: −4°C (minus 4 degrees) means 4 degrees below zero, the freezing point.

Fractional parts in Celsius readings, like other SI measurements, are given as decimal fractions. A reading halfway between 20°C and 21°C is 20.5°C .

Read the thermometer to give the temperature that matches each description.

1. Normal body temperature
2. Normal room temperature
3. A mild spring day
4. A day on which ice forms
5. Good skating weather
6. A very cold day



EXTRA PRACTICE

No rules or any other kind of measuring device allowed for this page!

Which measurement best describes the object?

1. **a** Mass of a small car: 20 000 g 500 kg 1 t
b Width of a small car: 5 m 2 m 1 m
c Mass of a bag of sugar: 5 kg 10 g 100 g
d Capacity of a carton of milk: 10 ℓ 1 ℓ 100 ml
e Height of a large dog: 10 cm 50 cm 150 cm

PROGRESS CHECK

Write the symbol for:

1. **a** ten kilograms **b** twenty-five centimetres **c** three hundred millilitres
d five metres **e** fifty grams

Write the preferred unit measure for the following:

2. **a** 0.001 ℓ **b** one thousand grams **c** one thousandth of a metre
d 0.01 m **e** 1000 m

CONSUMER PACKAGING

When you go shopping, look for items in *standard sizes*.

For example, have you ever noticed the measurement on a tube of toothpaste?

It comes only in sizes such as 25 ml, 50 ml, 100 ml, and 150 ml . The manufacturers of toothpaste have agreed to make their brands in the very same quantities.

Now compare these prices with the amounts in the container.

Brand A shampoo 83 ml for 69¢	Brand X toothpaste 50 ml for 47¢
Brand B shampoo 71 ml for 59¢	Brand Y toothpaste 50 ml for 46¢

When manufacturers make things in standard sizes, what advantages are there for the buyer? for the manufacturer? Can you think of items for which standard sizes wouldn't matter?

In what quantity does your mother or father buy the following items? (Check when you get home if you're not sure.)

COFFEE	**SALT**	**DETERGENT**
SUGAR	**TEA**	**FRUIT JUICE**

Which of these is the best buy? Why?

Why do manufacturers offer better prices on larger sizes? If you can save money by buying the largest package, why do people buy the smaller packages?

Standard sizes of some items are set by an agreement between their manufacturers. They agree that their customers will always use a particular size.

Choose the items that you think should have standard sizes.

SWIMMING POOLS BOOKS TIRES

ENVELOPES BALLOONS

GOLF BALLS HOUSES CARPETS

SHIRTS FOOTBALLS

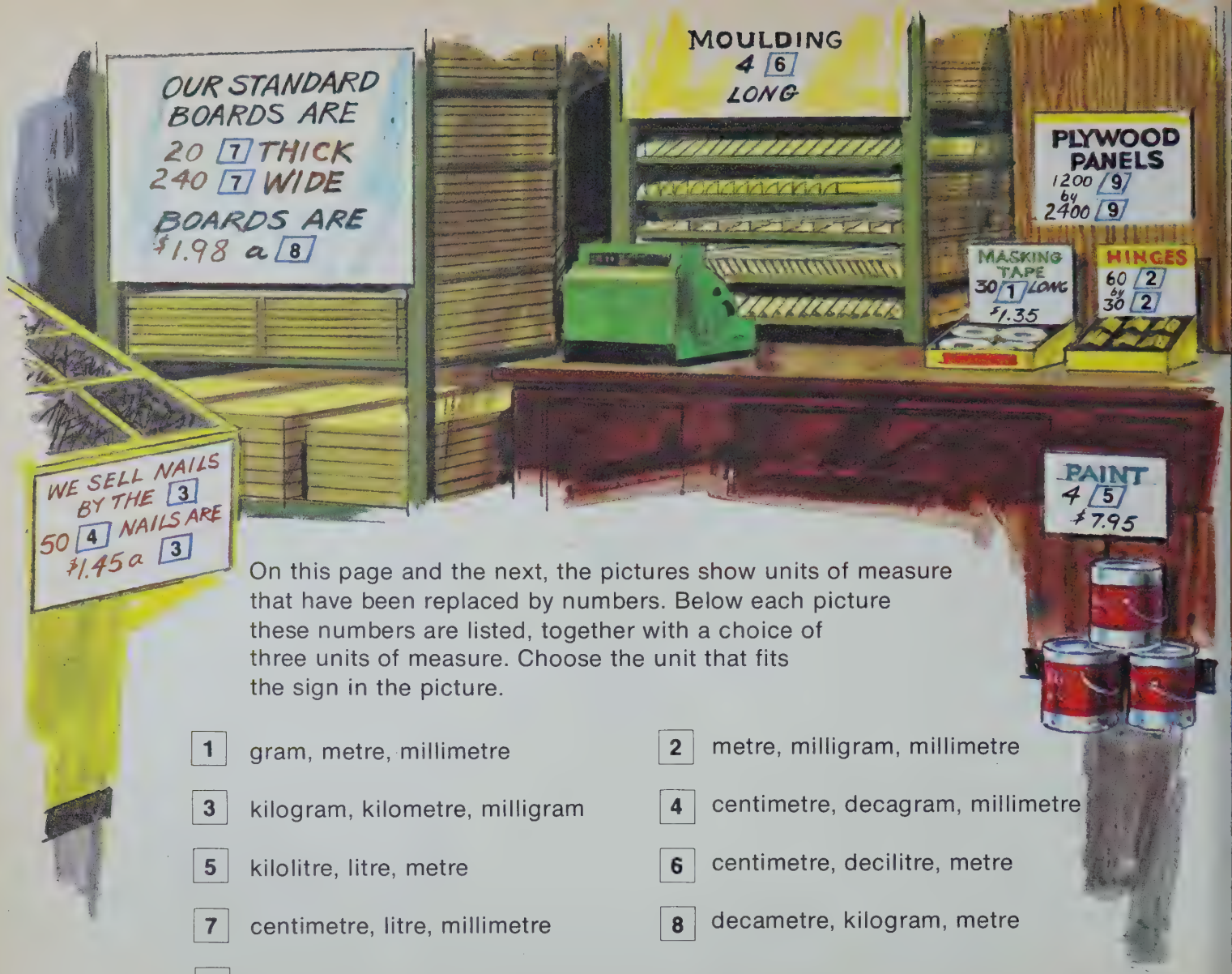
Should these items always be sold in standard sizes? Why? Can you think of other things that have standard sizes set by agreement?

Standard sizes make it easier to decide which is the "better buy" — but you can't always decide by price alone.
What other things might make you decide on a "better buy"?

CAN YOU SAVE MONEY?

Choose the better buy. (It's not *always* the larger size!) Tell how much you would save.

1. Toothpaste
   - **a** 25 ml — 44¢ *or* 50 ml — 89¢
   - **b** 50 ml — 89¢ *or* 150 ml — $2.09
2. Dog food
   - **a** 1 kg — 89¢ *or* 3 kg — $2.57
   - **b** 500 g — 47¢ *or* 1 kg — 95¢
3. Paint
   - **a** 1 ℓ — $1.29 *or* 5 ℓ — $6.20
   - **b** 500 ml — 75¢ *or* 1 ℓ — $1.35
4. Masking tape
   5 m — 39¢ *or* 15 m — 99¢
5. 20 mm tape
   5 m — 89¢ *or* 10 m — $1.59



On this page and the next, the pictures show units of measure that have been replaced by numbers. Below each picture these numbers are listed, together with a choice of three units of measure. Choose the unit that fits the sign in the picture.

1 gram, metre, millimetre

2 metre, milligram, millimetre

3 kilogram, kilometre, milligram

4 centimetre, decagram, millimetre

5 kilolitre, litre, metre

6 centimetre, decilitre, metre

7 centimetre, litre, millimetre

8 decametre, kilogram, metre

9 kilogram, metre, millimetre

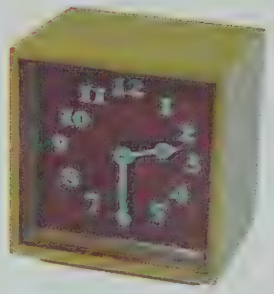
1	decilitre, gram, milligram	2	decalitre, kilogram, litre
3	decimetre, gram, hectogram	4	decagram, kilometre, metre
5	kilogram, kilolitre, tonne	6	gram, kilogram, kilolitre
7	centigram, gram, millilitre	8	centimetre, gram, milligram
9	decagram, decalitre, kilogram	10	decigram, kilogram, kilolitre

TIME TO THINK ABOUT TIME

You already know about common units for measuring time: seconds, minutes, hours, days, months, and years. Units such as *decades*, *centuries*, and *millennia* are used for measurements of longer periods of time. (Try to find out more about these.)

With the customary method of measuring time, we use a.m. and p.m. with numerals to show whether times are before or after noon.

Time expressed by 24-hour digital clocks does not need the usual abbreviations. A 24-hour clock tells time by showing four digits. Midnight is 00 00. Noon is 12 00. After 23 59 (one minute before midnight), the clock goes on to 00 00 again. The first two numbers show how many hours have passed since midnight. The other two show how many minutes have passed in the hour.



The photographs show an ordinary clock and a 24-hour digital clock. Do they give the same time? How would you write the times they show?

Where can you find 24-hour digital clocks? Where is this method of showing time used most?

WRITE THE TIME THAT MATCHES

a	Getting out of bed	08 30
b	Going to school	13 00
c	Lunchtime	15 30
d	End of lunchtime	20 30
e	End of school day	12 00
f	Eating dinner	07 30
g	Going to bed	23 59
h	Sound asleep	18 10

BEING PRECISE ABOUT TIME

There are special clocks and watches that can give time measurements more precisely than the ordinary watch or the 24-hour digital clock can. We use the decimal system to express fractional parts of a second.

1. A stopwatch measures time to the nearest tenth of a second (0.1 s).

a When he began practice, Peter ran the 200 m distance in a time of 23.4 s . The next day, his time for the distance was 23.1 s . By how much had he improved?

b On the day of the competition, he ran the 200 m race in 22.9 s . By how much had he improved since his first practice run?

2. Officials timing Formula I racing cars use clocks that measure hundredths of a second.

a A car turned one lap in 37.54 s and another lap in 37.13 s . How many seconds did it take for the two laps?

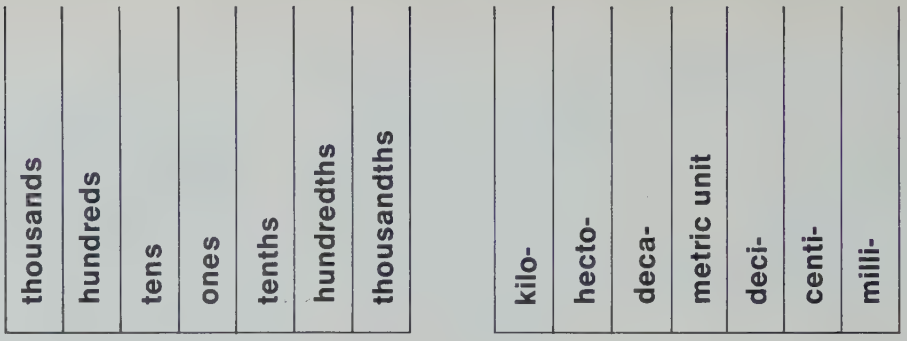
b What was the difference in time between the two laps?

3. Scientists and engineers may need to measure time to the nearest millisecond.

a What do you think a millisecond is? (Hint: Remember your prefixes.)

b How many digits to the right of the decimal point would millisecond measurements show?

What is the chart on the left? on the right? ARE THEY RELATED?



Decimals and measurement go together, thanks to the people who invented our metric system of measurement.

You know that 25 cm is 25 hundredths of a metre.
You can write this as a decimal fraction: 0.25 m .

You know that 225 ml is 225 thousandths of a litre.
You can write this in decimal form: 0.225 ℓ .

1. Replace the ? by a decimal fraction.

a 15 cm = ? m **b** 600 ml = ? ℓ **c** 755 mm = ? m **d** 250 mg = ? g
e 455 ml = ? ℓ **f** 85 cm = ? m **g** 5 cm = ? m **h** 5 ml = ? ℓ

Did you remember that 5 hundredths is written 0.05?
that 5 thousandths is written 0.005?

Here are more problems that may be tricky.

2. Give a decimal fraction for each answer.

a 5 mm = ? m **b** 55 ml = ? ℓ **c** 3 cm = ? m **d** 25 mg = ? g
e 50 ml = ? ℓ **f** 1 mm = ? m **g** 10 cm = ? m **h** 1 cm = ? m

Our measurement system makes it easy to change a measurement to a related unit.
You can change to a smaller unit by multiplying by 10, 100, 1000, and so on.
You can change to a larger unit by dividing by 10, 100, 1000, and so on.

Copy and complete these tables. Watch for the pattern in each. IT IS SIMPLE!

1.

cm	mm
5.5	55
2.1	?
17.5	?
20.8	?
36.0	?

2.

m	cm
2.50	250
1.25	?
10.65	?
20.01	?
36.10	?

3.

ℓ	ml
1.550	1550
2.125	?
3.300	?
5.675	?
8.080	?

4.

m	mm
1.695	1695
1.990	?
2.625	?
5.000	?
7.333	?

5.

cm	m
105	1.05
230	?
475	?
709	?
965	?

6.

mm	m
35	0.035
75	?
175	?
300	?
2400	?

7.

ml	ℓ
60	0.060
95	?
305	?
500	?
7575	?

8.

mm	cm
35	3.5
60	?
128	?
410	?
809	?

In the tables above, you kept the zeros or added zeros (when there were no ones or no tenths).

Tables 9 to 12 have some disappearing zeros. Watch for them in the patterns as you copy and complete the tables.

9.

cm	mm
0.2	2
0.8	?
0.9	?
1.2	?
10.5	?

10.

m	cm
0.01	1
0.65	?
0.90	?
5.50	?
12.05	?

11.

ℓ	ml
0.010	10
0.055	?
0.070	?
0.404	?
3.095	?

12.

g	mg
0.003	3
0.007	?
0.070	?
0.190	?
6.705	?

Did *you* discover when the zeros disappeared?

The problems on page 211 gave you a chance to find patterns when you change measurements.
You were working with the smaller units of measurement.
You have probably made discoveries that will help in changing measurements given in larger units.

Copy and complete the tables. Watch for the pattern in each. (There may be some disappearing zeros!)

1.

km	m
1.500	1500
3.650	?
7.885	?
0.025	?
0.002	?

2.

kg	g
0.625	625
2.000	?
0.400	?
1.055	?
0.075	?

3.

kl	ℓ
1.200	1200
0.500	?
0.100	?
0.005	?
1.000	?

4.

t	kg
1.010	1010
5.250	?
0.800	?
3.500	?
6.550	?

5.

g	kg
25	0.025
250	?
100	?
1250	?
275	?

6.

m	km
450	0.450
25	?
625	?
2600	?
5	?

7.

ℓ	kl
1220	1.220
200	?
30	?
484	?
3908	?

8.

kg	t
5000	5.000
3000	?
303	?
6809	?
7075	?

Replace the ? by the correct numeral. (You will need to remember all the patterns.)

9. **a** 1.750 km = ? m **b** 205 m = ? km **c** 350 g = ? kg **d** 0.045 t = ? kg

e 1.209 kl = ? ℓ **f** 500 g = ? kg **g** 175 cm = ? m **h** 225 ml = ? ℓ

i 6.75 m = ? cm **j** 0.350 km = ? m **k** 4040 kg = ? t **l** 3030 mg = ? g

THE SLIPPERY SLIDING DECIMAL POINT

Dennis Digit had problems with problems in changing measurements.
The decimal point always seemed to slip through his fingers.
His sister Dorothy showed him how to handle the slippery decimal point.
Perhaps you've already discovered the patterns Dennis didn't know.
Would you explain them the way Dorothy did?

OF COURSE YOU NEED TO KNOW YOUR MEASUREMENT UNITS.

You decide whether to multiply or divide, and by what number.

Multiply to change to a smaller unit. Divide to change to a larger unit.

MULTIPLY	*By 10*	24.5 cm = ? mm	*By 100*	99.5 m = ? cm	*By 1000*	5.890 t = ? kg
DIVIDE	*By 10*	9 mm = ? cm	*By 100*	405 cm = ? m	*By 1000*	65 ℓ = ? kl

Slide the decimal point: right, in multiplying; left, in dividing.

	One place for 10	*Two places for 100*	*Three places for 1000*
RIGHT	24.5 cm = 245 mm (The . isn't needed now.)	99.5 m = 9950 cm (You need to add 0.)	5.890 t = 5890 kg (0 was already there.)
LEFT	9 mm = 0.9 cm (0 is needed in the ones place.)	405 cm = 4.05 m	65 ℓ = 0.065 kl (0 is needed for ones and tenths.)

You don't need zero at the left of a whole number after you multiply.

0.8 cm = 8 mm 0.07 m = 7 cm 0.009 t = 9 kg

ARE YOU READY FOR THE PROBLEMS DENNIS LEARNED TO DO?

Try page 214.

Copy and complete the tables.

1.

Problem	× or ÷	10, 100, 1000?	Right or left?	How many places?	Answer
0.095 km = ? m	×	1000	Right	3	95 m
35 mm = ? cm	÷	10			
995 ℓ = ? kl					
6.40 m = ? cm					
8.775 t = ? kg					
2425 ml = ? ℓ					
35 cm = ? mm					
16 g = ? mg					

2.

mm	cm
48	?
7	?
?	12
558	?
?	2.5

3.

cm	m
?	0.085
?	0.9
679	?
?	2.8
5445	?

4.

m	km
?	1.9
78	?
495	?
?	8.86
?	9.406

5.

mg	g
95	?
?	0.750
2708	?
65	?
?	3.877

6.

g	kg
5956	?
56	?
?	0.508
384	?
?	1.758

7.

kg	t
?	2.004
522	?
7888	?
?	0.079
?	5.7

8.

ml	ℓ
?	0.049
?	4.49
799	?
4675	?
?	0.645

9.

ℓ	kl
614	?
?	6.982
86	?
?	7.79
9673	?

Here are some word problems in which you have to change measurements.

1. A restaurant served 200 g steaks.
Six people at a table ordered steaks.
How many grams of meat was that?
How many kilograms?

2. Beth had a roll of ribbon 10 m long.
She cut the ribbon into 20 cm lengths.
How many lengths did she cut from 1 m?
How many from the roll?

3. There was a sale on toothpaste.
The shopper bought 10 150 ml tubes, 5 100 ml tubes, and 5 50 ml tubes.
How many millilitres of toothpaste was that?
How many litres?

4. A heavy rainfall may measure 50 mm .
A heavy snowfall may measure 30 cm .
What is the difference between the two in millimetres? in centimetres?

5. In 1972, a skater from the Netherlands set new speed records for 3000 m and 5000 m .
How many metres did the events total?
How many kilometres?

6. The mass of a single telephone is about 2 kg .
The receiver alone has a mass of about 300 g .
What is the mass in grams of the other parts?

7. The mass of a carton of groceries was about 9 kg .
There were tinned foods and 5 packages of chips.
Each package of chips had a mass of 500 g .
How many grams of chips were there?
About how many grams of tinned foods?

CHECKOUT

1. **a** Give the place-value meaning of each prefix.

kilo- milli- deci- deca- centi- hecto-

b Write the word that each symbol stands for.

dag km t hl mg dal cm ml mm

c If we used metric prefixes with the word *dollar*, what word would be used to mean a thousand dollars? a hundred dollars? ten cents? one cent?

d If the symbol for *dollar* were d, what would the symbol dad mean?

2. Copy and complete the tables.

a

cm	m
?	0.39
59	?
259	?
?	6.50

b

mm	cm
6	?
?	0.9
?	16.5
398	?

c

g	kg
234	?
?	0.755
1234	?
?	6.905

Tell which of the units of measure is probably the best one.

3. A friend of mine is 165 ? (centimetres or decimetres) tall.

4. A sign reads: "Speed limit, 90 ? (metres or kilometres) an hour."

5. A roll of transparent tape is about 19 ? (centimetres or millimetres) wide.

6. One of the events at the Winter Olympics was ski jumping from a 70 ? (metre or kilometre) hill.

10 FRACTIONS

Ready or not – it is fraction time again.

You have learned so much about fractions. It doesn't seem possible there is still more ground to cover.

You know how to –

Name equivalent fractions for $\frac{1}{2}$: $\frac{1 \times 2}{2 \times 2} = \underline{?}$ $\frac{1 \times 3}{2 \times 3} = \underline{?}$

Rename fractions so that they have their simplest name. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \underline{?}$ $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \underline{?}$

Add fractions with common denominators: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \underline{?}$

and mixed numbers too: $4\frac{4}{7} + 2\frac{1}{7} = \underline{?}$ $2\frac{3}{8} + 1\frac{5}{8} = \underline{?}$

and decimals: $25.67 + 2.12 = \underline{?}$ $3.06 + 1.94 = \underline{?}$

Subtract fractions with common denominators: $\frac{7}{8} - \frac{1}{8} = \frac{7-1}{8} = \underline{?}$

and mixed numbers too. $5\frac{4}{5} - 3\frac{2}{5} = \underline{?}$ $10\frac{7}{8} - 9\frac{7}{8} = \underline{?}$

and decimals: $16.7 - 5.6 = \underline{?}$ $102.56 - 7.69 = \underline{?}$

You know a little about problems like this:

$\frac{7}{8} + \frac{1}{4} = \underline{?}$ $\frac{3}{4} - \frac{3}{5} = \underline{?}$ $4\frac{3}{4} + 1\frac{2}{3} = \underline{?}$ $1\frac{3}{4} \times 2 = \underline{?}$

Your goal is to know a lot about problems like that.

One of your last BIG problems with fractions was finding common denominators.

$\frac{5}{6} + \frac{1}{3} = \frac{?}{\_\_}$

A method of renaming sixths and thirds was explored.
You thought: Can $\frac{5}{6}$ be renamed as $\frac{?}{3}$? You said NO.
Then you thought: Can $\frac{1}{3}$ be renamed as $\frac{?}{6}$?
You said YES and you got the job done.
You found an equivalent fraction for $\frac{1}{3}$.

Focus just on the denominators 6 and 3.
You know that 3 is a factor of 6.
You know 6 is *not* a factor of 3; it is a multiple.
So save time. Look at the denominators only.

$\frac{1}{4} + \frac{1}{2} = \frac{?}{\_\_}$

Aha! 2 is a factor of 4. And of course 4 is a multiple of 2. Save time. You know you can find an equivalent for $\frac{1}{2}$. The equivalent must have the denominator 4.

$\frac{1}{3} + \frac{1}{9}$

Is 3 a factor of 9?
Which fraction will be renamed? Do it.
Rewrite the problem so that it has common denominators.

Your turn. Rewrite each problem so that it has common denominators. Don't find the sum.

	a	b	c	d
1.	$\frac{1}{8} + \frac{1}{2}$	$\frac{1}{4} + \frac{1}{12}$	$\frac{1}{5} + \frac{1}{10}$	$\frac{1}{9} + \frac{1}{3}$
2.	$\frac{5}{6} + \frac{5}{12}$	$\frac{5}{6} + \frac{2}{3}$	$\frac{3}{4} + \frac{7}{8}$	$\frac{1}{3} + \frac{7}{12}$

You should be ready for practice.
Addition problems can be written like this: $\frac{1}{2} + \frac{3}{4}$

Rename $\frac{2}{4}$ ← That should become a **Think** step.

Rewrite $\frac{2}{4} + \frac{3}{4}$ or $\frac{2+3}{4} = \underline{?}$

Decide which of those forms helps you the most.
Save time. Don't write them both. Take your pick.

1. a $\frac{1}{4} + \frac{3}{8}$ Write the problem.
Think the renaming.
$\frac{2}{8} + \frac{3}{8}$ Then rewrite. Then add.

b $\frac{1}{12} + \frac{3}{4}$ Write the problem.
Think the renaming.
$\frac{?}{?} + \frac{?}{?}$ Then rewrite and add.

You are really on your own now. Find the sums.

2. a $\frac{1}{2} + \frac{3}{8}$ **b** $\frac{3}{4} + \frac{1}{12}$ **c** $\frac{3}{10} + \frac{2}{5}$ **d** $\frac{1}{9} + \frac{2}{3}$ **e** $\frac{1}{6} + \frac{5}{12}$ **f** $\frac{1}{8} + \frac{3}{4}$

3. a $\frac{1}{2} + \frac{5}{6}$ **b** $\frac{2}{3} + \frac{5}{6}$ **c** $\frac{3}{4} + \frac{7}{8}$ **d** $\frac{2}{3} + \frac{7}{12}$ **e** $\frac{5}{8} + \frac{3}{4}$ **f** $\frac{1}{2} + \frac{7}{10}$

4. Look back. Find the sums that need to be renamed in simplest form. Get that job done now.

5. The sums in row three were all mixed numbers.
Sometimes this renaming takes the longest time.
It's time for practice on that skill.
Rename these as mixed numbers.

a $\frac{5}{4}$ **b** $\frac{6}{4}$ **c** $\frac{8}{4}$ **d** $\frac{11}{4}$ **e** $\frac{14}{4}$ **f** $\frac{8}{6}$ **g** $\frac{13}{6}$ **h** $\frac{18}{6}$

Try to find a shortcut for renaming fractions as mixed numbers.

$\frac{10}{3}$ can be renamed by thinking subtraction.

How many $\frac{3}{3}$s in $\frac{10}{3}$?

$\frac{10}{3} - \frac{3}{3} = \frac{7}{3}$ (1)

$\frac{7}{3} - \frac{3}{3} = \frac{4}{3}$ (2)

$\frac{4}{3} - \frac{3}{3} = \frac{1}{3}$ (3)

There are 3 and $\frac{1}{3}$ more, or $3\frac{1}{3}$.

Why not think division?

$\frac{10}{3}$ is the same as $3\overline{)10}$.
How many threes in 10?
There are 3 threes and one part of three remaining.
Using numerals, that is $3\frac{1}{3}$.

Practice

Rename these fractions as mixed numbers in simplest form.

1. **a** $\frac{5}{2}$ **b** $\frac{7}{3}$ **c** $\frac{9}{4}$ **d** $\frac{12}{5}$

2. **a** $\frac{15}{6}$ **b** $\frac{20}{7}$ **c** $\frac{17}{8}$ **d** $\frac{16}{9}$

The next ones take more time.

3. **a** $\frac{10}{4}$ **b** $\frac{14}{6}$ **c** $\frac{18}{8}$ **d** $\frac{22}{10}$

4. **a** $\frac{9}{6}$ **b** $\frac{12}{8}$ **c** $\frac{15}{10}$ **d** $\frac{30}{12}$

Practise addition again.

5. **a** $\frac{1}{2} + \frac{3}{4}$ **b** $\frac{5}{6} + \frac{5}{12}$ **c** $\frac{5}{8} + \frac{3}{4}$ **d** $\frac{2}{3} + \frac{5}{9}$

You know this:

$\frac{1}{3} + \frac{5}{6}$

$\frac{2}{6} + \frac{5}{6}$ or $\frac{2+5}{6} = \frac{7}{6}$, or $1\frac{1}{6}$

You could have written it in vertical form like this:

$$\begin{array}{r} \frac{1}{3} \longrightarrow \frac{2}{6} \\ +\ \frac{5}{6} \longrightarrow \frac{5}{6} \\ \hline \frac{7}{6}, \text{ or } 1\frac{1}{6} \end{array}$$

Many people like the vertical form best. Do you? If so, use it. The vertical form is almost a must for mixed numbers.

$$\begin{array}{rll} 4\frac{1}{3} & \textit{Rename} \longrightarrow & 4\frac{2}{6} \\ +\ 3\frac{5}{6} & \textit{Rewrite} \longrightarrow & 3\frac{5}{6} \quad \text{Now add.} \\ \hline & & 7\frac{7}{6} \end{array}$$

$\frac{7}{6} = 1\frac{1}{6}$

$7 + 1\frac{1}{6}$, or $8\frac{1}{6}$

Here's another example to study:

$$\begin{array}{rll} 7\frac{2}{3} & \textit{Rename} \longrightarrow & 7\frac{8}{12} \\ +\ 3\frac{3}{4} & \textit{Rename} \longrightarrow & 3\frac{9}{12} \quad \text{Now add.} \\ \hline & & 10\frac{17}{12} \end{array}$$

$10 + 1\frac{5}{12}$, or $11\frac{5}{12}$

1. You try some.

a $2\frac{3}{8} \rightarrow 2\frac{?}{8}$
$+\ 1\frac{1}{4} \rightarrow 1\frac{?}{8}$

b $3\frac{1}{3} \rightarrow 3\frac{?}{6}$
$+\ 2\frac{1}{2} \rightarrow 2\frac{?}{6}$

c $3\frac{1}{4} \rightarrow 3\frac{?}{20}$
$+\ 2\frac{4}{5} \rightarrow 2\frac{?}{20}$

d $3\frac{1}{5}$
$+\ 2\frac{1}{2}$

e $6\frac{1}{2}$
$+\ 2\frac{5}{8}$

f $4\frac{1}{5}$
$+\ 2\frac{3}{10}$

Bill worked $1\frac{1}{4}$ hours on Monday and $1\frac{1}{2}$ hours on Friday. He had to know how much time he worked before he could get paid. He added.

$$\begin{array}{r} 1\frac{1}{4} \\ +\ 1\frac{2}{4} \\ \hline 2\frac{3}{4} \end{array}$$

← Where did the $\frac{2}{4}$ come from?

Bill worked another $1\frac{1}{2}$ hours on Saturday. How much time now?

$$\begin{array}{r} 2\frac{3}{4} \\ +\ 1\frac{2}{4} \\ \hline 3\frac{5}{4} \end{array}$$

← Is $1\frac{2}{4}$ another name for $1\frac{1}{2}$?

← Now what? You finish it.

Try this one on your own.

$$\begin{array}{r} 1\frac{3}{4} \\ +\ 1\frac{3}{8} \\ \hline ? \end{array}$$

← Which number must be renamed?

You need a common denominator.

$$\begin{array}{r} 1\frac{6}{8} \\ +\ 1\frac{3}{8} \\ \hline \blacksquare\blacksquare \end{array}$$

And you need to rename again. →

1. Find a common denominator and then add.

a $1\frac{1}{2}$ *Rename* → $1\frac{2}{4}$

$+\ 1\frac{1}{4}$ $\qquad +\ 1\frac{1}{4}$

$?$

b $1\frac{2}{3}$ *Rename* → $1\frac{?}{6}$

$+\ 2\frac{1}{6}$ $\qquad +\ 2\frac{1}{6}$

$?$

2. You're really on your own this time.

a $3\frac{3}{10}$ $\quad +\ 1\frac{1}{5}$

b $2\frac{3}{4}$ $\quad +\ 1\frac{1}{8}$

c $1\frac{1}{3}$ $\quad +\ 2\frac{5}{6}$

d $2\frac{1}{3}$ $\quad +\ 3\frac{5}{6}$

e $4\frac{5}{8}$ $\quad +\ 2\frac{1}{4}$

Are you ready? Remember that curve ball in the subtraction game. Start with problems that have common denominators.

Try this one.

$$\begin{array}{r} 3\frac{1}{3} \\ -\,1\frac{2}{3} \\ \hline \end{array}$$

Can you subtract the fractions?

When this problem comes up in whole numbers, you rename—a ten as ten ones, for example. Can $3\frac{1}{3}$ be renamed? Rename one of the whole numbers as $\frac{3}{3}$. How many thirds then?

$$\begin{array}{r} 3\frac{1}{3} \\ -\,1\frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 2\frac{1}{3}+\frac{3}{3} \\ -\,1\frac{2}{3} \\ \hline \end{array} \quad \textit{Rewrite} \quad \begin{array}{r} 2\frac{4}{3} \\ -\,1\frac{2}{3} \\ \hline \end{array}$$

O.K. now

Try these. You'll have to rename.

LOOK

1. $6\frac{1}{4} \longrightarrow 5\frac{5}{4}$

$$\begin{array}{r} -\,2\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} -\,2\frac{3}{4} \\ \hline \end{array}$$

NO! O.K. now.

2. $8\frac{2}{5} \longrightarrow 7\frac{7}{5}$

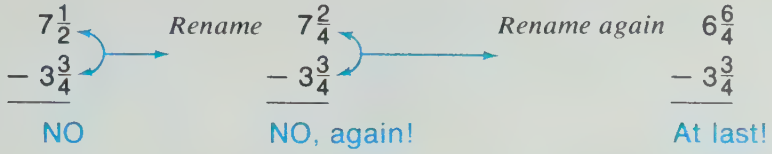
$$\begin{array}{r} -\,1\frac{4}{5} \\ \hline \end{array} \qquad \begin{array}{r} -\,1\frac{4}{5} \\ \hline \end{array}$$

3. $\begin{array}{r} 3\frac{4}{7} \\ -\,1\frac{5}{7} \\ \hline \end{array}$ **4.** $\begin{array}{r} 6\frac{1}{5} \\ -\,2\frac{4}{5} \\ \hline \end{array}$ **5.** $\begin{array}{r} 8\frac{1}{4} \\ -\,2\frac{3}{4} \\ \hline \end{array}$ **6.** $\begin{array}{r} 2\frac{1}{6} \\ -\,1\frac{5}{6} \\ \hline \end{array}$

Ask about these if you have forgotten what to do.

7. $6-2\frac{3}{4}$ **8.** $5-3\frac{2}{3}$ **9.** $5-3\frac{1}{3}$ **10.** $4-3\frac{4}{5}$

If you thought that last curve ball was a good one, just wait. This is the last pitch in the subtraction game. Don't worry. You will win the game.



First you had to rename to find a common denominator. Then you had to rename the fraction part of the mixed number so that you could subtract.

Take it one step at a time.

1. Find a common denominator and then subtract.

a $4\frac{2}{3}$ —Rename→ $4\frac{4}{6}$

$-\ 2\frac{1}{6}$ $-\ 2\frac{1}{6}$

? You do it.

b $5\frac{3}{4}$ —Rename→ $5\frac{3}{4}$

$-\ 2\frac{1}{2}$ $-\ 2\frac{2}{4}$

? You do it.

2. You're on your own. You will have to rename at least two fractions.

a $2\frac{1}{4}$ $-\ 1\frac{2}{3}$

b $4\frac{3}{8}$ $-\ 2\frac{3}{4}$

***c** $1\frac{1}{12}$ $-\ \frac{3}{4}$

***d** $2\frac{1}{3}$ $-\ \frac{7}{10}$

Here is a simple-looking problem: $\frac{1}{3}+\frac{1}{4}$ ← The troublemakers again.

Can $\frac{1}{3}$ be renamed as $\frac{?}{4}$? Can $\frac{1}{4}$ be renamed as $\frac{?}{3}$?

It Can't Be Done! Or Can It?

Both fractions must be renamed.

Can both $\frac{1}{3}$ and $\frac{1}{4}$ be renamed as $\frac{?}{6}$? as $\frac{?}{8}$? as $\frac{?}{9}$? as $\frac{?}{12}$?

Here is the problem again: $\frac{1}{3}+\frac{1}{4}=\frac{?}{}$

Rename and *rewrite*. $\frac{4}{12}+\frac{3}{12}=\frac{?}{12}$

Use the common denominator and add.

Look at another example: $\frac{2}{5}+\frac{1}{2}=\frac{?}{}$

Can both be renamed as $\frac{?}{2}$? as $\frac{?}{5}$? as $\frac{?}{10}$?

Here's the problem again: $\frac{2}{5}+\frac{1}{2}=\frac{?}{}$

Rename and *rewrite*. $\frac{4}{10}+\frac{5}{10}=\frac{?}{}$

Use the common denominator and add.

Try some on your own. It won't hurt a bit. Go trial-and-error method.
Find the common denominator. Rewrite the problem. Do *not* find the sums.

	a	b	c	d
1.	$\frac{1}{4}+\frac{1}{5}$	$\frac{1}{3}+\frac{1}{2}$	$\frac{1}{2}+\frac{1}{7}$	$\frac{1}{3}+\frac{1}{5}$
2.	$\frac{1}{5}+\frac{1}{6}$	$\frac{1}{7}+\frac{1}{3}$	$\frac{1}{8}+\frac{1}{3}$	$\frac{1}{4}+\frac{1}{7}$

Maybe you had good luck on these. But you don't have to depend on luck.
Go on to the next page.

Let's Get Organized

$\frac{1}{4} + \frac{1}{7} = \underline{?}$ What's a common denominator? (There is more than one.)

$\frac{1}{4}$ can't be renamed $\frac{?}{7}$.
And $\frac{1}{7}$ can't be renamed $\frac{?}{4}$.

You can write equivalent fractions for each.

$\frac{1}{4} = \frac{2}{8}\ \ \frac{3}{12}\ \ \frac{4}{16}\ \ \frac{5}{20}\ \ \frac{6}{24}\ \ \frac{7}{28}$

$\frac{1}{7} = \frac{2}{14}\ \ \frac{3}{21}\ \ \frac{4}{28}$

You can stop here.
Use 28 as the common denominator.

The problem: $\frac{1}{4} + \frac{1}{7}$

Rename and *rewrite*. $\frac{7}{28} + \frac{4}{28}$ And now you can find the sum.

There is another way. Just multiply the denominators.

$\frac{1}{4} + \frac{1}{7}$ (×)

You know that 4 will be a factor of 28.
And 7 is a factor of 28.

1. Go back to the problems in rows 1 and 2 on the page before.
Compare this method with the one you used.
Did you come up with the same common denominator?

2. Use any method you like to find common denominators.
Add. Make sure each sum has its simplest name.

a $\frac{2}{8} + \frac{1}{2}$ **b** $\frac{3}{4} + \frac{1}{5}$ **c** $\frac{1}{4} + \frac{2}{3}$ **d** $\frac{4}{5} + \frac{1}{2}$

e $\frac{5}{6} + \frac{1}{2}$ **f** $\frac{2}{5} + \frac{1}{3}$ **g** $\frac{1}{2} + \frac{3}{7}$ **h** $\frac{4}{5} + \frac{2}{3}$

There are no new rules for subtraction.
Find the common denominator and **GO!**

	a	b	c	d
1.	$\frac{3}{4} - \frac{1}{5}$	$\frac{2}{3} - \frac{1}{2}$	$\frac{4}{5} - \frac{1}{3}$	$\frac{5}{6} - \frac{1}{5}$
2.	$\frac{7}{8} - \frac{2}{3}$	$\frac{6}{7} - \frac{1}{4}$	$\frac{1}{2} - \frac{2}{7}$	$\frac{5}{6} - \frac{2}{5}$

Maybe word problems would be more fun. They will all require subtraction.

Tim the Timer went wild with the stopwatch.

3. He was supposed to time a runner for $1\frac{1}{3}$ laps of the track. He started talking and did not stop the watch until the runner had covered $1\frac{1}{2}$ laps. How much too far did the runner go?

4. Tim was supposed to time 3 horses as they crossed the finish line. He became excited and stopped the watch while the horses were still running. The first horse beat the third horse by $1\frac{1}{4}$ lengths. The second horse beat the third horse by $\frac{1}{2}$ a length. By how much did the first horse beat the second horse?

5. Tim worked for $5\frac{1}{2}$ hours that day. He spent $2\frac{3}{4}$ hours at the track. How many hours did he spend timing other sports?

1. Choose a common denominator from the list.
Rename the fractions and compute.

a	$\frac{1}{2} + \frac{1}{3}$	3, 6, 9, 12	**b**	$\frac{1}{5} + \frac{1}{2}$	5, 10, 15, 20	**c**	$\frac{3}{4} - \frac{1}{3}$	4, 8, 12, 16
d	$\frac{5}{6} - \frac{1}{4}$	6, 12, 18, 24	**e**	$\frac{2}{3} + \frac{5}{8}$	8, 16, 24, 32	**f**	$\frac{1}{3} - \frac{1}{4}$	4, 8, 12, 16
g	$\frac{3}{10} + \frac{3}{4}$	10, 20, 30, 40	**h**	$\frac{3}{4} - \frac{1}{5}$	5, 10, 15, 20	**i**	$\frac{5}{6} + \frac{2}{9}$	9, 18, 27, 36

***2.** What is the source of the numbers on each list?

PROGRESS CHECK

Add. Rename the sum as a mixed number if you can.

(1.) $\frac{3}{5} + \frac{4}{5}$ (2.) $\frac{1}{2} + \frac{5}{4}$ (3.) $2\frac{1}{8} + 1\frac{1}{2}$

(4.) $\frac{2}{5} + \frac{1}{2}$ (5.) $\frac{1}{4} + \frac{5}{6}$ (6.) $3\frac{2}{3} + 1\frac{3}{4}$

Subtract.

(7.) $\frac{9}{10} - \frac{7}{10}$ (8.) $\frac{5}{6} - \frac{2}{3}$ (9.) $3\frac{1}{2} - 1\frac{1}{6}$

(10.) $\frac{4}{5} - \frac{2}{3}$ (11.) $\frac{5}{6} - \frac{2}{9}$ *(12.) $4\frac{1}{2} - 2\frac{1}{7}$

A

$\frac{3}{4}$ pie $\frac{3}{4}$ pie $\frac{3}{4}$ pie

How many pies in all?

B

$\frac{2}{3}$ page $\frac{2}{3}$ page $\frac{2}{3}$ page

How many pages in all?

You have had enough work with addition for a while.
Skip the drawings. You know you can multiply.

$\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}$

$4 \times \frac{3}{5}$

The easy part of multiplication is that you don't have to worry about a common denominator. Why not?

But you will still use your renaming skills. $4 \times \frac{3}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$ or **?**

1. $4 \times \frac{3}{5}$ was an easy problem. Here are some more easy ones to do. Multiply and give the simplest name for the product.

a $2 \times \frac{1}{5}$ **b** $10 \times \frac{1}{6}$ **c** $\frac{5}{6} \times 6$ **d** $\frac{7}{8} \times 8$ **e** $3 \times \frac{5}{6}$ **f** $\frac{2}{3} \times 9$

g $8 \times \frac{3}{4}$ **h** $6 \times \frac{1}{4}$ **i** $\frac{4}{5} \times 4$ **j** $3 \times \frac{3}{6}$ **k** $5 \times \frac{5}{8}$ **l** $10 \times \frac{7}{10}$

m Were all your products greater than 1? Can you find one example when you take a whole number times a fraction and the product is less than 1?

1. If you multiply two fractions that are less than 1, will the product be greater or less than 1? Let these problems help you answer.

a $\frac{2}{3} \times \frac{1}{6} = \frac{2 \times 1}{3 \times 6} = \frac{2}{18}$ This can be renamed. Do it. The product of this problem is less than 1.

b $\frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{3}{12}$ Rename the product. This product is also less than 1.

c $\frac{9}{10} \times \frac{9}{10} = \frac{9 \times 9}{10 \times 10} = \frac{81}{100}$ That product cannot be renamed. But it is still less than 1.

2. Keep on. Multiply. Write the simplest name for each product.

a $\frac{1}{4} \times \frac{1}{2}$ **b** $\frac{1}{3} \times \frac{2}{5}$ **c** $\frac{2}{5} \times \frac{1}{6}$ **d** $\frac{5}{6} \times \frac{2}{5}$

e $\frac{3}{4} \times \frac{2}{3}$ **f** $\frac{4}{5} \times \frac{4}{5}$ **g** $\frac{0}{2} \times \frac{1}{2}$ **h** $\frac{5}{6} \times \frac{4}{5}$

i Why will the product of two fractions that are less than 1 also be less than 1?

3. Do you think you have to learn something new to complete a problem such as $1\frac{1}{2} \times \frac{1}{4}$?

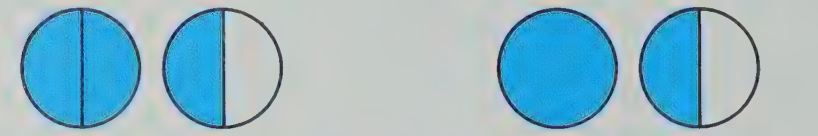
Try the old rules.
Rename $1\frac{1}{2}$ as a fraction; then multiply. $\frac{3}{2} \times \frac{1}{4} = \frac{3 \times 1}{2 \times 4} = \frac{?}{}$

4. Review renaming mixed numbers.

a $3\frac{1}{3}$ **b** $2\frac{3}{4}$ **c** $4\frac{2}{5}$ **d** $6\frac{1}{6}$ **e** $1\frac{7}{10}$ **f** $5\frac{1}{2}$

Here's some help if you are having a hard time with renaming.

Renaming a mixed number is not hard.
You know that $\frac{3}{2} = 1\frac{1}{2}$, so you should also know that $1\frac{1}{2} = \frac{3}{2}$.



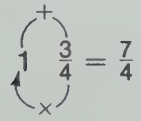
$\frac{3}{2} = \frac{2}{2} + \frac{1}{2}$ → $1 + \frac{1}{2}$, or $1\frac{1}{2}$

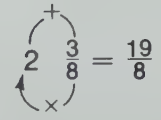
Look at another.

$3\frac{1}{2}$ **Think**
How many $\frac{1}{2}$s in 1?
How many $\frac{1}{2}$s in 2?
How many $\frac{1}{2}$s in 3?
How many $\frac{1}{2}$s in $3\frac{1}{2}$?

$2\frac{2}{3}$ **Think**
How many $\frac{1}{3}$s in 1?
How many $\frac{1}{3}$s in 2?
How many $\frac{1}{3}$s in $2\frac{2}{3}$?

Here's a shortcut. If you can work it out, you can use it.





Stick with the top method if this doesn't make sense.

1. Practise with these.

a $2\frac{1}{3}$ **b** $1\frac{1}{6}$ **c** $3\frac{7}{8}$ **d** $6\frac{7}{10}$ **e** $2\frac{1}{2}$ **f** $1\frac{3}{4}$

g $1\frac{5}{6}$ **h** $3\frac{1}{6}$ **i** $2\frac{3}{10}$ **j** $3\frac{3}{8}$ **k** $2\frac{2}{3}$ **l** $2\frac{3}{5}$

1. Finish these multiplication problems.

a $\frac{3}{4} \times 1\frac{1}{2}$

Rename $\frac{3}{2}$

Rewrite $\frac{3}{4} \times \frac{3}{2} = \frac{3 \times 3}{4 \times 2} = \frac{9}{8}$ Rename again. That's your job.

b $3\frac{1}{8} \times \frac{3}{5}$

Rename $\frac{25}{8}$

Rewrite $\frac{25}{8} \times \frac{3}{5} = \frac{25 \times 3}{8 \times 5} = \frac{75}{40}$

$\frac{40}{40} = 1, \frac{80}{40} = 2$ Nope! So $\frac{75}{40} = 1\frac{35}{40}$ or ?

c $1\frac{1}{3} \times \frac{5}{6}$

Rename $\frac{4}{3}$

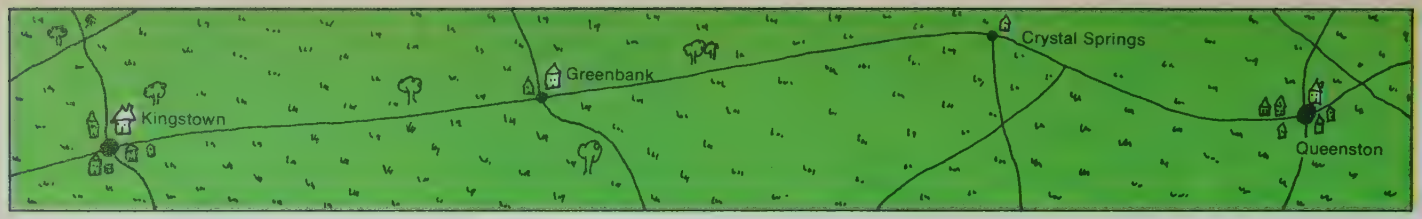
Rewrite $\frac{4}{3} \times \frac{5}{6} = \frac{4 \times 5}{3 \times 6} =$? Your turn to finish.

You are on your own.

d $\frac{1}{4} \times 5\frac{1}{2}$ **e** $2\frac{3}{4} \times \frac{1}{6}$ **f** $\frac{1}{2} \times 3\frac{1}{2}$ **g** $2\frac{2}{5} \times \frac{5}{6}$

h $2\frac{2}{3} \times \frac{3}{8}$ **i** $\frac{1}{7} \times 1\frac{2}{5}$ **j** $1\frac{1}{2} \times \frac{2}{3}$ **k** $\frac{3}{4} \times 2\frac{1}{2}$

2. Be brave. Try this one on your own. $2\frac{1}{2} \times 1\frac{2}{3}$



It takes $1\frac{1}{3}$ hours to drive from Kingstown to Queenston. How long should it take for the round trip?

$2 \times 1\frac{1}{3}$

Rename $\frac{4}{3}$

Rewrite $2 \times \frac{4}{3} = \frac{2 \times 4}{3} = \frac{8}{3}$ or $2\frac{2}{3}$

You will be doing a lot of renaming.

Look at some more.

$\frac{1}{3} \times 1\frac{3}{4}$

Rename $\frac{7}{4}$

Rewrite $\frac{1}{3} \times \frac{7}{4}$

You might want to write it as $\frac{1 \times 7}{3 \times 4}$. Anyway, you are ready to multiply.

1. Finish these multiplication problems.

a $\frac{1}{4} \times 1\frac{3}{4}$

Rename $\frac{7}{4}$

Rewrite $\frac{1}{4} \times \frac{7}{4}$ or $\frac{1 \times 7}{4 \times 4} = \underline{?}$

b $5\frac{1}{2} \times \frac{1}{3}$

Rename $\frac{11}{2}$

Rewrite $\frac{11}{2} \times \frac{1}{3}$ or $\frac{11 \times 1}{2 \times 3} = \underline{?}$

You can make the rename step and the rewrite step just one step.

c $1\frac{5}{8} \times \frac{1}{4}$

Rename and rewrite $\frac{13}{8} \times \frac{1}{4}$ or $\frac{13 \times 1}{8 \times 4} = \underline{?}$

d $\frac{3}{5} \times 1\frac{1}{8}$

Rename and rewrite $\frac{3}{5} \times \frac{9}{8}$ or $\frac{3 \times 9}{5 \times 8} = \underline{?}$

e $1\frac{3}{4} \times \frac{9}{8}$ $\frac{7}{4} \times \frac{9}{8}$ or $\frac{7 \times 9}{4 \times 8} = \underline{?}$

Don't write both of these.
Decide which form helps you most. Use that one.

Watch for the patterns in these problems.
Think about the problems within a set.
How are they alike? How are they different?

SET 1

	a	b	c	d	e
1.	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times 1$	$\frac{1}{2} \times \frac{3}{2}$	$\frac{1}{2} \times 2$	$\frac{1}{2} \times \frac{5}{2}$
2.	$4 \times \frac{1}{5}$	$4 \times \frac{2}{5}$	$4 \times \frac{3}{5}$	$4 \times \frac{4}{5}$	$4 \times \frac{5}{5}$
3.	$3 \times \frac{1}{2}$	$3 \times \frac{1}{3}$	$3 \times \frac{1}{4}$	$3 \times \frac{1}{5}$	$3 \times \frac{1}{6}$

SET 2

1.	$1\frac{1}{2} \times 2$	$1\frac{1}{2} \times 3$	$1\frac{1}{2} \times 4$	$1\frac{1}{2} \times 5$	$1\frac{1}{2} \times 6$
2.	$1\frac{1}{7} \times 5$	$1\frac{2}{7} \times 5$	$1\frac{3}{7} \times 5$	$1\frac{4}{7} \times 5$	$1\frac{5}{7} \times 5$

Only for the very brave.

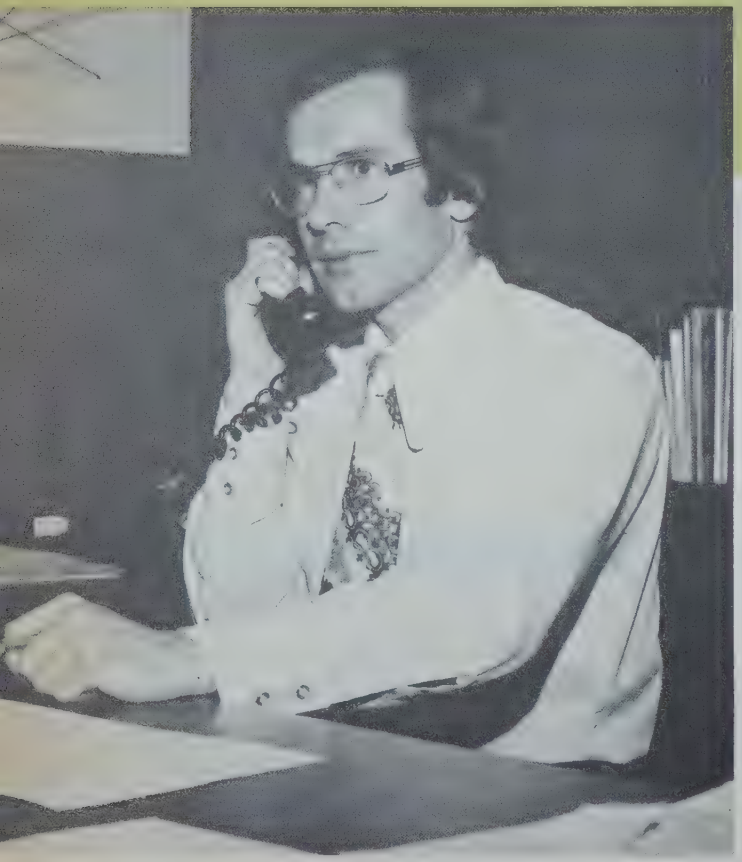
*3.	$1\frac{1}{10} \times 10$	$1\frac{1}{10} \times 20$	$1\frac{1}{10} \times 100$	$1\frac{1}{10} \times 200$	$1\frac{1}{10} \times 1000$

PROGRESS CHECK

You're on your own. Make sure every product is in simplest form.

(1.) $2 \times 1\frac{2}{3}$ (2.) $2\frac{1}{4} \times 5$ (3.) $3\frac{3}{4} \times 3$ (4.) $1\frac{1}{3} \times 3$ (5.) $1\frac{1}{2} \times 10$

Are all problems that multiply a whole number and a mixed number that bad? Look at some real-life situations.



Colin Cannon works a $6\frac{1}{2}$-hour day, 5 days a week. How many hours does he work in each two-week pay period?
There are 10 days in the pay period. So:

$$10 \times 6\frac{1}{2} = 10 \times \frac{13}{2} = \frac{10 \times 13}{2} = \frac{130}{2}$$

How much is 130 halves?

THINK 2 halves = 1 You know this.
3 halves = $1\frac{1}{2}$ $2\overline{)3}$
10 halves = 5 $2\overline{)10}$
100 halves = 50 $2\overline{)100}$
110 halves = $2\overline{)110}$ or 55
120 halves = $2\overline{)120}$ or 60
130 halves = $2\overline{)130}$ or 65 hours

GOOD GRIEF! How can you take a shorter way?
Look at the product $\frac{130}{2}$

THINK $2\overline{)130}$ right away.

Try another problem.

Mrs. Threadneedle takes about $14\frac{3}{4}$ hours to make a fancy costume for the school play. She has to sew 4 costumes, all alike. About how many hours must she allow for her sewing?

$$4 \times 14\frac{3}{4} = 4 \times \frac{59}{4} = \frac{4 \times 59}{4}$$

HOLD EVERYTHING!

This is a real-world problem. You wouldn't do this computation. You would estimate.
$14\frac{3}{4}$ hours is about 15 hours. $4 \times 15 = 60$.
She needs to allow about 60 hours.

It is possible for you to find a problem like $34 \times 25\frac{1}{2}$. It is possible to do it in the same way you have done the other multiplication problems.
But look at another way:

$$\begin{array}{rl} 34 & \\ \times\ 25\frac{1}{2} & \\ \hline 17 & \frac{1}{2} \times 34 \\ 170 & 5 \times 34 \\ 680 & 20 \times 34 \\ \hline 867 & \end{array}$$

The way the problem is written makes sense when you have such large numbers. Try just a couple of them.

1.

$$\begin{array}{rl} 56 & \\ \times\ 45\frac{1}{2} & \\ \hline \blacksquare\blacksquare & \frac{1}{2} \times 56 \\ \blacksquare\blacksquare\blacksquare & 5 \times 56 \\ \blacksquare\blacksquare\blacksquare\blacksquare & 40 \times 56 \\ \hline ? & \end{array}$$

2.

$$\begin{array}{rl} 48 & \\ \times\ 32\frac{1}{2} & \\ \hline \blacksquare\blacksquare & \frac{1}{2} \times 48 \\ \blacksquare\blacksquare & 2 \times 48 \\ \blacksquare\blacksquare\blacksquare\blacksquare & 30 \times 48 \\ \hline ? & \end{array}$$

3.

$$\begin{array}{rl} 63 & \\ \times\ 25\frac{1}{3} & \\ \hline \blacksquare\blacksquare & \frac{1}{3} \times 63 \\ \blacksquare\blacksquare\blacksquare & 5 \times 63 \\ \blacksquare\blacksquare\blacksquare\blacksquare & 20 \times 63 \\ \hline ? & \end{array}$$

There is only one more type of problem to look at.
Grit your teeth and get ready.

1. The hotel cook used $3\frac{2}{3}$ dozen eggs to serve breakfast to his customers. At lunch he used half as many again for omelettes. How many dozen eggs did he use in all?

The math sentence for this problem is $1\frac{1}{2} \times 3\frac{2}{3} = \underline{?}$

You have been lucky so far. All you have had to do is rename and multiply. Do you suppose it will work again?

try it

$$1\frac{1}{2} \times 3\frac{2}{3}$$

Rename $\frac{3}{2} \times \frac{11}{3}$ or $\frac{3 \times 11}{2 \times 3} = \frac{33}{6} = \underline{?}$

That wasn't so bad after all.

2. Try these.

a $1\frac{1}{2} \times 1\frac{1}{2}$ **b** $1\frac{1}{5} \times 1\frac{1}{2}$ **c** $2\frac{1}{2} \times 1\frac{1}{3}$ **d** $2\frac{2}{3} \times 2\frac{1}{4}$ **e** $3\frac{3}{4} \times 1\frac{3}{8}$

f $4\frac{1}{2} \times 3\frac{1}{3}$ **g** $3\frac{1}{3} \times 1\frac{1}{3}$ **h** $4\frac{1}{6} \times 3\frac{3}{5}$ **i** $2\frac{1}{2} \times 1\frac{3}{5}$ **j** $1\frac{1}{10} \times 1\frac{1}{2}$

3. Multiply each number by $1\frac{1}{4}$.

a $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{5}{6}$ **e** $1\frac{1}{4}$ **f** $1\frac{1}{5}$ **g** $3\frac{1}{2}$ **h** $2\frac{2}{3}$

CHECKOUT

1. Write simplest names for these fractions.

a $\frac{12}{15}$ **b** $\frac{8}{14}$ **c** $\frac{6}{9}$ **d** $\frac{10}{12}$ **e** $\frac{10}{25}$ **f** $\frac{12}{16}$

2. Rename each fraction as a mixed number. (Simplest form, please.)

a $\frac{3}{2}$ **b** $\frac{6}{4}$ **c** $\frac{10}{4}$ **d** $\frac{12}{9}$ **e** $\frac{18}{8}$ **f** $\frac{16}{6}$

3. Rename each mixed number as a fraction.

a $2\frac{2}{3}$ **b** $1\frac{7}{8}$ **c** $3\frac{3}{4}$ **d** $2\frac{2}{5}$ **e** $4\frac{1}{2}$ **f** $1\frac{10}{11}$

4. Add. Make sure each sum has its simplest name.

a $\frac{2}{7} + \frac{8}{7}$ **b** $\frac{7}{8} + \frac{5}{8}$ **c** $\frac{4}{9} + \frac{1}{3}$ **d** $\frac{1}{2} + \frac{5}{8}$

e $\frac{3}{4} + \frac{1}{3}$ **f** $\frac{1}{6} + \frac{1}{10}$ **g** $2\frac{1}{4} + 1\frac{1}{5}$ **h** $3\frac{1}{4} + 1\frac{1}{3}$

5. Subtract.

a $\frac{5}{6} - \frac{1}{6}$ **b** $\frac{5}{8} - \frac{3}{8}$ **c** $\frac{3}{4} - \frac{1}{8}$ **d** $\frac{7}{9} - \frac{2}{3}$

e $\frac{4}{5} - \frac{1}{2}$ **f** $\frac{3}{4} - \frac{1}{3}$ **g** $4\frac{3}{5} - 1\frac{1}{5}$ **h** $3\frac{1}{4} - 1\frac{3}{4}$

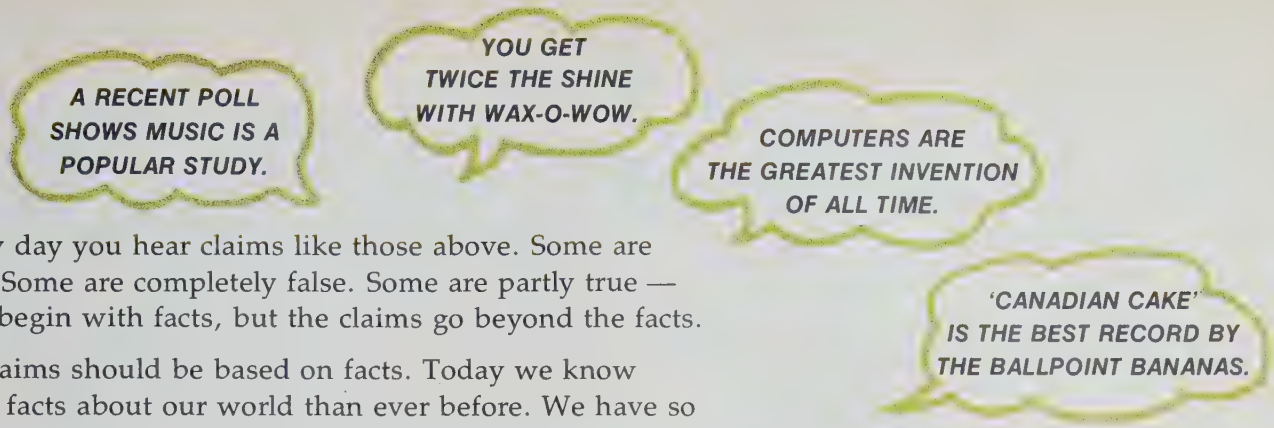
6. Multiply.

a $3 \times \frac{1}{2}$ **b** $\frac{2}{5} \times 4$ **c** $9 \times \frac{1}{3}$ **d** $\frac{2}{3} \times 6$

e $\frac{3}{4} \times \frac{2}{3}$ **f** $\frac{1}{2} \times \frac{4}{5}$ **g** $\frac{3}{8} \times 1\frac{1}{4}$ **h** $2\frac{5}{6} \times 1\frac{3}{4}$

11

FINDING, ORGANIZING, AND REPORTING INFORMATION


A RECENT POLL SHOWS MUSIC IS A POPULAR STUDY.
YOU GET TWICE THE SHINE WITH WAX-O-WOW.
COMPUTERS ARE THE GREATEST INVENTION OF ALL TIME.
'CANADIAN CAKE' IS THE BEST RECORD BY THE BALLPOINT BANANAS.

Every day you hear claims like those above. Some are true. Some are completely false. Some are partly true — they begin with facts, but the claims go beyond the facts.

All claims should be based on facts. Today we know more facts about our world than ever before. We have so much more information that people talk about the "knowledge explosion." Much of the information is stored in computers, machines that can find, organize, and report facts.

You don't want to depend on reports from people or machines. *You* want to find the facts, make the decisions, and THINK for yourself.

is to learn more about
— finding and collecting information
— organizing the information you find
— reporting what your information shows

1. Suppose someone says, "Music is the most popular subject at our school." He should have some **data** (facts) to show that his claim is true.

   a Suppose there are 300 pupils at your school. Suppose you ask ten of them and each says, "I like Music best." Would you agree that Music is the most popular subject at your school?

   b What if six say they like Music best and four say they dislike it?

   c Would you have to ask all three hundred to find out whether Music was most popular?

   d What would be a reasonable number to ask?

Think of ads that claim products are

Were any data used to show the claims were true? What kind of data? **Do you believe it?**

1. Do you know what is meant by "you get twice the shine with Wax-O-Wow"? Twice what? Compared with what?

2. Many radio stations play "the top ten." How do they decide which records are the top ten? Is record No. 1 the same for every station?

3. Some advertisers say, "We're Number 1." What does this mean?

4. For sports fans. Who do you think is the best player in the sport you like best? Why? Did you use data in deciding (matches won, goals scored, or points gained)?

5. "There are 100 more students at Central School than at West School. So Central's school band will be better." Must this be true? Why might West's band be better?

If you count the cars passing your home from 5:00 p.m. to 5:15 p.m., you have some *data* about the traffic. The number of people in your family is *data* about your family.

There are lots of ways to collect data. Tallying is a simple way to keep track of things being counted.

VOTE FOR CLASS PRESIDENT

Bill 𝍸 𝍸 𝍸 = 15

CINDY 𝍸 𝍸 𝍸 | = 16

One mark for each vote.

Every fifth mark a diagonal.

1. Jim Larsen was helping count the supplies in the school storeroom. Here's his tally of the packages of paper.

a White 𝍸 𝍸 𝍸 𝍸 ||||

b Blue |||

c Green 𝍸 𝍸 ||

d Pink 𝍸 𝍸 𝍸

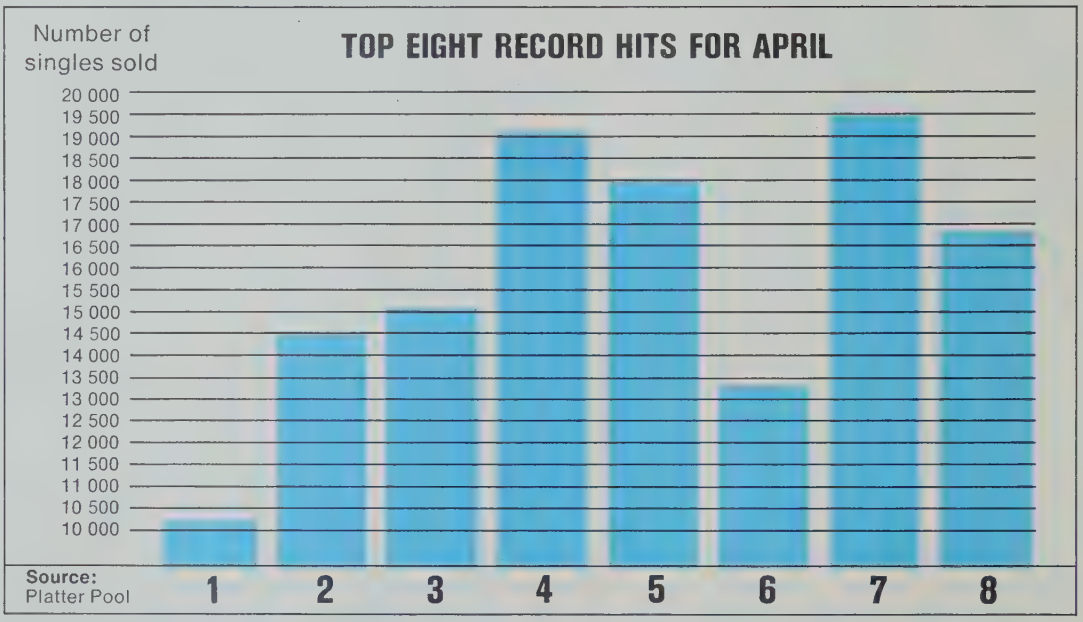
Write these tallies as standard numerals.

2. A week later a new paper supply arrived. Tally the number of each kind. Then write a standard numeral for each.

Green	Green	Blue	Pink	Blue	White	Pink	Blue	White	White	Blue	White	Pink	Green
Blue	Blue	Green	White	Green	Blue	Green	Blue	White	Blue	Blue	Pink	White	Blue

Collecting data is not enough. You need to organize and report facts. Charts and graphs are used to organize information. One way of reporting information is through pictures. Few words are needed if the right picture is picked.

What does this graph tell? What doesn't this graph tell?



Graphs and charts are used to—

1. present information,
2. show relationships,
3. catch the reader's attention, and
4. make information easily available.

Start a search. Look in newspapers and magazines.
Get permission to cut them out. Bring them to class.

Every graph needs to have certain labels.

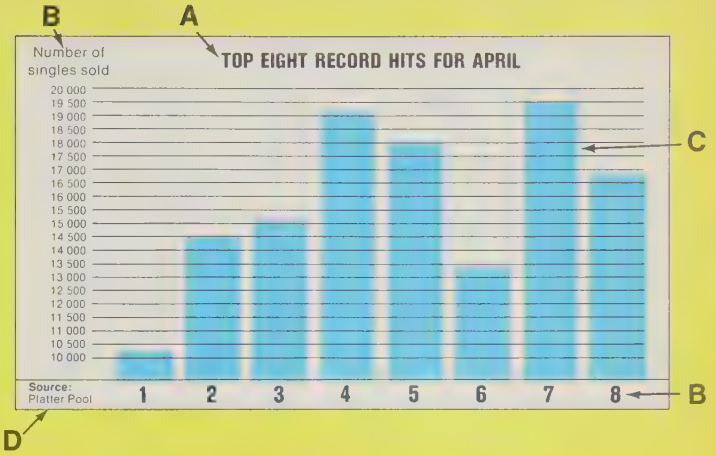
A It must have a title.

B You must know what the numbers refer to on the vertical scale and the horizontal scale.

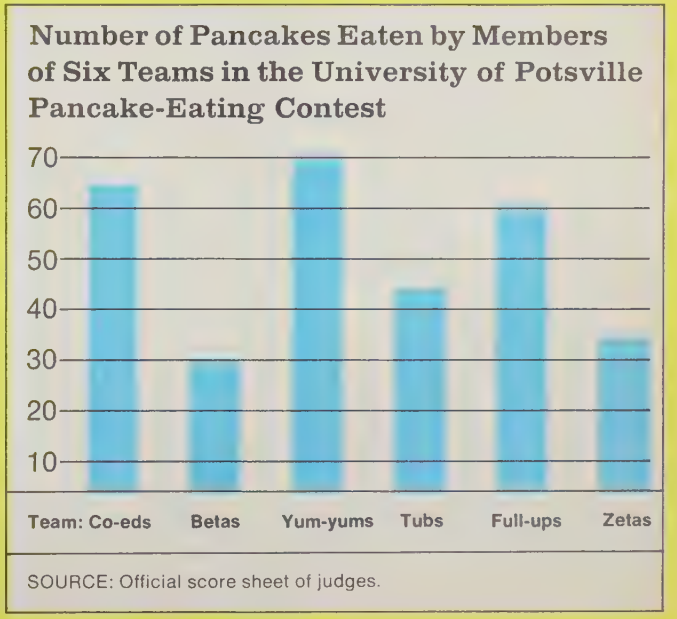
C It needs some sort of grid so that you can read it easily.

D It should have the source of this information stated.

E It should have complete information. This graph doesn't. You don't know the year or title of the records in the graph.



Does this graph have all the necessary information?



1. Mr. Kaplan asked his class to graph the number of people in their families.

What a mess! Julia said there were 18 in her family: mother, father, 6 children, 2 cats, 1 dog, 7 gerbils.

Victor didn't know whether to count his grandmother. She had lived at his house since 1952.

There was lots of confusion. Barb lived alone with her mother. But her aunt was visiting them. Two families were staying together because of a fire. One boy didn't live with any of his relatives.

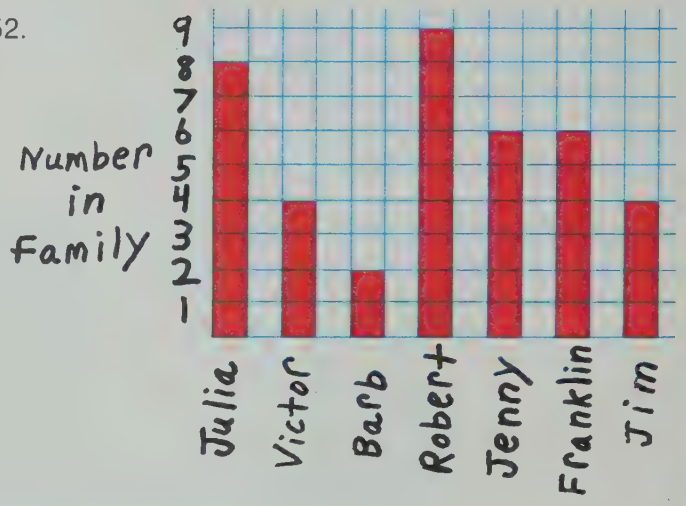
Finally the class agreed on what they meant by "family." They made a bar graph. Part of their graph is shown at the right.

a What's the smallest family shown? What's the largest?

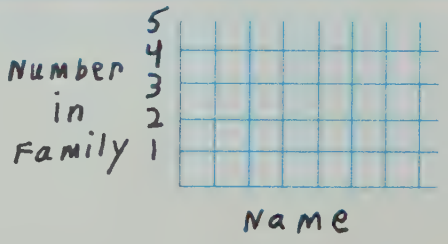
b What information is missing from the graph?

When you count,

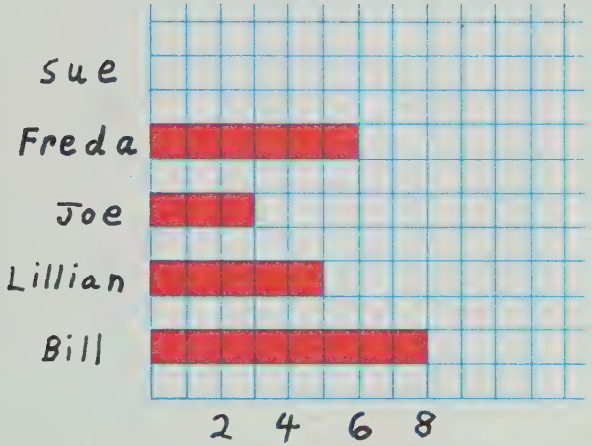
decide first what it is you want to count.



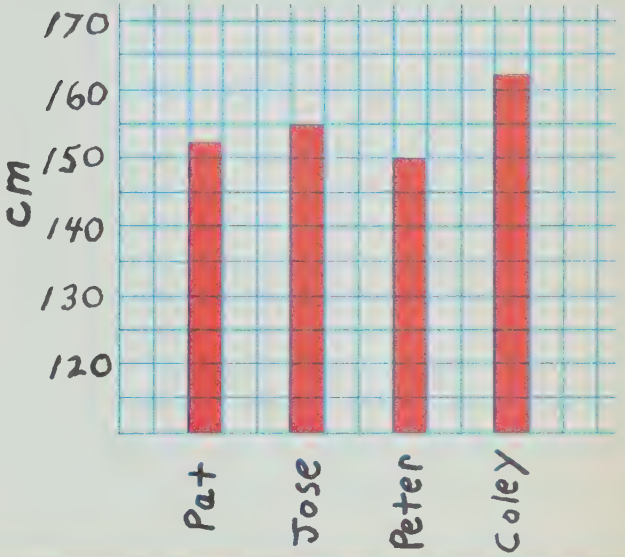
2. The graph from Mr. Kaplan's class is called a bar graph. Make a bar graph for family size in your class (or a smaller group within your class). You will need to make a grid like the one on the right. Put names along the bottom. Above each name, shade in one square for each family member. Make the graph as large as necessary.



1. Bar graphs can go across. This bar graph shows the number of years each member of the Reid family has spent in school. Make a graph like this for your family, relatives, or friends. Each rectangle across should stand for the same number of years.

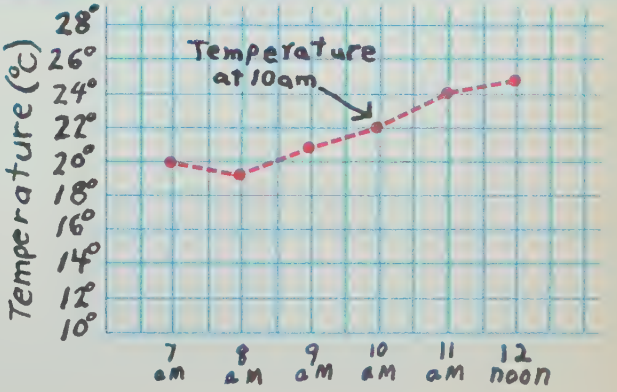


2. Bar graphs can go up. Make a bar graph showing heights of your classmates. (On a big sheet of paper, the bars can be the same heights as the people.)



Here is a different kind of graph. The dots show the temperature at different times. At 7 a.m. it was 20°. What was it at 10 a.m.? at noon?

The dots are connected by a broken line. The line helps show how the temperature changed. But the temperature readings were made only on the hour.

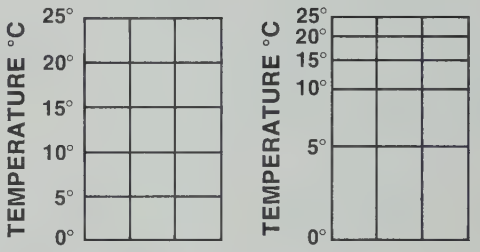


Sometimes you can use either kind of graph to show something.

1. Make a line graph showing the outside temperature at the same time each day for a week. (Use a thermometer at home or on a building, or use a radio or television report.) Or make a line graph showing the outside temperature each hour for at least five hours.

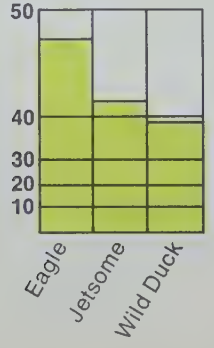
2. Collect at least six examples of line or bar graphs. Look in newspapers, textbooks, or reference books.

3. People sometimes change the spacing on graphs. Sometimes they leave off parts of graphs. There might be a good reason for doing so. Or maybe the reason is to fool someone!

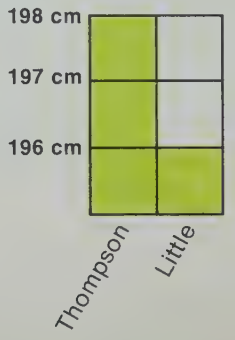


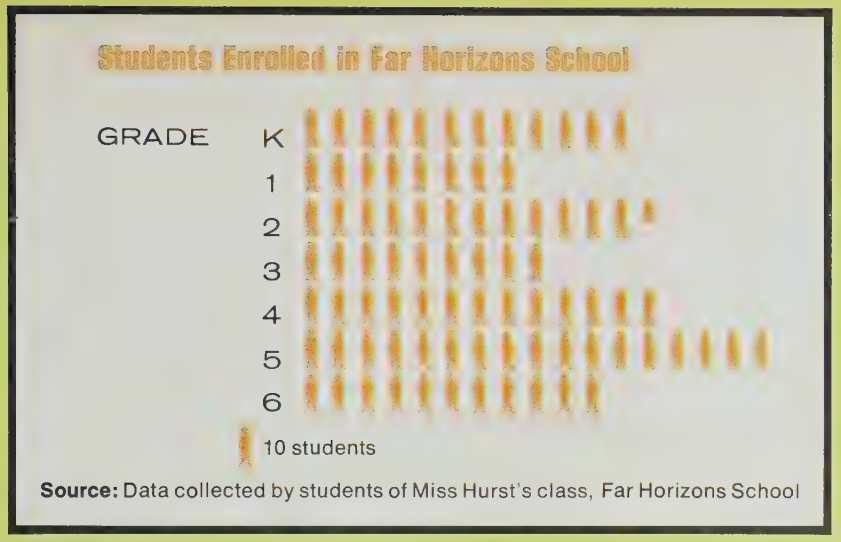
Equally spaced Not equally spaced Why not?

a 50 flights on 3 different airlines were checked. The graph shows how many were on time. Is this a good graph? Why?



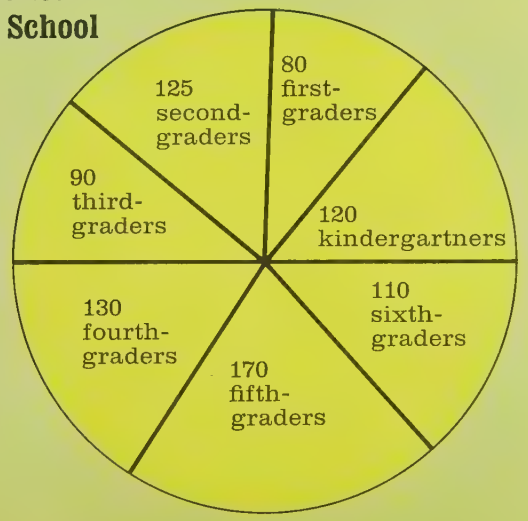
b The graph makes Tiny Thompson look a lot taller than Midge Little. Is he really that much taller?





Graphs can have pictures to replace bars.

What information does this picture graph have?



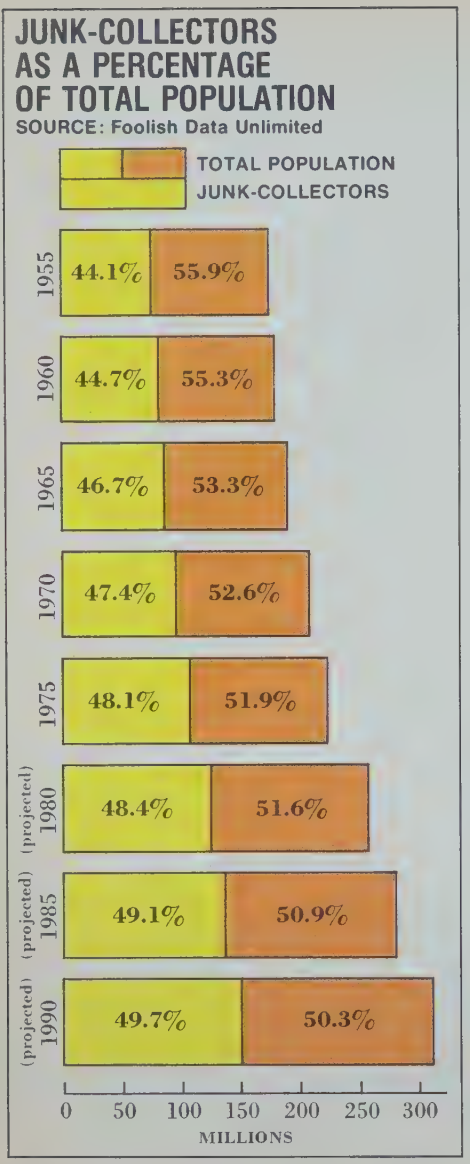
The same information could have been presented this way. This is a

CIRCLE GRAPH

1. Use either graph to answer these questions:

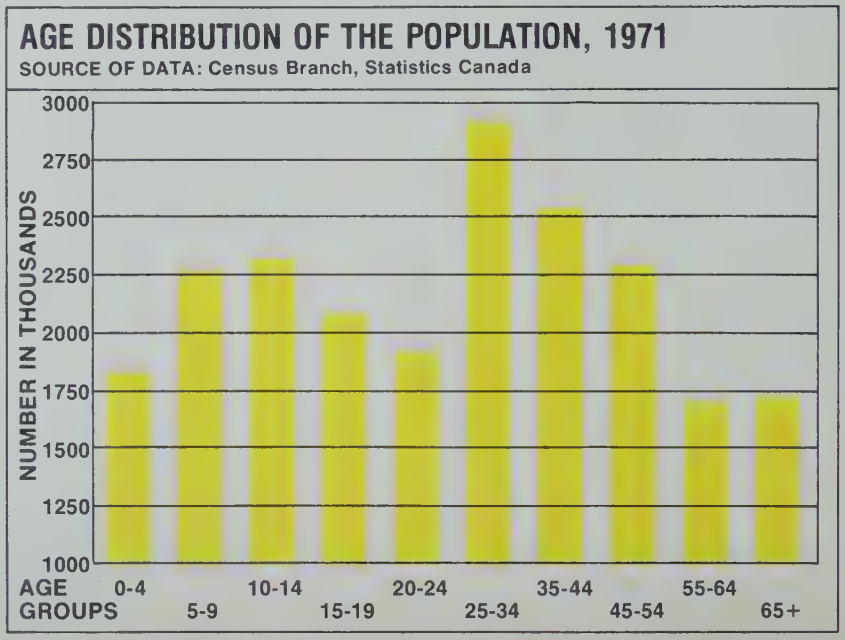
   a Which grade had the most students? the fewest?

   b How many students in all were enrolled in the school?



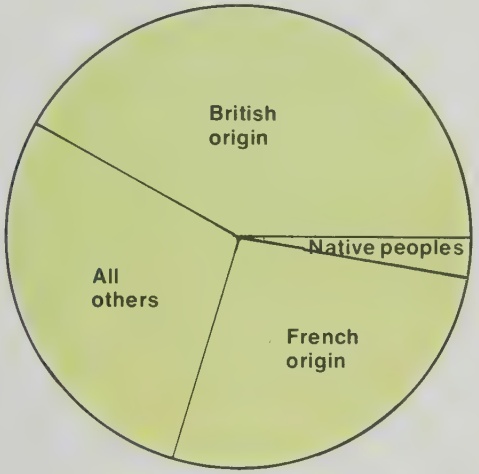
Study the graphs on this page and the next. Does each of these graphs have—

1. A title? What is it?
2. Numbers on the horizontal and vertical scale? What do they refer to?
3. A grid? Can you read it easily?
4. The source of information stated? Is it a source you trust?



NATIONAL ORIGINS OF THE POPULATION

DATA: Canadian Statistical Review
Third Quarter, 1974

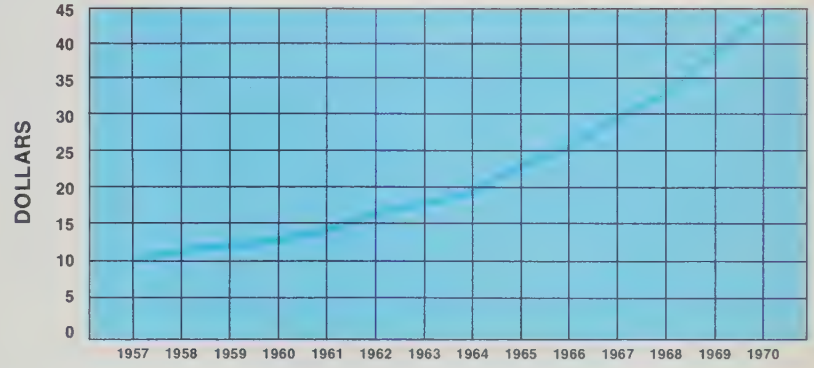


Number in millions:

British, over 9.6
French, over 6
Native peoples, about 0.5
Others, about 5.9

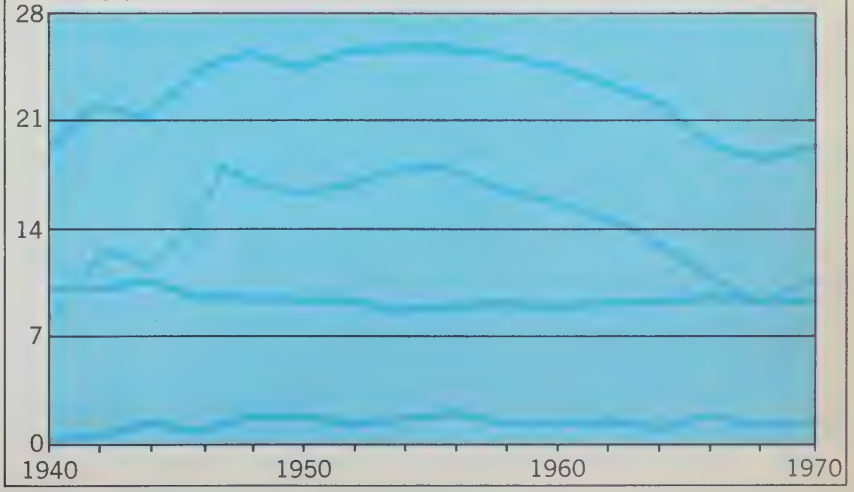
COST PER PATIENT-DAY, PUBLIC HOSPITALS, 1957-1970

DATA: Health Programs Branch, Department of National Health and Welfare



RATES OF NET GROWTH BY BIRTH, DEATH AND IMMIGRATION

Per 1000 population



Data are numbers collected by observing events or counting things. But data alone usually aren't very interesting. We use graphs to make pictures of data. Now we'll look at some other ways of presenting data.

Statistics is the study of data: how to collect data, what data mean, and how to use data. The idea of "average" is important in statistics.

You've probably heard the word *average* before: average size, average speed, average height, and so on. We'll look at a type of average called the *mean*. (That's short for *arithmetic mean*. People usually think of the mean when they say "average.")

A STUDENT WITH SOME KNOWLEDGE SAID,

I can sometimes use my head.
I can feel cross; I can grin.
I'm not fat, nor am I thin.
I'm not short, nor am I tall.
I'd say I'm average, all in all.

Here's a set of numbers:

9, 8, 11, 5, 2
Find their sum.

9
8
11
5
2
35

5 numbers

Divide the sum by the number of numbers added. The quotient is the *mean* of the numbers in the set.

sum

$35 \div 5 = 7$

mean

The mean of a set of numbers is one way to describe the size of the numbers. It's somewhere between the smallest number and the biggest number. Does the mean of a set of numbers have to be one of the numbers in the set?

1. Find the mean of each set of numbers.

a 5, 7, 18 **b** 12, 3, 0, 10, 15 **c** 25, 17, 46, 4

d 100, 300, 600, 100 **e** 0, 6, 25, 5 **f** 4, 5, 9, 8, 11, 25, 15

2. What is the mean height of these five children?
Pat, 145 cm; John, 155 cm; Linda, 145 cm; Sheila, 140 cm; Mike, 150 cm .

a Is anyone's height equal to the mean height?

b How many are taller than the mean height?

c How many are shorter?

3. Pam, Pat, and Paul worked in a shop.
They made these sales.

Pam: 5¢, 15¢, 20¢, 5¢, 5¢

Pat: 5¢, 5¢, 10¢, 10¢, $3.25

Paul: 25¢, 15¢, 5¢, 10¢, 10¢

Find the average sale for each of the three.

a Why is the one average so different from the other two?

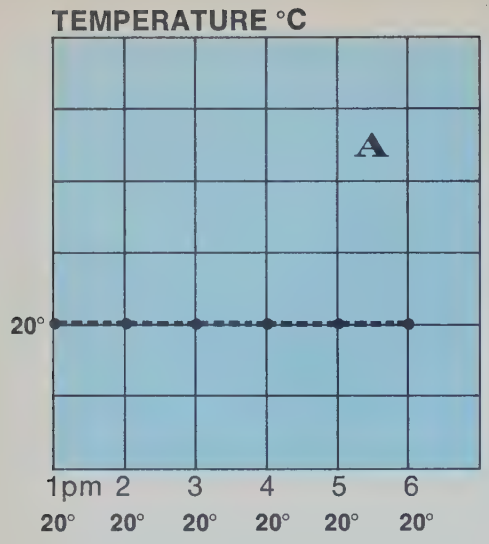
b What is the average of all the sales?

4. Look back at problems you did early in this chapter. Calculate the mean of at least two sets of data. (For example, what's the mean family size?)

5. Find the word *average* in ads, newspaper articles, and other places. Is *average* used to mean the mean? Can you tell?

6. For sports fans. How is each of these found?

a A batting average **b** A hockey-player's goal average **c** A fullback's rushing average



1. "The average temperature yesterday was 20°." Do you know exactly what that means? Or do you need more facts?

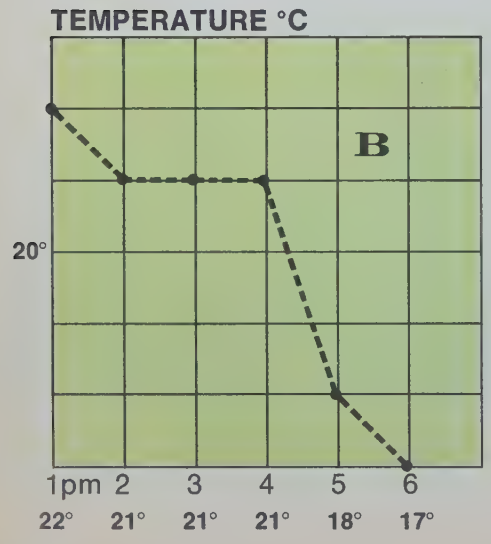
   a What does *yesterday* mean? (Daytime? 24 hours?)

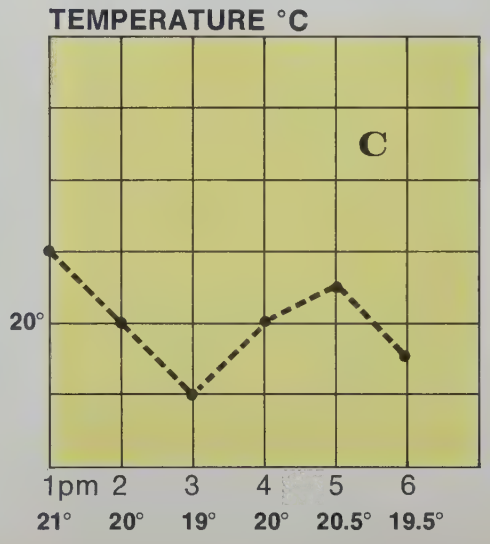
   b How often was the temperature recorded? (Hourly? Three times?)

   c Where was it recorded? (City? Country? Shore?)

   d When was it recorded?

2. Graphs A, B, and C show temperatures taken at six different times. Find the mean temperature for each set of data. If two sets of data have the same mean, do their graphs look alike?





Here are the heights of five children:
Pat, 145 cm; John, 155 cm; Linda, 145 cm;
Sheila, 140 cm; Mike, 150 cm . Who's the shortest? Who's the tallest? Subtract the height of the shortest from the height of the tallest. Your result is the *range* of these data.

140 cm, 145 cm, 145 cm, 150 cm, 155 cm

$$\begin{array}{r} 155 \text{ cm} \\ -\,140 \text{ cm} \\ \hline 15 \text{ cm} \end{array} \leftarrow \text{range}$$

The range of a set of numbers shows the difference between the smallest number and the largest number in the set.

1. Find the range of each set of numbers.

a 5, 7, 18

b 14, 3, 0, 10, 15

c 25, 17, 46, 5

d 100, 300, 600, 100

e 0, 7, 25, 15

f 4, 7, 9, 8, 11, 25, 15

2. How wide do you think the board in your classroom is? Everyone in the class should write down a guess (to the nearest centimetre).

a Tally the guesses.

b Make a bar graph showing the tally.

c Find the range of the guesses.

d Find the mean of the guesses.

Actually measure the board.
How close is the mean to the measurement?

1. A traffic engineer wanted to know how many people caught buses at the train station. She studied four buses. Each leaves the station ten times a day, on the hour. Here are the figures.

PASSENGERS EACH TIME											
Bus	7 am	8 am	9 am	10 am	11 am	12 noon	1 pm	2 pm	3 pm	4 pm	Total
A	21	22	25	25	24	20	21	19	18	18	213
B	25	39	60	60	60	57	55	49	52	47	504
C	5	9	13	11	12	17	60	60	42	12	241
D	4	6	53	60	60	60	35	10	7	6	301

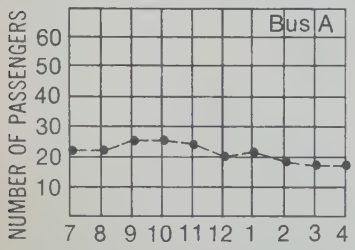
Bus capacity is 60.

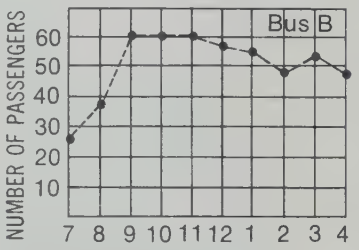
a Tally the number of times each bus left the station with 60 passengers.

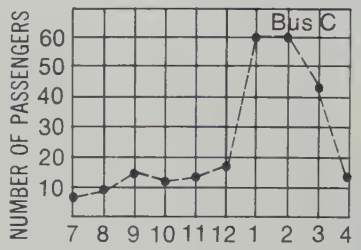
b What is the passenger range for each bus?

c What is the mean number of passengers for each bus?

2. Here are some graphs the traffic engineer prepared.







a Compare the graphs. Why is bus A's line almost flat? Why is there a big hump in bus C's graph? What might cause such a hump?

b Make a graph showing the data for bus D.

Not every kind of information can be put into a graph.

1. Charts can give a lot of information. They save time, too.
What does this chart tell?

ANSWERS CORRECT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
THEN YOUR SCORE IS	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

This chart was made to use when a test had 20 questions. What would your score be if you got all 20 answers correct? Here are the number of correct answers that ten people got. You write the score each will get.

a Amy—17
b Bob—19
c Carol—18
d Dennis—20
e Ellen—20
f Frank—12
g Glen—16
h Helen—15
i Ida—2
j Jack—0

(Ida and Jack had a bad day. They got 20 correct on the next test, though.)

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2. What information does this chart give?

a What good is the information?
b Who might use this information?
c If a store were out of No. 4 tins, what could you buy instead?
d If a store were out of No. 5 tins, would three No. 1 tins be enough?

APPROXIMATE TIN SIZES		
TIN SIZE	MASS	HEIGHT
No. 1	170 g	8 cm
No. 2	225 g	8 cm
No. 3	300 g	10 cm
No. 4	450 g	12 cm
No. 5	575 g	16 cm
No. 6	800 g	21 cm

For research

Look at some tins of food at home. Try to find the size. Tell what is in the tins. Tell how much the label says is inside.

From CHILTON

SATURDAYS ONLY

Read Down	533	21	535	29	31	33	35
	AM	AM	AM	AM	AM	PM	PM
CHILTON Lv (Albert St. Station)	5.10	6.50	7.55	9.40	10.40	11.40	12.40
Kedzie Ar	5.19	6.59	8.04	9.49	10.49	11.49	12.49
Oak Park (Marion St.)	5.27	7.06	8.11	9.56	10.56	11.56	12.56
River Forest	....	....	....	....	....	....	....
Maywood	5.30	7.10	8.15	10.00	11.00	12.00	1.00
Melrose Park	5.32	7.12	8.17	10.02	11.02	12.02	1.02
Bellwood	5.35	7.15	8.20	10.05	11.05	12.05	1.05
Berkeley	5.38	7.18	8.23	10.08	11.08	12.08	1.08
Elmhurst	5.41	7.21	8.26	10.11	11.11	12.11	1.11
Villa Park	5.45	7.24	8.29	10.14	11.14	12.14	1.14
Lombard	5.49	7.28	8.33	10.18	11.18	12.18	1.18
Glenn Ellyn	5.54	7.32	8.37	10.22	11.22	12.22	1.22
College Avenue	....	7.35	8.40	10.25	11.25	12.25	1.25
Wheaton	5.59	7.38	8.43	10.28	11.28	12.28	1.28
Winfield	....	7.42	8.47	10.32	11.32	12.32	1.32
East Hilton Ar	6.10	7.47	8.52	10.37	11.37	12.33	1.37
HILTON Ar	6.18	7.55	9.00	10.45	11.45	....	1.45
	AM	AM	AM	AM	AM	PM	PM

Does anyone in your family ride a train to work? If so, you have seen one of these charts. You have to have good eyes to read charts like this. This type of chart is called a schedule.

1. If you leave Chilton at 10:40 a.m., when would you get to Wheaton?

2. Could you use this schedule if you wanted to go from Maywood to Elmhurst?

3. Could you use this if you were going on Sunday?

4. Pretend you are in the Chilton train station. Pick a place to go. Tell when you will go and when you will get there. Stay there at least one hour. Continue by train to Hilton. Tell when you will go and when you will get there.

Some charts look hard to read. Small print is not the only problem. You may need special information to decode the chart.

The chart below lists some flights from Edmonton to other cities. The letters after each city show that the times are Standard: EST (Eastern), CST (Central), MST (Mountain), and PST (Pacific).

D	A	✕	#	✈	†	①②③④⑤⑥⑦
Edmonton MST						
Toronto/Hamilton EST						
01.50	07.10	B	164	JET	-	①②③④⑤⑥⑦
09.25	14.45	L	106	L10	-	①②③④⑤⑥⑦
15.10	20.30	D	154	L10	-	①②③④⑤⑥⑦
Via Winnipeg						
15.55	23.05	D	232-264	JET-L10	2	①②③④⑤⑥⑦
Vancouver PST						
08.50	09.15		283	JET	-	①②③④⑤⑥⑦
10.00	10.25		275	JET	-	①②③④⑤⑥⑦
15.30	15.55		273	JET	-	①②③④⑤⑥⑦
17.35	18.00	S	281	JET	-	①②③④⑤⑥⑦
20.00	20.25		287	JET	-	①②③④⑤⑥⑦
20.10	20.35		237	JET	-	①②③④⑤⑥⑦
20.50	21.15		293	JET	-	①②③④⑤⑥⑦
Windsor/Detroit EST						
Via Toronto						
01.50	10.15	B	164-307	JET	1	①②③④⑤⑥⑦
09.25	18.15	L	106-325	L10-JET	1	①②③④⑤⑥⑦
Via Winnipeg						
13.55	20.30	D	284-206	JET	1	①②③④⑤⑥⑦
Winnipeg CST						
07.05	10.25	B	212	JET	1	①②③④⑤⑥⑦
10.10	13.30	L	272	JET	1	①②③④⑤⑥⑦
13.55	16.35		284	JET	-	①②③④⑤⑥⑦
15.55	19.15	D	232	JET	1	①②③④⑤⑥⑦
22.40	01.20	S	296	JET	-	①②③④⑤⑥⑦

Courtesy of Air Canada

Each symbol that heads a column has a meaning.

1. The letters D and A head columns that give times of departure and arrival. Both times are given on the 24-hour clock, beginning at midnight.

   You reach Toronto at 7:10 a.m. (07 10 on the 24-hour clock). When did you leave Edmonton?

2. A knife and a fork are the symbol for meals on the flight. Letters tell which meals: B, breakfast; S, snack; L, lunch; D, dinner.

   Which meal are you served if you leave for Winnipeg at 7:05 a.m.? at 10:10 a.m.?

3. The symbol # heads a column of flight numbers. Two numbers show that passengers change flights.

   At which airport do passengers to Windsor/Detroit change to flight 307 or 325?

4. A tiny plane heads a column describing the aircraft. L10 means a Lockheed 1011.

   How many flights to Toronto use *only* L10s?

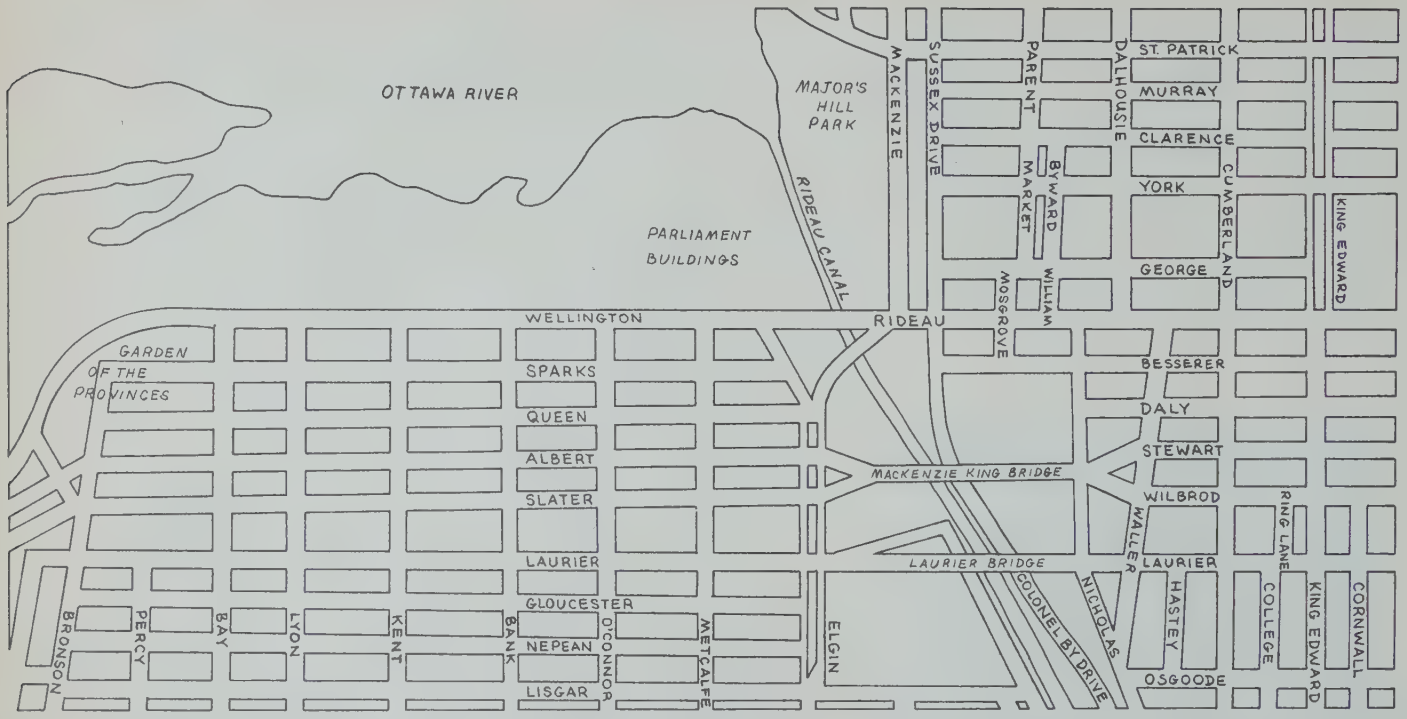
5. The dagger symbol is over numbers telling how many stops. The mark - means a non-stop flight.

   Are all flights to Vancouver non-stop?

6. Circled numbers ending each row tell days of departure, from 1 for Monday to 7 for Sunday.

   Do all flights listed here depart daily?

A street map like this one — Downtown Ottawa — can give you useful information.



1. If you wish to visit the Parliament Buildings, which street must you get to?
2. If you go right across the Rideau Canal from Wellington, which street are you on?
3. What park is situated near the canal? near the curve of Wellington Street?
4. The map names two bridges across the canal. What are the names?
5. Which streets must you cross to go from Rideau to St. Patrick along Dalhousie? from Sussex Drive to King Edward along St. Patrick?
6. Which streets must you cross to go from Wellington to Laurier along Kent? from Kent to Elgin along Laurier?

Collect some maps.
Make sure one of them is a road map.
Look on your maps to see if there is a

LEGEND.

The legend below contains information you need to understand an airline schedule. The legend at the right — for a highway map — is quite different.

Which legend, airline or highway, helps to find out about:

a distances? **b** equipment? **c** boundaries?
d times? **e** stops? **f** ferries?

LEGEND/LÉGENDE

D — Departure/Départ
A — Arrival/Arrivée
✕ — Meals/Repas
\# — Flight/Vol
✈ — Equipment/Équipement
† — Stops/Escales
▶ — Begins/Commence
◀ — Ends/Se termine

① — Monday/Lundi
② — Tuesday/Mardi
③ — Wednesday/Mercredi
④ — Thursday/Jeudi
⑤ — Friday/Vendredi
⑥ — Saturday/Samedi
⑦ — Sunday/Dimanche

15.03.75 Day • Month • Year
Jour • Mois • Année

Origin City/Ville d'origine, e.g. Toronto
Destination City/Ville de destination, e.g. Montréal

747 — Boeing 747
L10 — Lockheed 1011
727 — Boeing 727

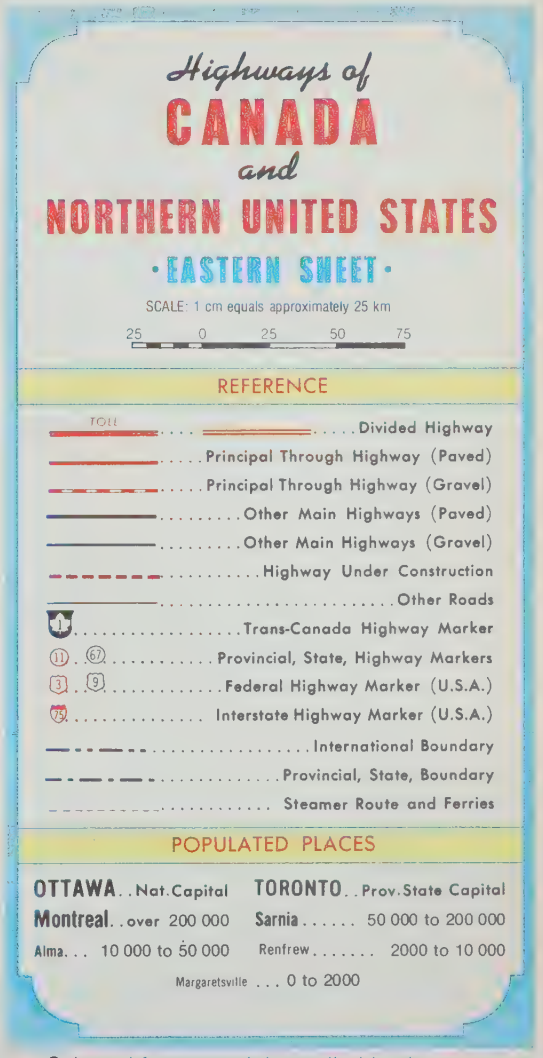
JET — DC8/DC9
PRP — Propellor/Avion à hélices

B — Breakfast/Petit Déjeuner
L — Lunch/Déjeuner
D — Dinner/Dîner
S — Snack/Collation

All Flights are shown in the 24-hour clock.
Tous vols indiqués sur cadran de 24 heures.

0700 — 7 AM
1200 — 12 Noon
1900 — 7 PM
2015 — 815 PM

Courtesy of Air Canada


Highways of
CANADA
and
NORTHERN UNITED STATES
• EASTERN SHEET •
SCALE: 1 cm equals approximately 25 km
25 0 25 50 75
REFERENCE
TOLL
Divided Highway
Principal Through Highway (Paved)
Principal Through Highway (Gravel)
Other Main Highways (Paved)
Other Main Highways (Gravel)
Highway Under Construction
Other Roads
Trans-Canada Highway Marker
Provincial, State, Highway Markers
Federal Highway Marker (U.S.A.)
Interstate Highway Marker (U.S.A.)
International Boundary
Provincial, State, Boundary
Steamer Route and Ferries
POPULATED PLACES
OTTAWA . . Nat. Capital
Montreal . . over 200 000
Alma . . . 10 000 to 50 000
TORONTO . . Prov. State Capital
Sarnia . . . . . 50 000 to 200 000
Renfrew . . . . . 2000 to 10 000
Margaretsville . . . 0 to 2000

Selected from material supplied by the Department of Energy, Mines and Resources
SI usage adopted for scale and populations

A map scale in the legend tells what distance is represented by a given length on the map.

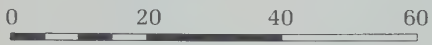
SCALE OF KILOMETRES

1. The bar on this scale is 4 cm long. The scale says that every 4 cm on the map represents 400 km . What does 1 cm on the map represent?

APPROXIMATE SCALE IN KILOMETRES

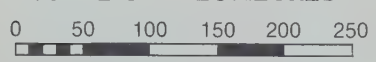
2. This bar is 7 cm long. The scale says that 7 cm represent how many kilometres?

One centimetre equals approximately 10 km



3. One centimetre would represent 10 km if the scale were on a map. How many kilometres would be represented by 2 cm? by 5 cm? by 10 cm?

SCALE OF KILOMETRES



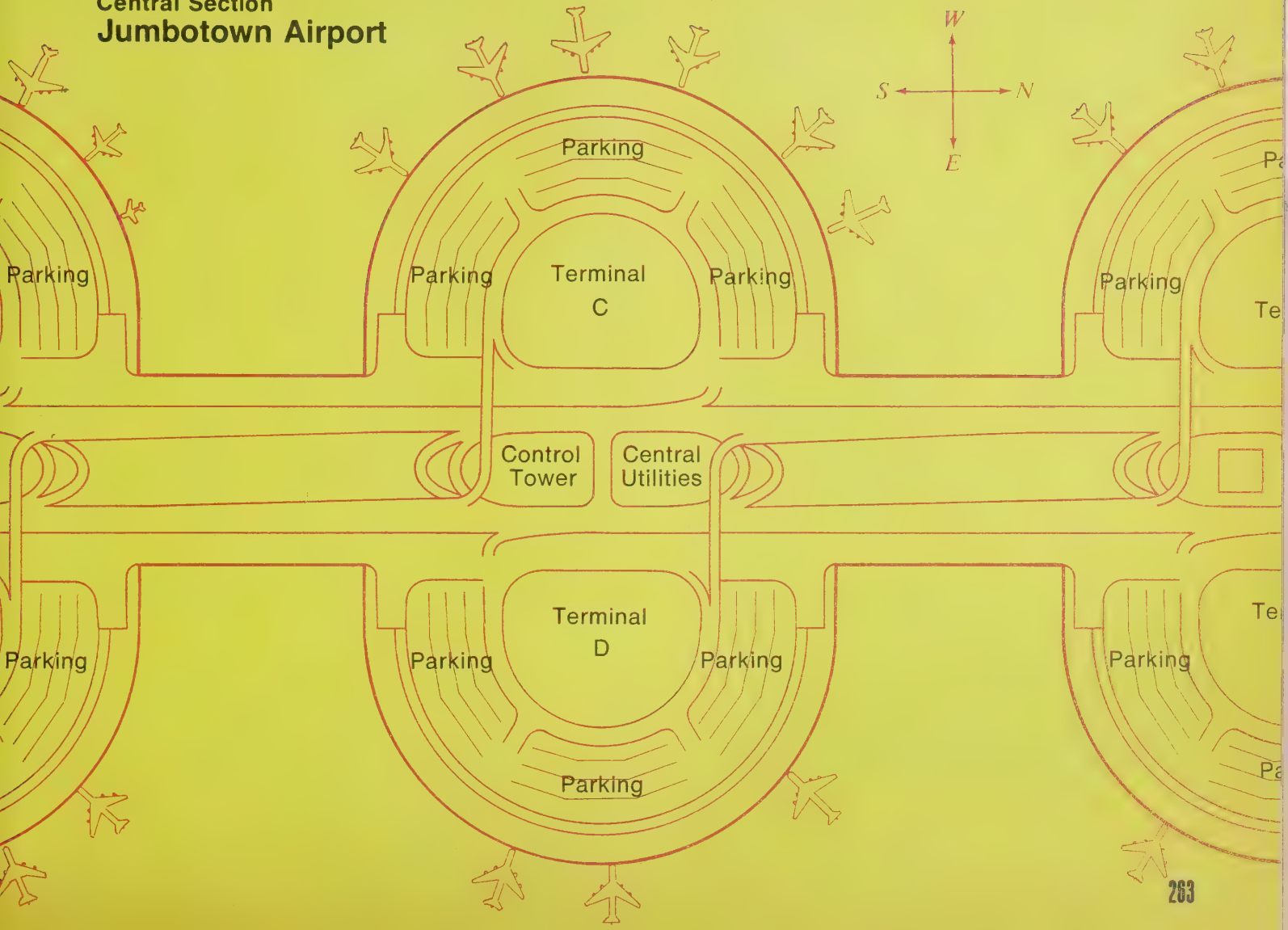
4. If this scale were marked on a map, how many kilometres would be represented by 2 cm? by 5 cm? by 10 cm?

5. If you use your centimetre rule, you will find that 1 cm represents 12.5 km . How many kilometres are represented by 2 cm? by 3 cm? by 5 cm?

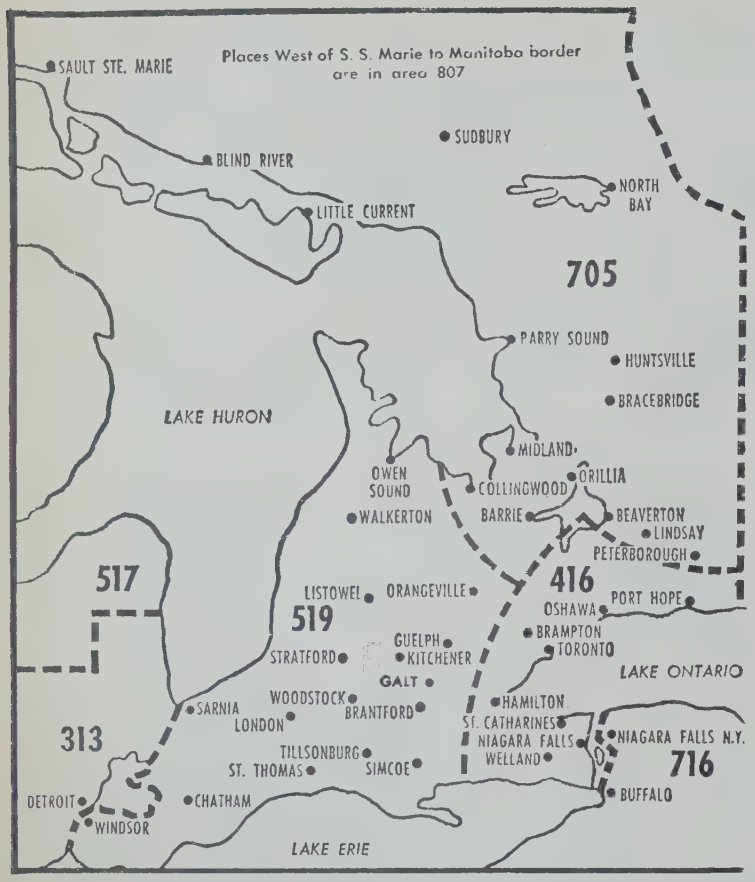
Not all maps are scaled maps. Their purpose is to tell you location, not size or distance.

Central Section

Jumbotown Airport



This is part of an area code map in a telephone directory for Windsor, Ontario.



AREA CODE LIST FOR SOME PLACES YOU CAN DIAL

Please Do Not Dial Area Code for Places in Area Code 519.

Courtesy of Bell Canada

1. What is an area code?

2. When would you use an area code?

3. If you lived in Windsor, what area code would you dial to call someone in Sault Ste. Marie? in Toronto? in Detroit?

4. When would you use a postal code?

5. Postal codes include three letters as well as three numbers. Write AC for area code or PC for postal code for each code below.

   a 807 **b** M5P 1E8 **c** T5N 1L2
   d E3B 5H1 **e** 403 **f** 506

6. What is your area code? postal code?

7. You will need a telephone directory for these answers. Find the area code for each city in the list.

   a St. John's, Newfoundland
   b Sherbrooke, Quebec
   c Moose Jaw, Saskatchewan
   d Grande Prairie, Alberta
   e Nanaimo, British Columbia

You've been using statistics, with data organized in graphs and charts. Statistics helps in understanding what's happened. Another kind of mathematics, called *probability*, helps in understanding what's going to happen in the future.

Suppose you flip a coin. It might land with heads up. What other outcome is possible? You have 1 chance in 2 to get tails.

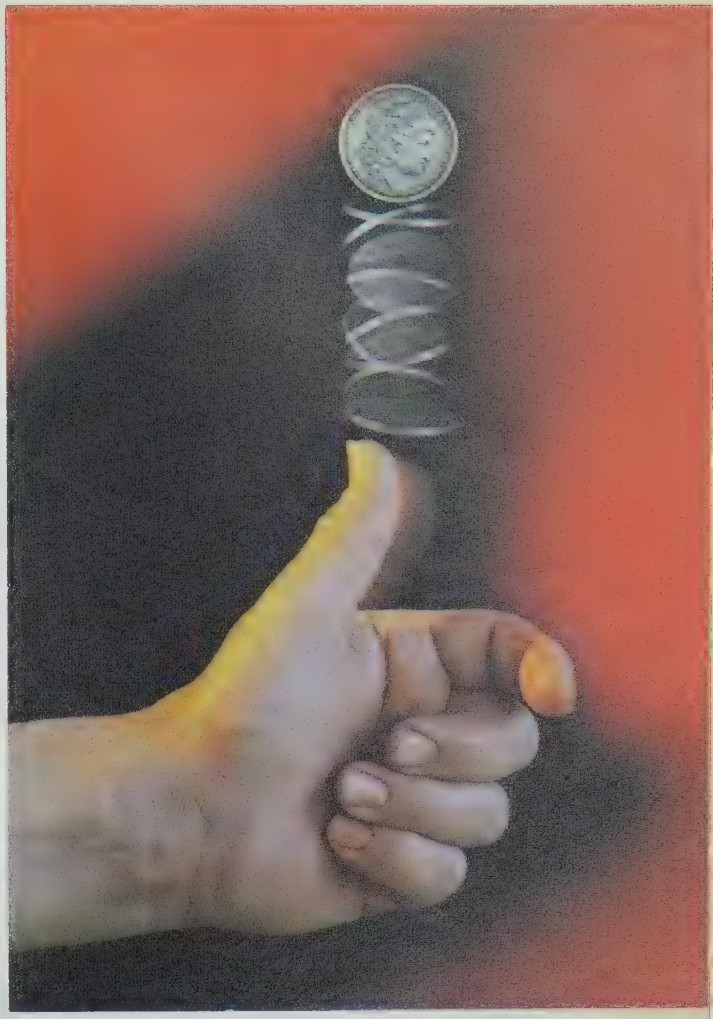
Suppose there are 15 orange jelly beans in a bag. If you reach in without looking, what kind will you get?

Suppose instead that there are 14 orange jelly beans and 1 green jelly bean in the bag. If you reach in without looking, which kind do you think you'll get? *Must* you get that kind? You have only 1 chance in 15 to get green. The outcome is more likely to be orange.

When you have time, try flipping a coin 10 times. Does it land heads up 5 out of the 10 times? Should it?

Try flipping it 20 times. Does it land heads up 10 out of the 20 times? Should it?

The more times you flip it, the more likely it will land heads up half of the time. Why?



Spinners are used in many games. This one has a shaded part and a plain part. Do you know where it will stop? Do you think it's more likely to stop on plain or on shaded?

1. Mark an X on one side of a six-sided pencil. Roll the pencil 12 times. Tally the number of times the X ends up on the top. Is this about what you expected? Repeat 12 more times. Is the result the same?

Plain 𝍸 𝍸 𝍸 𝍸
X 𝍸

2. Put 8 red and 2 black checkers in a bag. (Use disks or other things if you wish. But these should be two different colors.) Draw a checker without looking. Look at it and tally the color. Then put it back in the bag. Repeat for a total of 30 draws. How often did you draw red? black? Is this what you expected?

Black 𝍸 𝍸
Red 𝍸 𝍸 𝍸

3. Flip two coins at the same time. Write "1" if you get two heads. Write "0" if you didn't. Keep writing 0 or 1 until you've flipped both coins 36 times.

0 0 0 1 0 0 1 0 0 0 0 0 0 0 1 1 0

What's the longest string of 0s before you get a 1? What's the shortest string of 0s?

In the preceding section you could tell the chances of something happening just by looking at the possibilities. For example, if there's an X on one side of a six-sided pencil, there's 1 chance in 6 of rolling the X.

Sometimes it's impossible to tell the chances ahead of time. Then we might use statistics to see what happened in the past. This could show what's likely to happen in the future.

If you flip a spoon, it can land with the bowl up or down. Which do you think is more likely? Actually flip a spoon 12 times. Tally the results. Repeat 12 more times. Are the results about the same? If you flipped the spoon 12 more times, what result will you get?

Predictions about the future are usually based on what happened in the past. Sometimes statistics is used in predicting. Often predictions about what a lot of people will do are based on what a few people *have* done (or say they *will* do).

Weather predictions are based on information about past weather.

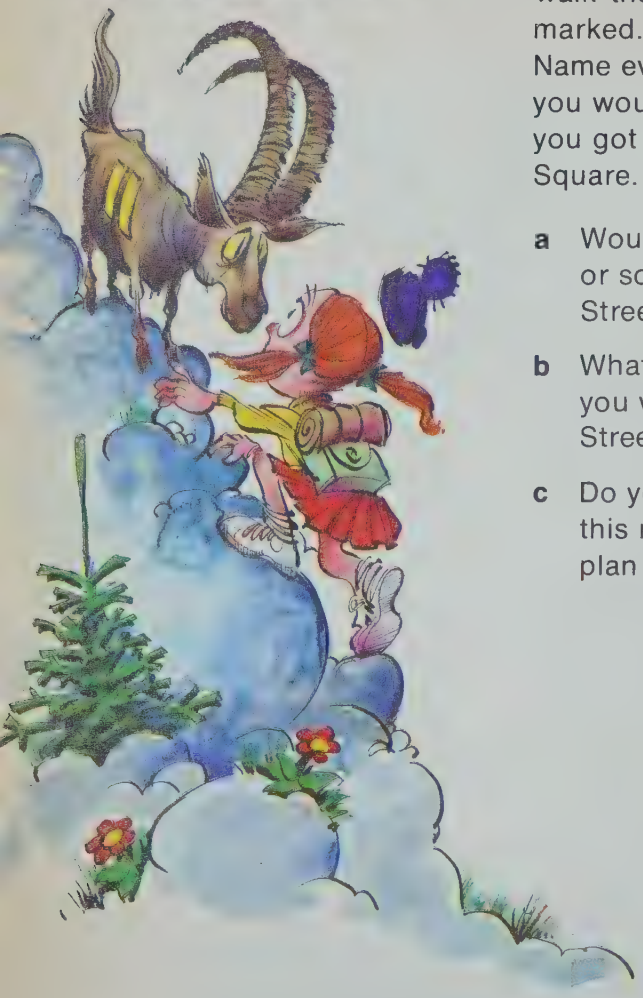
Before an election a small number of people are asked who they'll vote for. Predictions are then made about what will happen when everyone votes.

Temperature and Precipitation Data

	Av. Dates of Freezing Temperatures (0°C or Lower)		Av. Number of Days of Precipitation (All Forms)
	Last in Spring	First in Autumn	
Prairie Provinces —			
Manitoba —			
Churchill	June 22	Sept. 12	141
The Pas	May 28	Sept. 20	128
Winnipeg	May 25	Sept. 21	121
Saskatchewan —			
Regina	May 27	Sept. 27	114
Saskatoon	May 27	Sept. 15	103
Swift Current	May 28	Sept. 19	112
Alberta —			
Beaverlodge	May 22	Sept. 7	129
Calgary	May 28	Sept. 12	113
Edmonton	May 14	Sept. 19	121
Medicine Hat	May 17	Sept. 20	89

Canada Year Book 1973

CHECKOUT

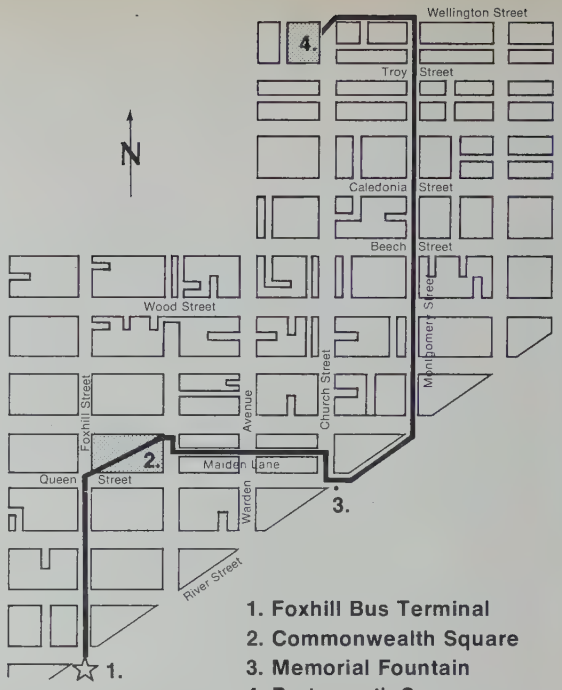


1. Pretend you are going to walk the path that is marked. Start at the star. Name every street that you would walk before you got to Portsmouth Square.

a Would you walk north or south on Foxhill Street?

b What direction would you walk on River Street?

c Do you know from this map how far you plan to walk?



1. Foxhill Bus Terminal
2. Commonwealth Square
3. Memorial Fountain
4. Portsmouth Square

2. A company offers to give you 10¢ if you find "10¢" printed under its bottle cap. You get 25¢ if it says "25¢," and so on, up to $25. But most bottle caps are blank. If you managed the company, would you print "$25" on many caps? Would 10-cent caps be more common than 25-cent caps?

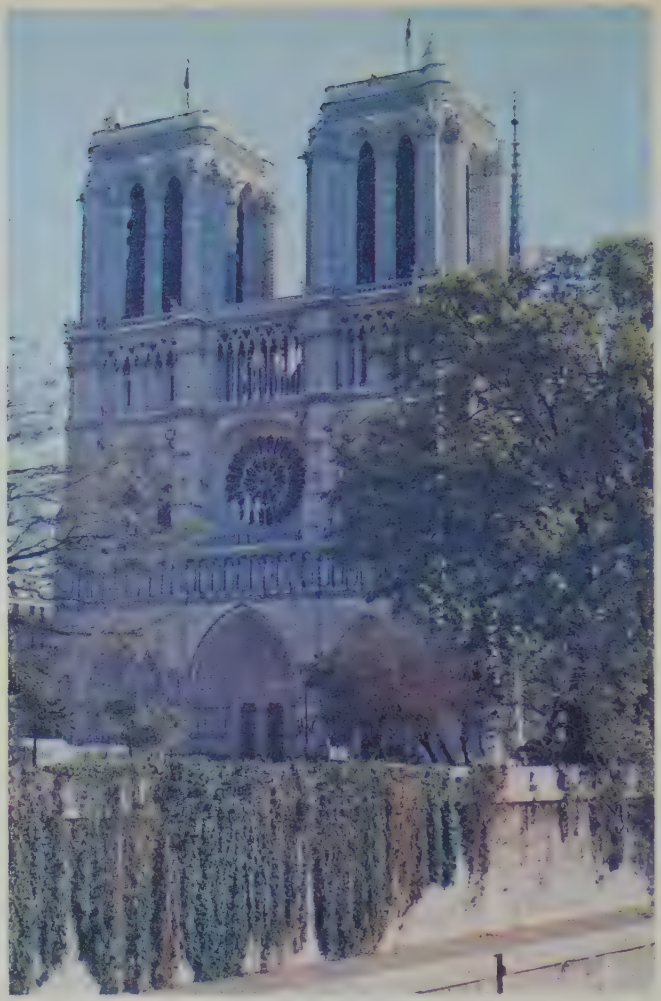
3. Here are ten test scores. There were 25 questions on the test. The scores are the number of questions each person got right. Find the range of the scores. Find the mean (average) of the scores.

20, 18, 24, 8, 15, 12, 17, 23, 24, 21

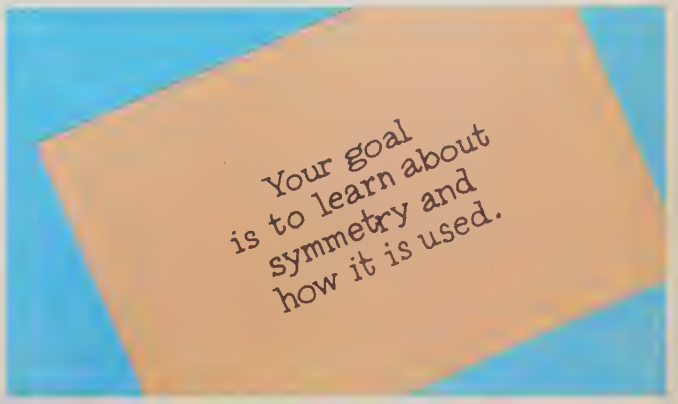
12

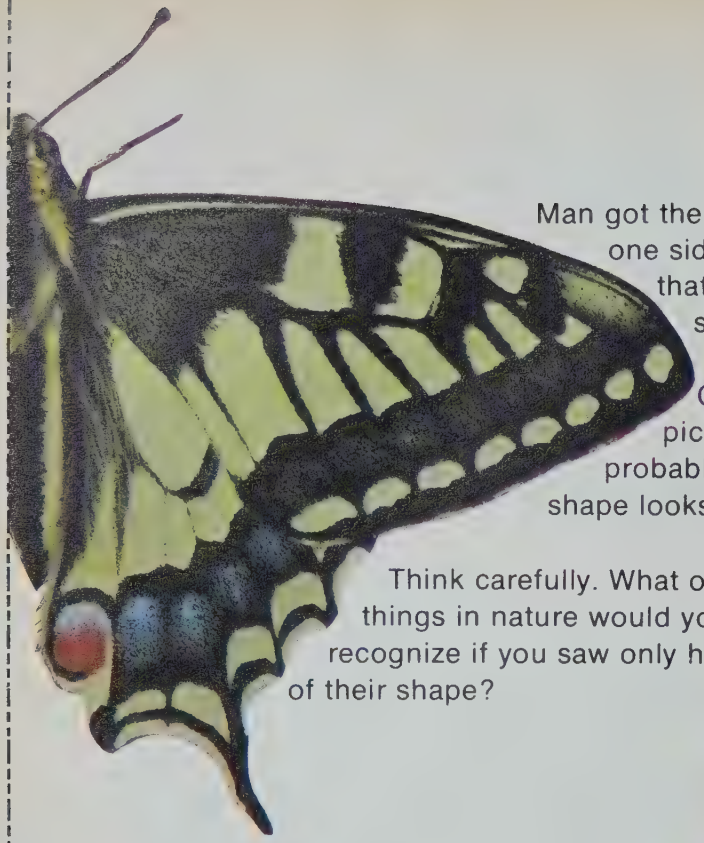
GEOMETRY SYMMETRY

This style of building has been popular for cathedrals over centuries. Why do you think so many people find this style beautiful? What is pleasing about its looks?



Some very modern buildings have one part of their style in common with this very old style of building. What is it? There is an idea of geometry that can help you answer the last question. It's called symmetry. The idea is used a lot.





Man got the idea of balancing one side of an object so that it matches the other side from nature.

Only half of a shape is pictured on the left. But you probably know what the full shape looks like.

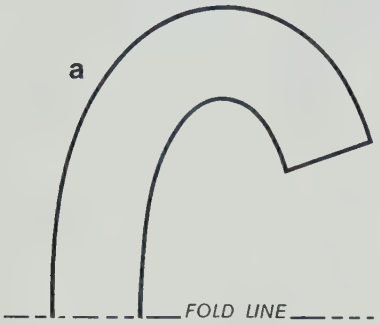
Think carefully. What other things in nature would you recognize if you saw only half of their shape?

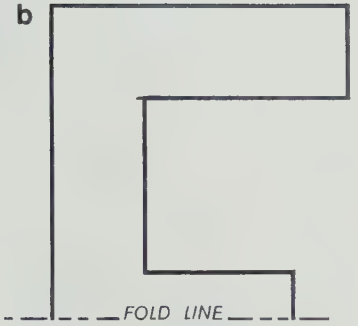
Cover half of this creature with a sheet of paper. If you saw only half, would you know what it was?

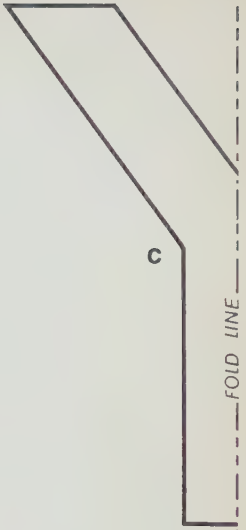
Shape is a big clue that helps you know what an object is.

Think again. What man-made objects would you recognize if you saw only half of their shape? Remember—you have no clues other than shape.

1. Half of each of these figures is folded under the figure itself. What would each figure look like it it were unfolded and you could see the whole shape?



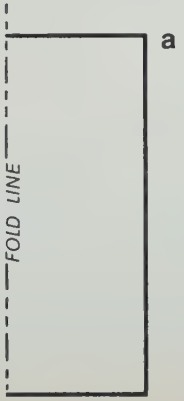


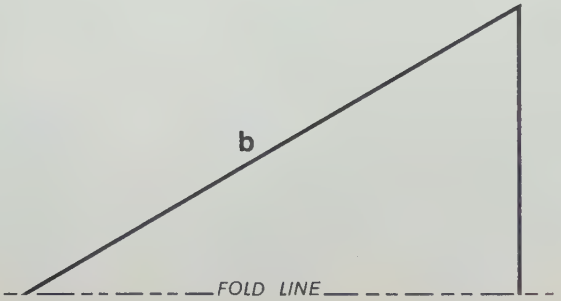


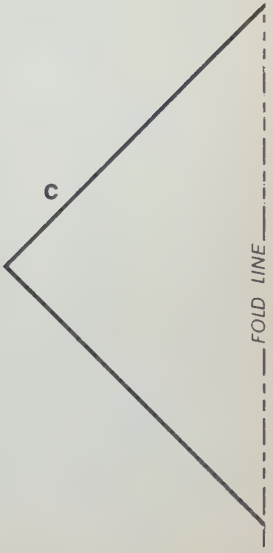
2. Name other letters of the alphabet you think can be cut from folded paper. Name some that cannot. Prove it.

3. Half of each of these figures is folded under itself. What would each figure be if it were unfolded?

Could figure **a** and figure **c** be the same size and shape?







Here are some designs to cut from folded paper. Trace each design, and then fold your paper along the dotted line. Before you cut, try to guess the shape you will have when you open the cutout. Did you guess right?

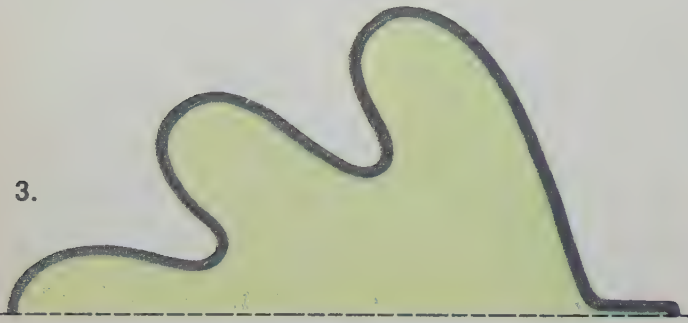
1.

FOLD LINE

2.

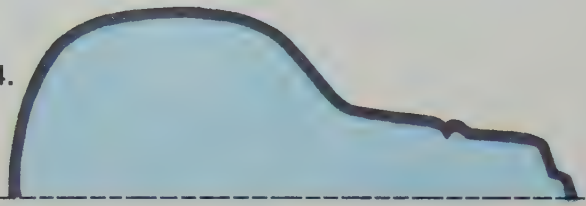
FOLD LINE

3.



FOLD LINE

4.



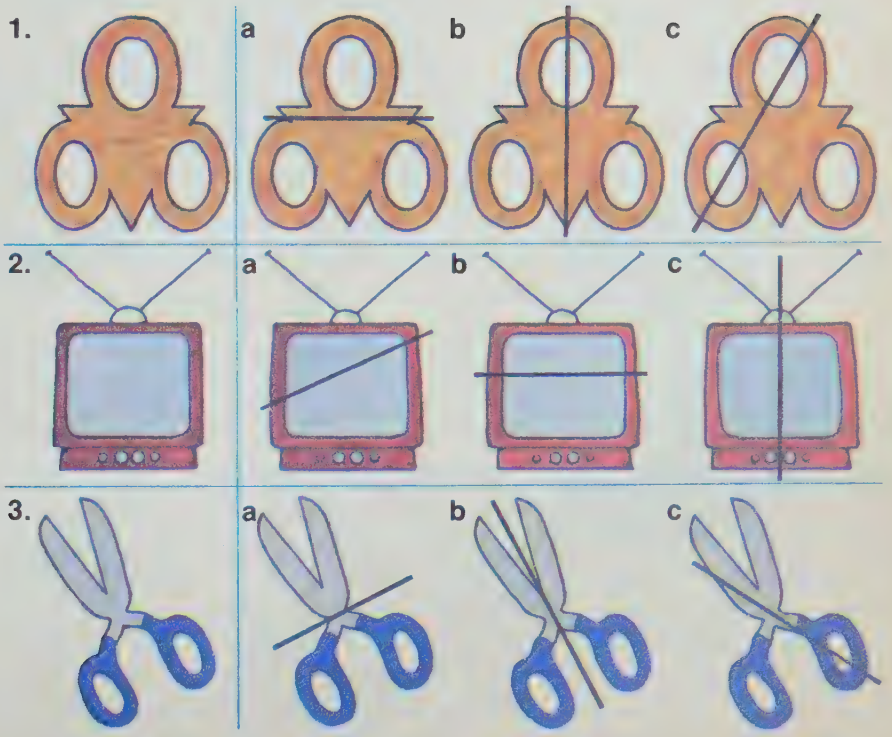
FOLD LINE

The shapes you have been cutting out look the same when they are flipped over because they have two halves that match each other exactly.
Such shapes are called **symmetrical** shapes.

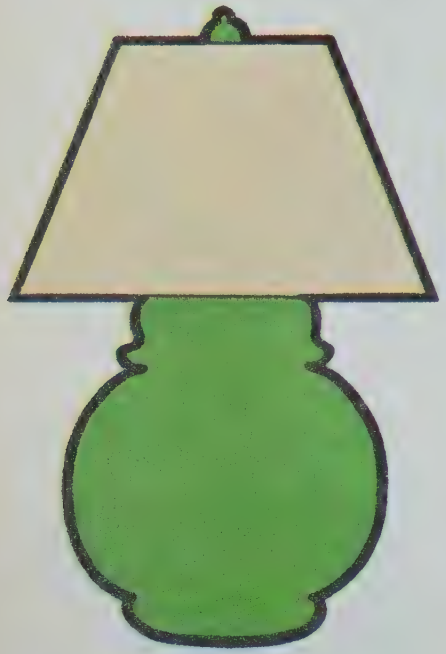
The lines where they can be folded to make matching parts are called

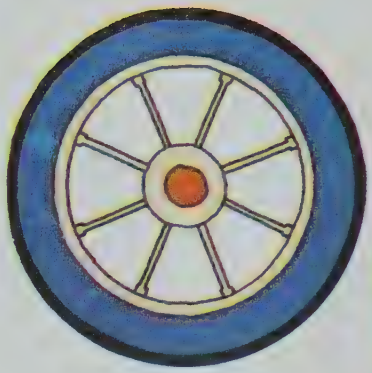
lines of symmetry.

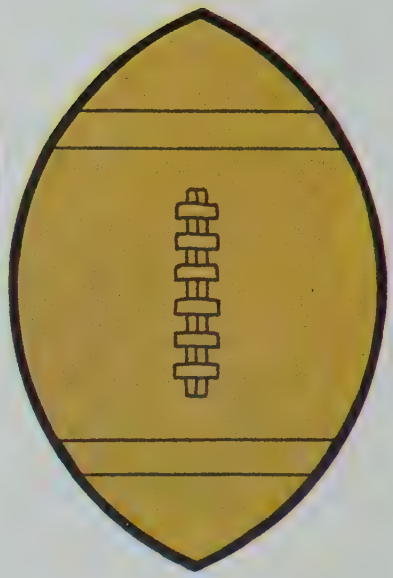
Which figure on the right shows the line of symmetry for the figure on the left?



1. Here are some other shapes that have lines of symmetry. To find a line of symmetry, imagine where the shape could be folded. You can test your ideas by tracing the shape on another sheet of paper and then folding it.







2. Find some other shapes that are symmetrical. If possible, bring pictures or drawings to class. Try to find at least five things in your classroom, five things in your home, and five things outdoors that look symmetrical.

Things to think about.

1. Is the front of your school symmetrical? Is the front of your home symmetrical? Find some buildings that are symmetrical and some that are not.

2. Are cars symmetrical? Look at one from several different sides and ask the question each time.

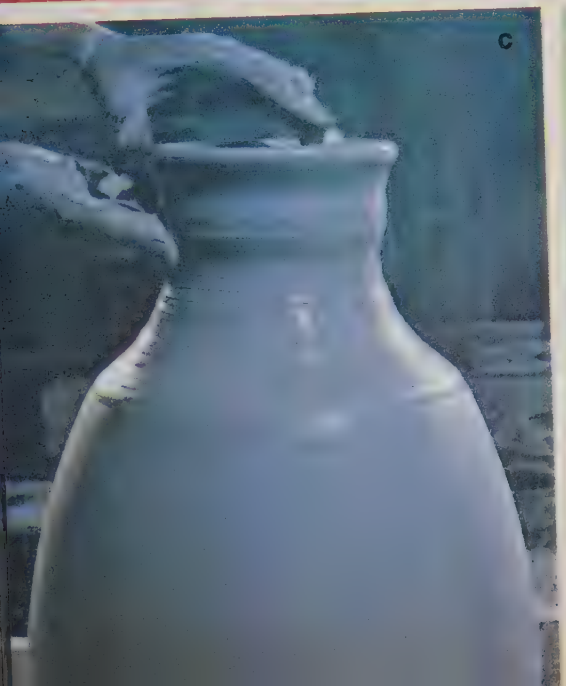
3. Study some fruits and vegetables from different sides. Find some that appear symmetrical and some that do not.

4. Some things seem to be symmetrical in some ways but not in others. Look at your classroom door. Would you consider it symmetrical? In every way? What about the cover of this book? traffic signs? telephones?

5. Look at dogs, cats, birds, and other animals. Are they symmetrical?

6. There are many things in real life that appear symmetrical. This fact is very important to scientists. Can you think of any animal that is not symmetrical in at least some way? Do you see any ways in which a symmetrical structure is helpful to an animal?

1. Are you symmetrical? Do you have matching parts on the two sides of your body? Are your two hands exactly the same? What about your feet or your eyes? Take a good look at yourself and other people.



1. Which of these appear to be symmetrical?
2. Which are symmetrical? (Test by tracing.)
3. Which are man-made? Which are things found in nature?
4. Most natural objects would not pass the strict rules of *exact* matching that we use with our paper designs. Try tracing some real leaves, shells, or other natural objects that appear symmetrical. Do the halves always match exactly when you fold the tracing?

1. Color a piece of paper on one side. Fold it in half. Cut out a shape. Now open out the cutout and the piece of paper from which it was cut. Smooth them out. Fit the cutout back into the hole in the paper. How many ways can you fit the shape into the hole? If the shape is cut carefully, there will always be at least two ways to put the cutout back. Why do you think this is true?

2. Get a sheet of clean paper. Trace the shape of the cutout on the paper. Punch a small hole near the edge of your open cutout. Then fit it back into the outline you have just traced. Mark a point on the tracing through the hole.

3. Flip your cutout. Fit it back into the traced outline. Mark a point through the hole again. Is the second point you marked in the same place as the first one?

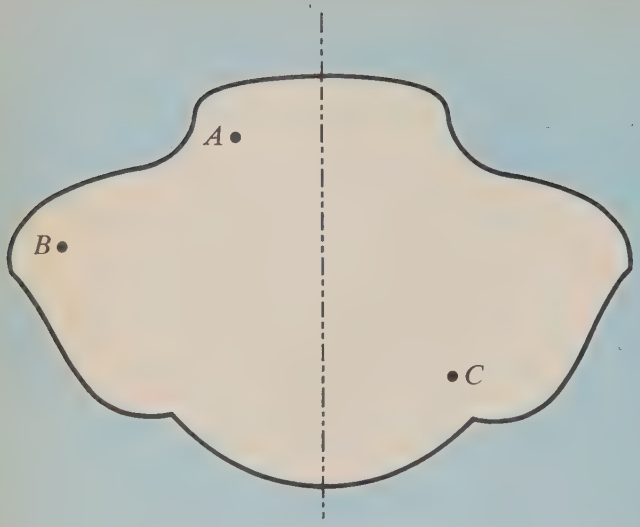
4. Get a ruler. Connect the two points with a *straight* line segment. What is the distance from the point on the left to the fold line? What is the distance from the fold line to the point on the right? How many square corners are formed at the point where your segment passes through the fold line?

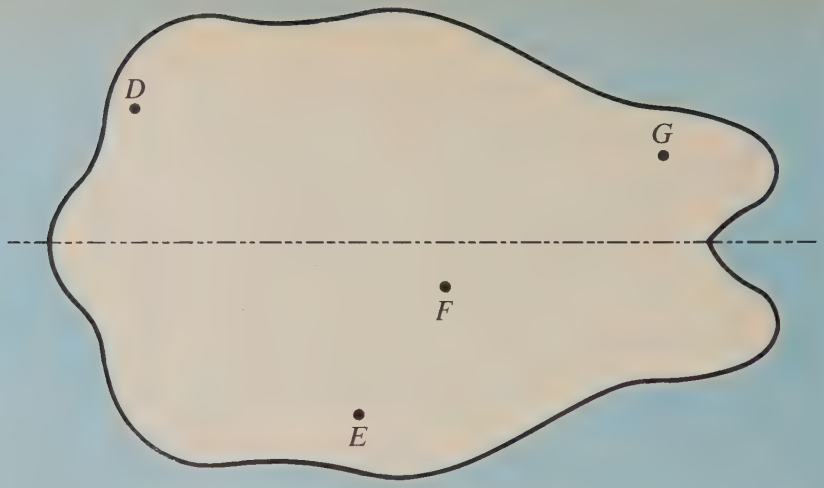
5. Punch another hole on one side of your cutout. Locate the point on your tracing. Flip the cutout. Locate the second point. Connect the two points with a straight line segment. How far is it from the left point to the fold mark? How far is it from the fold mark to the right point? Is this line segment longer or shorter than the first one?

6. Now try to punch a hole in the cutout that will locate *only one* point when the cutout is flipped.

7. Have you got a place on your cutout to punch one more time? This time fold the cutout. Punch the hole so that is goes through both sides of the cutout. Open the cutout. Locate these two new points on your tracing. Flip the cutout. Do you have to locate two new points? Why?

Repeat this activity with other cutouts if you have time.





1. Trace the two shapes above on a sheet of paper. Don't forget the points that are marked. Then find the matching point for each marked point.

2. Use your tracings. Pick two matching points and draw a *straight* line segment connecting them. Does your line cross the line of symmetry? Connect all the matching points in the same way.

3. Study all the lines drawn between matching points. Does each cross the line of symmetry? Is each perpendicular to the line of symmetry?

4. If you have drawn them carefully, each pair of lines will be the same distance apart along their entire length. Do you remember what lines like this are called?

Name of point	Distance of point from line of symmetry	Distance of matching point from line of symmetry	Distance of point from matching point

5. Can you find any pair of matching points joined by a line that is not parallel to the connecting lines of other pairs?

6. Measure the distance between point A and the line of symmetry. Record it on a chart like the one above.

Measure the distance between the matching point and the line of symmetry. Finally, measure the distance between point A and its matching point. Record all your findings.

7. Repeat this for other points until you find some patterns. Then make up some rules to describe the patterns you have found. Test your rules to see if they work for all points on any symmetrical figure.

YOU WILL NEED
SEVERAL SMALL SHEETS OF PAPER AND SCISSORS.

1. Fold a piece of paper in half.
Make sure straight sides match.
Cut off one corner. ⟶
Take the corner.
What shape do you think it will be?

Open it.

- **a** Measure. How many sides are the same length?
- **b** Match. How many angles are the same size?
- **c** Is the fold mark a line of symmetry?
- **d** Are there other lines of symmetry?

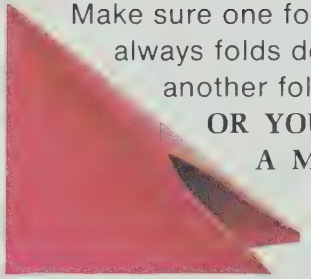
Experiment. How can you cut a folded sheet of paper and be sure each side will be the same length?

2. Use another piece of paper.
Fold in half.
Fold again so that the folded side comes together.
Measure 3 cm up from the folded corner.
Mark a point.
Measure 3 cm out on the other folded side.
Mark another point.
Connect the points with a straight line segment.
Cut along the lines.
Use the corner.
What shape do you think you will get?

Unfold.

- **a** Measure. How many sides are the same length?
- **b** Match. How many angles are the same size?
- **c** Are the fold marks lines of symmetry?
- **d** Are there others? How many?

DO THE SAME SORT OF THING AS YOU DID ON THE LAST PAGE. THIS TIME FOLD SO THAT YOU WILL HAVE EIGHT PARTS.



Make sure one folded edge always folds down to match another fold.

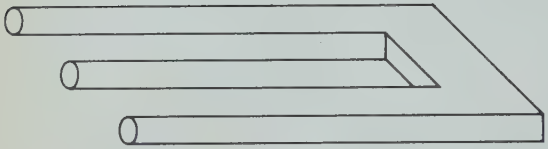
OR YOU WILL HAVE A MESS.

1. Mark the same length on each of the folded sides. Connect those points with a straight line segment. Cut along the line. Use the corner piece. Open it.
   - **a** How many sides?
   - **b** What is the length of each side?
   - **c** How many angles?
   - **d** Does a fold mark cut through each angle?
   - **e** Is each fold mark a line of symmetry?
   - **f** Are there other lines of symmetry? How many?

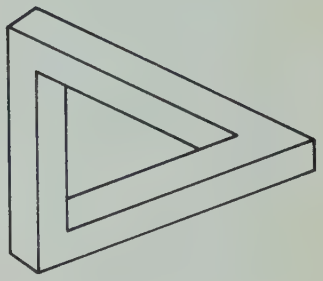
2. Fold another piece of paper in eight parts. This time make some different designs. Your first try will probably look like a paper doily. Keep on. You get some great designs. And each will be symmetrical.

Many optical illusions
are tricks played on your eyes
because of symmetry.

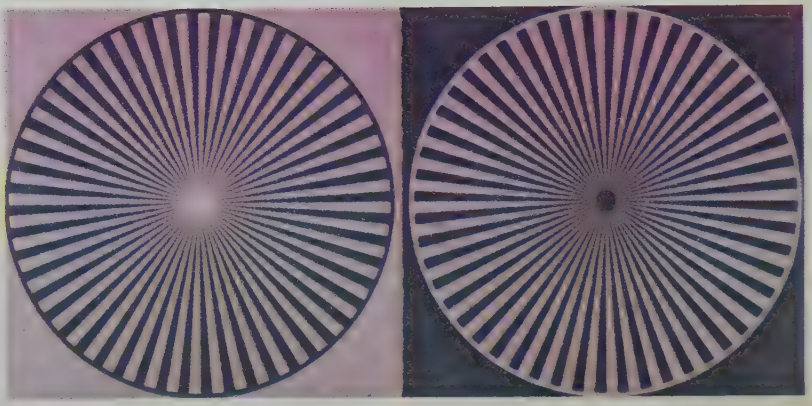
Do you see a vase?
Do you see two faces?



Look for two different images
in each of these pictures.



Collection, The Museum of Modern Art, New York.
Larry Aldrich Foundation Fund. Photograph
Sandak, Inc.



MEASURE

TRACE

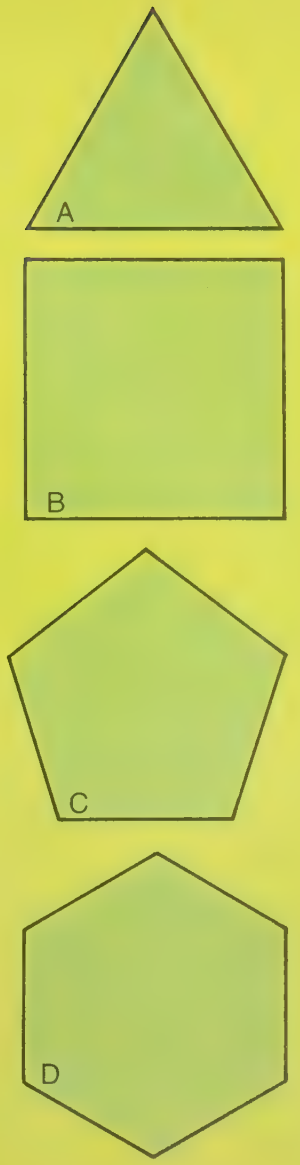
FOLD

ANSWER THESE QUESTIONS ABOUT EACH SHAPE

1. How many sides?
2. Are they the same length?
3. About how long is each side?
4. About how far is it around all the sides?
5. How many angles?
6. Are the angles the same size?
7. Is the figure symmetrical?
8. How many lines of symmetry?

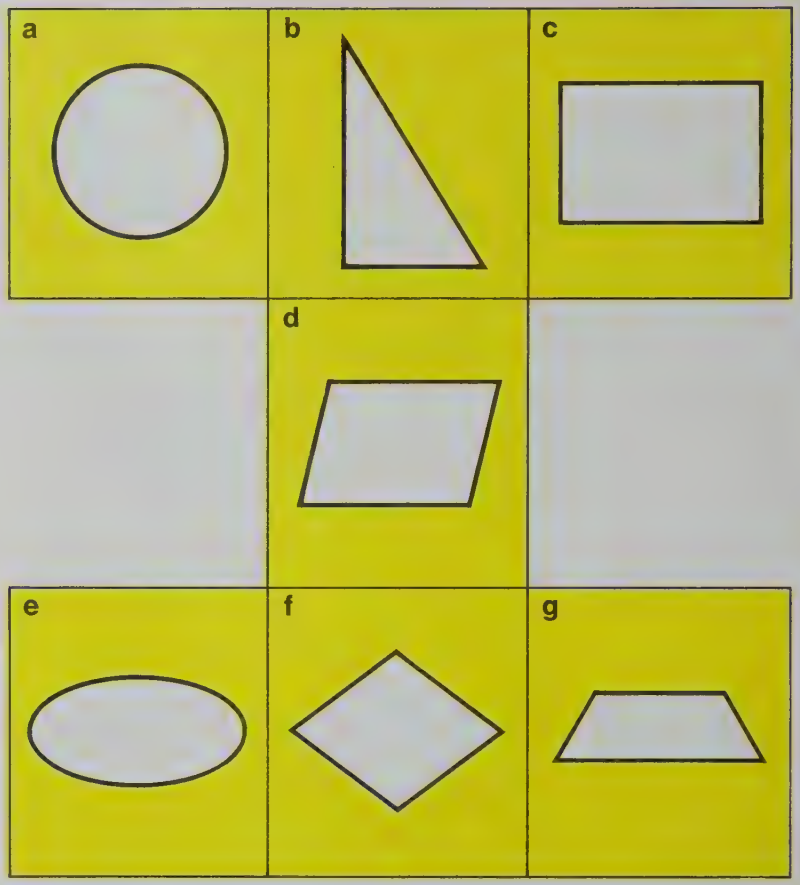
DO YOU REMEMBER—

that these are plane figures?
that they are called regular figures?
the name given to each shape?
What are the names?



CHECKOUT

1. Name the shapes that are symmetrical. Trace and fold if you want to make sure.

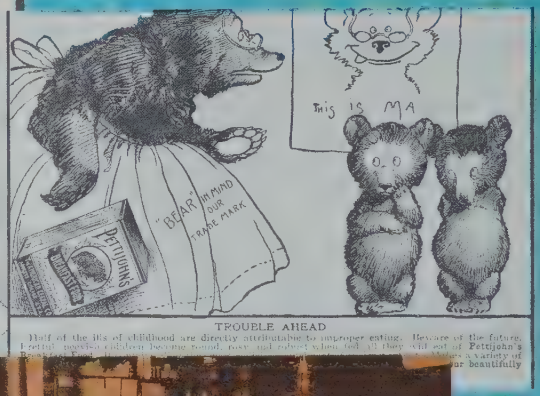


2. Which of the shapes above have only one line of symmetry?

3. Which of the shapes above have more than two lines of symmetry?

13

COMPUTATION



TROUBLE AHEAD

Half of the ills of childhood are directly attributable to improper eating. Beware of the future. Letting growing children become round, rosy and robust when half of their still eat of Pettijohn's [illegible] a variety of [illegible] ar beautifully

Your goal is to review your computational skills and have some fun, too!

There were no supermarkets in the early 1900s. There were very few department stores. Everything a family needed was found at the general store. Many foods were sold from big barrels. (They didn't have fancy packages in those days.) People used to gather at the general store and talk. These talks were called cracker-barrel sessions. They talked of things in the past and things in the future.

You can think of this chapter as a sort of math general store. It presents number ideas from the past. It also has some number ideas for the future. It has other things, too. There are three types of pages in this chapter.

pages. These pages will give everyone a chance to share ideas and discuss what's coming next.

Very Important Problems pages. You will work through these pages on your own. If you show that you know how to do them, you won't have to do many. If you don't know how to do them, you will get some help and have a chance to practise.

pages. These are scattered throughout the chapter. They are some interesting extra activities you can do when you have time.

1. What is the largest number you can find on these two pages?

2. What is the smallest number you can find?

3. Find the roman numerals. Write them down. Then rewrite them as standard numerals.

4. Find all the numbers that are in measurement situations. Write them. Tell whether the unit of measure is length, mass, capacity, time, temperature, or money.

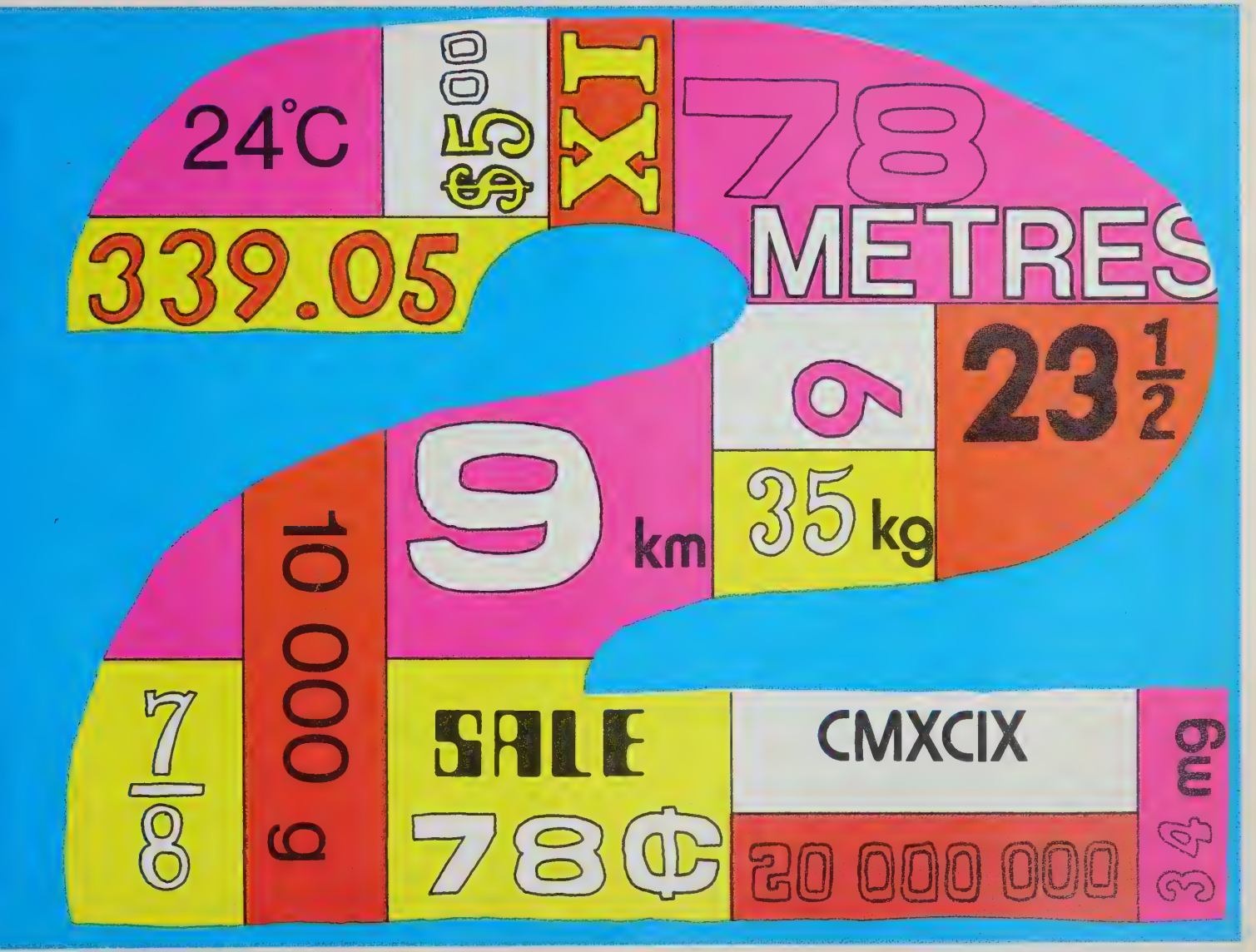
5. Write the fractions you can find.

And here is a real challenge!

As soon as mathematics class is over, try not to say any number of any kind until tomorrow at this same time. Keep track of the times you had to say a number. Jot down the reason you had to say it.

Remember — no numbers! No talk of time, money or temperature. No street numbers, or bus numbers, or TV channels, or phone numbers either.
Get the idea?

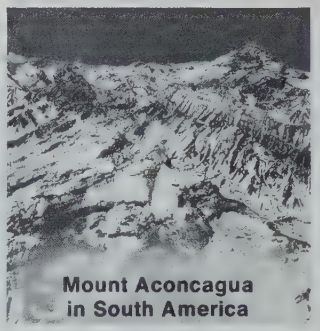
Good luck!


24°C
5 000
EXIT
78
339.05
METRES
9
23 1/2
9 km
35 kg
10 000 9
7 1/8
SALE
78¢
CMXCIX
20 000 000
34 mg

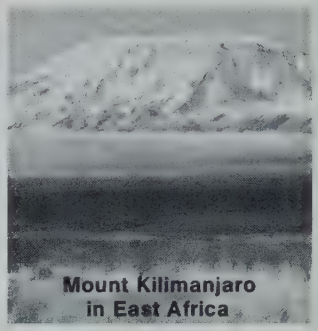
Very Important Problems

Pictured below are several of the tallest mountains in the world. Their exact heights are given. Usually the heights of mountains are given as rounded numbers. That job is left to you.

8948 m



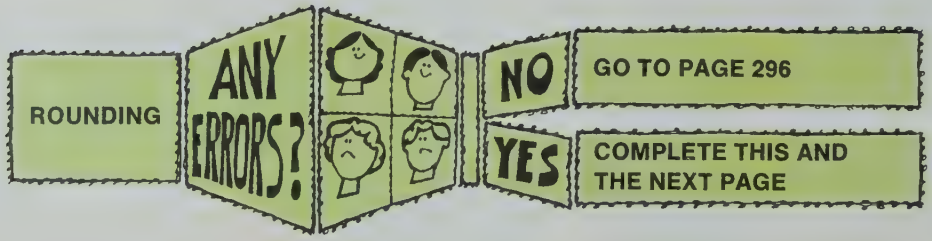
6960 m



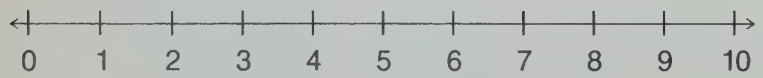
5895 m

a Round each height to the nearest ten.
b Round each height to the nearest hundred.
c Round each height to the nearest thousand.

Check your answers with the key on page 336.



Look at the number line below.



Which numbers on the line are closer to 0 than to 10?
Which are closer to 10 than to 0?
Is 5 closer to 0 or to 10?

When rounding, always round 5 up to 10.

rounding

When rounding to the nearest ten, look at the ones. If there are 5 or more ones, round up. If there are less than 5, round down.

For example: 27 rounds to 30. 495 rounds to 500. 3574 rounds to 3570.

Now it's your turn. Round these numbers to the nearest ten.

1. 9 **2.** 43 **3.** 89 **4.** 104 **5.** 644 **6.** 898

7. 2350 **8.** 4788 **9.** 73 **10.** 10 515 **11.** 17 **12.** 666

Is $153 closer to $100 or to $200? When rounding to the nearest hundred, look at the number to the right of the hundreds. If there are 5 or more tens, round up. If there are less than 5 tens, round down.

For example: 237 rounds to 200. 71 rounds to 100. 4672 rounds to 4700.

Your turn again. Round to the nearest hundred.

13. 508 **14.** 381 **15.** 35 **16.** 299 **17.** 650 **18.** 1783

19. 1870 **20.** 992 **21.** 6430 **22.** 12 745 **23.** 57 **24.** 4673

When rounding to the nearest thousand, look at the numbers to the right of the thousands.

For example: 23 532 rounds to 24 000. 4499 rounds to 4000.

Round these numbers to the nearest thousand.

25. 1570 **26.** 3845 **27.** 2220 **28.** 6399

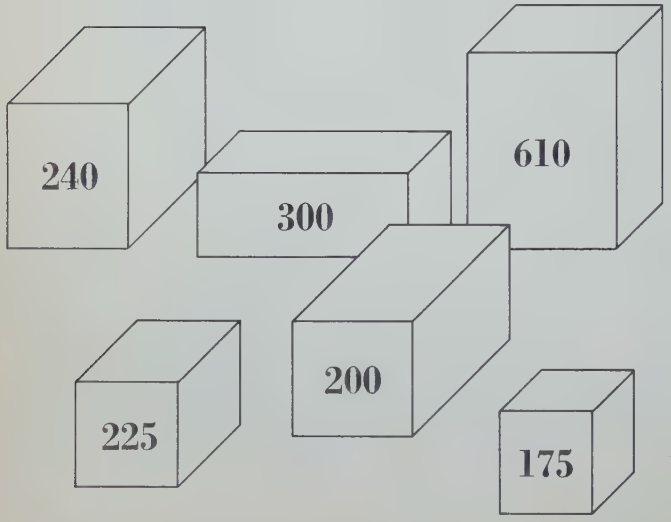
29. 7904 **30.** 15 038 **31.** 34 762 **32.** 84 360

Can you round to the nearest ten-thousand? Try it with these numbers.

33. 18 724 **34.** 33 786 **35.** 25 000 **36.** 76 369

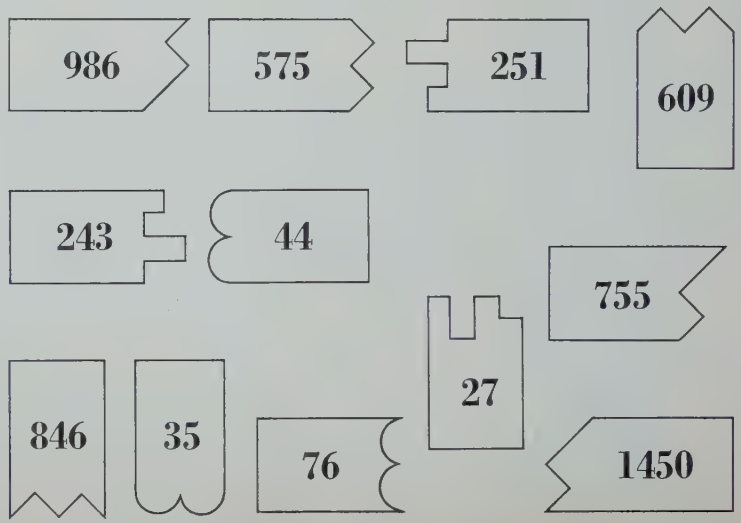
Mixed Bag

1. You have a boat that can carry 900 kg of cargo. There are 6 boxes that need to be taken across the lake. What boxes can you take at one time so that you have to make only two trips?



2. Here's another easy puzzle. Use estimates to help you. Find two pieces that fit together, and name each one of these products.

2660 515 214 6561 744 430



Graduation Barrel Session

You use rounding to estimate answers to addition problems.

Jay wanted to buy these things:
Book $2.98
Shirt $3.59
He had $5.00. Did he have enough money?
$2.98 rounds to $3.00
$3.59 rounds to $3.60
No, he didn't have enough money.
You don't need pencil and paper for that.

Using rounded numbers, estimate and answer these questions.

1 He needed about 500 trading stamps.
He had 185 in one book.
And 335 in another book.
Did he have about 500?

2 They hoped about 700 people would vote.
372 people voted in one place.
229 people voted in another place.
Did about 700 people vote?

3 There were about 100 chairs in the room.
36 parents planned to come.
24 students would attend.
And 9 teachers would be there.
Were there enough chairs?

4 Are these reasonable answers?

a $395 + 475 = 760$ **b** $525 + 675 = 1110$

Very Important Problems

Estimate the sums. Then compute exact sums.

a $\begin{array}{r} 375 \\ +\ 195 \\ \hline \end{array}$ **b** $\begin{array}{r} 5267 \\ +\ 1956 \\ \hline \end{array}$ **c** $\begin{array}{r} 525 \\ 296 \\ +\ 713 \\ \hline \end{array}$

Check your answers with the key on page 336.

An estimate will not tell you if your answer is right. It will tell you if your answer is reasonable.

695 rounds to 700
+ 225 rounds to 200
900 is a good estimate.
The answer is 920.
It is reasonable.

Here are some completed problems. Some are right. Some are unreasonable. Round the numbers computed. Then estimate to find which answers are not reasonable. Do not compute.

1. $\begin{array}{r} 476 \\ +\ 341 \\ \hline 817 \end{array}$ **2.** $\begin{array}{r} 222 \\ 993 \\ +\ 316 \\ \hline 5131 \end{array}$ **3.** $\begin{array}{r} 11 \\ 198 \\ +\ 224 \\ \hline 533 \end{array}$ **4.** $\begin{array}{r} 8825 \\ +\ 2914 \\ \hline 11739 \end{array}$

5. $\begin{array}{r} 326 \\ 555 \\ +\ 189 \\ \hline 970 \end{array}$ **6.** $\begin{array}{r} 75 \\ 284 \\ +\ 407 \\ \hline 666 \end{array}$ **7.** $\begin{array}{r} 4008 \\ +\ 8986 \\ \hline 12994 \end{array}$ **8.** $\begin{array}{r} 51294 \\ +\ 3414 \\ \hline 55706 \end{array}$

Mixed Bag

You can add the first three counting numbers in your head: $1 + 2 + 3 = 6$

Now add the first ten counting numbers in your head.
No pencil and paper, please.
That's not quite so easy, right?

1	Look at them in a column.	1
2	Can you add them now?	2
3		3
4	The column on the right shows	4
5	an easier way of doing it.	5
6	$1 + 10 = 11$	6
7	$2 + 9 = 11$	7
8	$3 + 8 = 11$	8
9	$4 + 7 = 11$	9
10	$5 + 6 = 11$	10

How many 11s? What's the product?
What's the sum of the first ten counting numbers?

Try to find shortcuts for adding these numbers.

1.
$$\begin{array}{r} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ +\ 7 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ +\ 10 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ +\ 12 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 2 \\ 4 \\ 6 \\ +\ 8 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ +\ 12 \\ \hline \end{array}$$

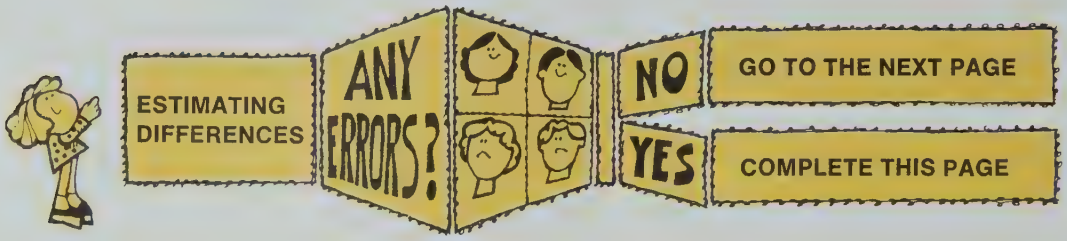
6. Experts only! Find the sum of the first 28 counting numbers without writing all the numbers down. Now try the first 39. **GOOD LUCK!**

Very Important Problems

Estimate the differences. Then compute exact differences.

a	b	c
705 − 496	6020 − 2856	4250 − 987

Check your answers with the key on page 336.



An estimate will tell you if your answer is reasonable.

963 rounds to 1000
− 225 rounds to 200
800 is a good estimate.
The exact answer is 738. Is it reasonable?

Here are some completed problems. Some are right. Some are unreasonable. Round; then estimate to find which answers are not reasonable. Do not compute.

1.	2.	3.	4.
824 − 387 = 537	601 − 225 = 376	8523 − 3918 = 4605	7030 − 942 = 6988

5.	6.	7.	8.
2725 − 1826 = 1899	5104 − 3313 = 1791	426 − 149 = 377	1403 − 649 = 854

You use rounding to estimate answers to problems.
Here is an example:

Mrs. White bought 17 tins of soup at 31¢ a tin.
The sale totalled $8.50 on the cash register.
Was the register working properly?

You can do this problem in your head. **Watch!**

17 rounds to 20.
31 rounds to 30.
How much is 20×30?
Is 850 close to 20×30?

Was the register working properly? Check to make sure.
Multiply 17×31 with pencil and paper.

An estimate will not tell you if your answer is right.
It will tell you if your answer is reasonable.

Are these reasonable?

1. Belle earns $0.75 an hour. She worked 12 hours one week. She said she earned about $75.00 that week.

Is that reasonable?

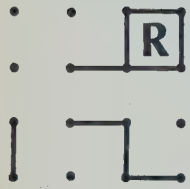
2. Bob had 7 boxes. Each box had 750 clips in it. He said he had less than 5000 clips.

Is that reasonable?

3. Mr. Jones drove 37 km every day to get back and forth from work. He said that he drove over 200 km each week just going to and from work.

Is that reasonable?

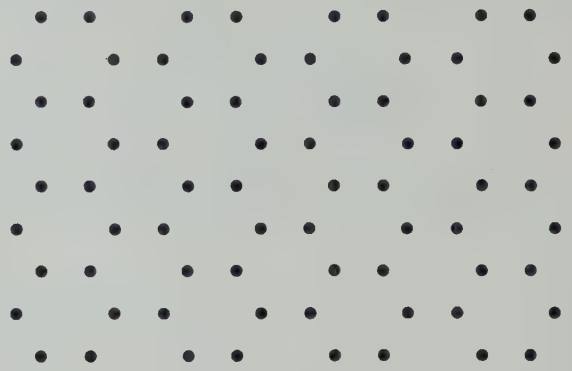
Mixed Bag



You may already know how to play the game of Dots. Each player connects two dots with a vertical or horizontal line segment. Eventually a player will complete a square. When he does, he puts his initials inside it and gets an extra turn. The player who makes the most squares wins.

You can play another version of Dots with regular hexagons instead of squares.

Copy or trace the following pattern of points. Connect the dots with horizontal or diagonal line segments only.



When a player forms a hexagon, he puts his initial inside and gets an extra turn.
Example:

Why is this pattern necessary for a game of Dots with regular hexagons?

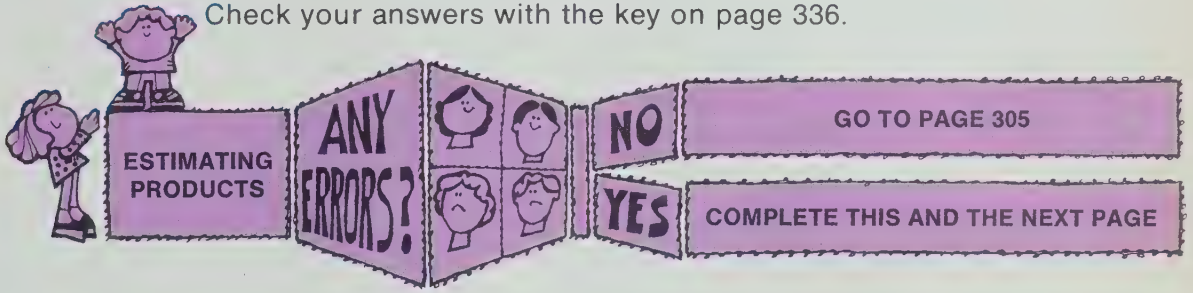
Can you design a pattern for a game of Dots with a polygon with three sides?

Very Important Problems

Estimate the products. Then compute the exact products.

Item	Number of boxes	Number in each box	Estimated product	Exact product
Paper hats	12	12	100	144
Music makers	18	36	**a** ?	**b** ?
Balloons	237	72	**c** ?	**d** ?
Licorice sticks	578	144	**e** ?	**f** ?

Check your answers with the key on page 336.



When estimating in multiplication, you round both factors and then multiply. But do you always know which place to round to?

Your goal is to estimate the answer in your head.
Use numbers you can multiply without using pencil and paper.

$$\begin{array}{rcl} 538 & \text{rounds to} & 500 \\ \times\ 74 & \text{rounds to} & 70 \end{array}$$

You can do this in your head.

Then when you compute the exact answer, you'll know if it's reasonable.
Would an answer close to 350 be reasonable?
Would an answer close to 3500 be reasonable?
What number should the exact answer be close to?

Do these in your head. Write the answers only.

1. 30×10	**2.** 50×20	**3.** 70×40	**4.** 200×50	**5.** 300×80
6. 500×70	**7.** 900×60	**8.** 300×200	**9.** 700×400	**10.** 800×700

First round the numbers. Then write the product of the two rounded numbers. Do not find the exact product.

11. 24×13	**12.** 57×49	**13.** 84×75	**14.** 115×38	**15.** 250×95
16. 473×46	**17.** 851×99	**18.** 777×129	**19.** 501×291	**20.** 699×549

Use rounding to estimate the product. Then compute the exact answer.

21. 34×12	**22.** 97×45	**23.** 54×48	**24.** 125×43	**25.** 383×71
26. 899×99	**27.** 715×14	**28.** 481×149	**29.** 809×305	**30.** 555×499

These look like ***awful*** problems, but you'll be surprised. Multiply and see what happens.

1. $\begin{array}{r} 12345679 \\ \times \quad 9 \\ \hline \end{array}$ **2.** $\begin{array}{r} 12345679 \\ \times \quad 18 \\ \hline \end{array}$ **3.** $\begin{array}{r} 12345679 \\ \times \quad 27 \\ \hline \end{array}$

4. What do you think the product will be if you multiply by 36? You'd better check to make sure.

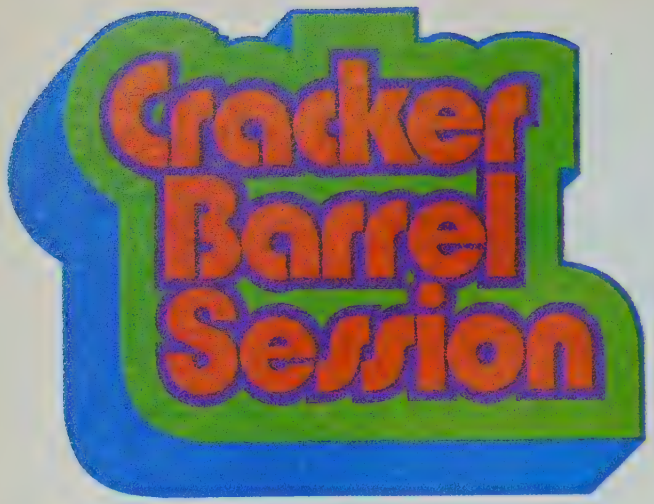
5. If the pattern holds, what will the product be if you multiply by 45? 54? 63? 72? 81?

The poor digit 8 was almost ignored in the problems above. That's not fair. Complete this computation and see what happens.

6. $(9 \times 9) + 7 = \underline{?}$ **7.** $(98 \times 9) + 6 = \underline{?}$ **8.** $(987 \times 9) + 5 = \underline{?}$

9. If the pattern holds, what would be the next complete sentence in order?

10. Only one other digit has not been considered. That's zero. You can take care of that in a hurry. Zero times any number equals what number? Now all digits are present and accounted for.



You can use estimation for a quick quotient that does not need to be exact. For example:

Cyclists are planning a tour that will cover 579 km . They plan on cycling about 88 km a day. How many days will the tour take?

Find an estimated answer without using pencil and paper.

Is this what you did?

$88\overline{)579}$ Round 88 to 90.

Think about how many 90s in 579?

$6 \times 90 = 540$ and $7 \times 90 = 630$.

So the tour will take between 6 and 7 days.

Now compute to find the exact answer.
What is it? Is the estimated answer close?

Estimate these answers. Do not compute.

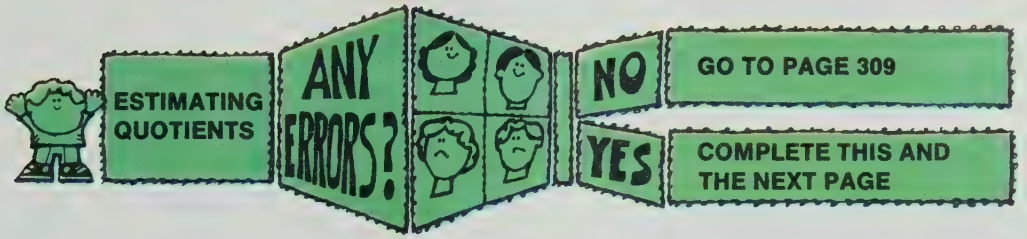
1. About how many 20s in 120?
2. About how many 25s in 120?
3. About how many 20s in 1200?
4. About how many 25s in 1200?
5. About how many 50s in 500?
6. About how many 52s in 500?
7. About how many 57s in 500?
8. About how many 70s in 4800?
9. About how many 70s in 4900?
10. About how many 70s in 500?
11. About how many 47s in 250?
12. About how many 53s in 250?
13. About how many 58s in 250?
14. About how many 62s in 250?
15. About how many 85s in 6300?
16. About how many 89s in 6300?
17. About how many 93s in 6300?
18. About how many 95s in 6300?

Very Important Problems

Here are some other plans for cycling trips.
First estimate the days needed to complete the trip.
Then compute the exact number.
(Days for stopovers are not included.)

Kilometres per day	Length of trip	Number of days (estimated)	Number of days (exact)
45	135	2	3
79	237	**a** ?	**b** ?
63	756	**c** ?	**d** ?
99	1089	**e** ?	**f** ?

Check your answers with the key on page 336.



$30\overline{)720}$ About how many 30s in 720?
If you say 20, you're right. $20 \times 30 = 600$
But there may be more than 20.
If you say 30, you're also right. $30 \times 30 = 900$
But there are fewer than 30.
The answer is between 20 and 30.

Estimate the quotients. Do not compute.

1. $40\overline{)320}$ **2.** $50\overline{)450}$ **3.** $20\overline{)1800}$ **4.** $60\overline{)3000}$ **5.** $30\overline{)900}$

6. $90\overline{)550}$ **7.** $70\overline{)500}$ **8.** $50\overline{)4100}$ **9.** $80\overline{)5000}$ **10.** $60\overline{)7000}$

NOW FOR ESTIMATION AND COMPUTATION

$27\overline{)6264}$ Estimate. About how many 30s in 6264?

$$\begin{array}{r} 200 \\ 27\overline{)6264} \\ 5400 \\ \hline 864 \end{array}$$

Was this your estimate?

200×27

remain

Estimate again.
How many 30s in 864? → 20

$$\begin{array}{r} 200 \\ 27\overline{)6264} \\ 5400 \\ \hline 864 \\ 540 \\ \hline 324 \\ 270 \\ \hline 54 \\ 54 \\ \hline 0 \end{array}$$

(20×27)

Estimate again. How many 30s in 324?

(10×27) Where does the 10 go?

You don't have to estimate this.

(2×27) Where does the 2 go?

What is the final quotient?

Division is a lot of work. You have to know how to estimate. You have to know how to multiply. You have to know how to subtract. And you have to add, too.

Pick any six of these division problems to complete.

1. $17\overline{)541}$ **2.** $21\overline{)684}$ **3.** $62\overline{)471}$ **4.** $78\overline{)8325}$ **5.** $56\overline{)1573}$

6. $20\overline{)8071}$ **7.** $91\overline{)3763}$ **8.** $88\overline{)7541}$ **9.** $69\overline{)7892}$ **10.** $33\overline{)3994}$

Mixed Bag

3	?	4	?	6	=	2
?		?		?		?
2	?	3	?	4	=	10
?		?		?		?
7	?	6	?	2	=	11
=		=		=		=
13	?	6	?	4	=	23

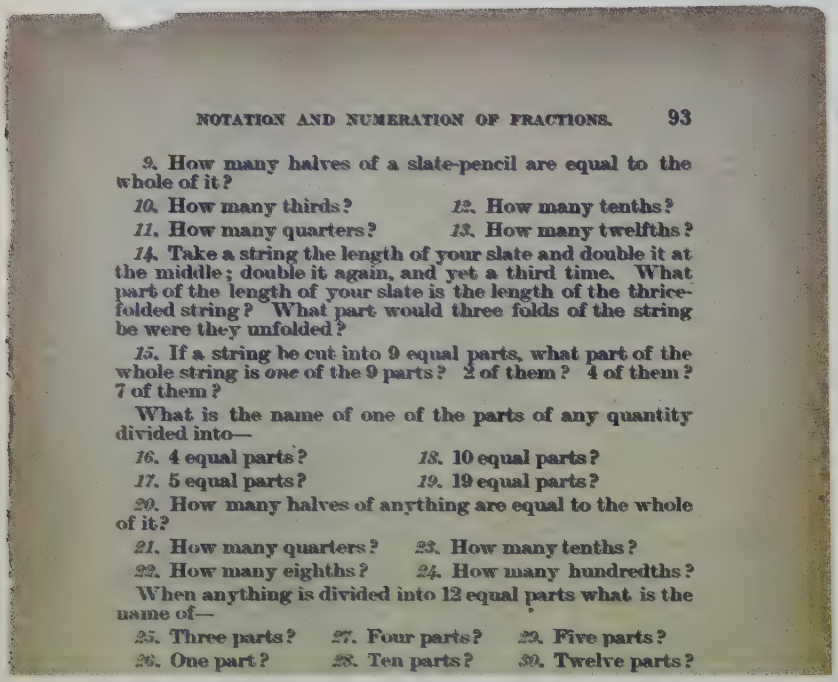
Copy this puzzle on paper. All you have to do to complete the puzzle is put operation symbols in the empty boxes. You can't put them any old place. You must complete a true sentence in every row or column that has numbers.

?	×	?	÷	?	=	4
×		−		×		+
?	×	?	÷	?	=	16
÷		+		−		−
?	+	?	+	?	=	11
=		=		=		=
3	×	1	+	6	=	9

This one is not so easy. Copy this puzzle too. All you have to do in this one is find the correct numbers to place in each empty box. You must complete a true sentence in each row and column that has numbers.

How would you get along with these questions?

They take up half a page in a book used in some schools before 1900. Beginning on page 92, work in fractions filled 38 pages.



NOTATION AND NUMERATION OF FRACTIONS. 93

9. How many halves of a slate-pencil are equal to the whole of it?

10. How many thirds? 12. How many tenths?

11. How many quarters? 13. How many twelfths?

14. Take a string the length of your slate and double it at the middle; double it again, and yet a third time. What part of the length of your slate is the length of the thrice-folded string? What part would three folds of the string be were they unfolded?

15. If a string be cut into 9 equal parts, what part of the whole string is *one* of the 9 parts? 2 of them? 4 of them? 7 of them?

What is the name of one of the parts of any quantity divided into—

16. 4 equal parts? 18. 10 equal parts?

17. 5 equal parts? 19. 19 equal parts?

20. How many halves of anything are equal to the whole of it?

21. How many quarters? 23. How many tenths?

22. How many eighths? 24. How many hundredths?

When anything is divided into 12 equal parts what is the name of—

25. Three parts? 27. Four parts? 29. Five parts?

26. One part? 28. Ten parts? 30. Twelve parts?

1. What are some of the differences between this mathematics book and yours?
2. Which problem gives instructions that you cannot follow in a classroom today?
3. Do you think you can answer these questions as well as a pupil then was expected to do? If you want to try, write the number of the question and the answer.

People who use recipes that their grandparents liked need to know fractions.
Old cookbooks always included many measurements that used fractions.
With metric cookery, you may think you need to know only decimal fractions.
You might be surprised! Read the recipe from a cookbook using metric measures.

BUTTER TARTS

Preheat oven to 190°C.
Prepare pastry; roll out half. Cut with a floured cookie cutter and line 15 medium-sized muffin cups. Do not prick.
Pour boiling water over
125 ml raisins
Let soak (5 minutes). Drain.
Stir together
50 ml butter or margarine
125 ml lightly packed brown sugar
Blend in
250 ml corn syrup
2 eggs
15 ml lemon juice
5 ml vanilla
Fold in raisins. Fill pastry-lined tart shells 2/3 full.
Bake in preheated 190°C oven for 20 to 25 minutes. Makes 15 tarts.

1. What fraction of the pastry is rolled out to line the muffin cups?

2. What fraction of the tart shell is filled with the recipe mixture?

3. Suppose that the muffin tin has only 12 cups. What fraction of the ingredients would be enough for 12 tarts? (Give the simplest name for the fraction.)

4. For 12 tarts, how many millilitres would you need of:
   a raisins? **b** butter? **c** brown sugar?
   d corn syrup? **e** lemon juice? **f** vanilla?

5. What would you do about a fractional part of 2 eggs?

Now that you have done the questions, you might like to try the recipe at home.

Recipe from *How to Metricook*, courtesy of Maple Leaf Mills Limited

Very Important Problems

Below is part of an application form for a job at Wilson's Warehouse. They want a boy or a girl to help stock parts two hours after school. To get the job, you must be able to add and subtract fractions.

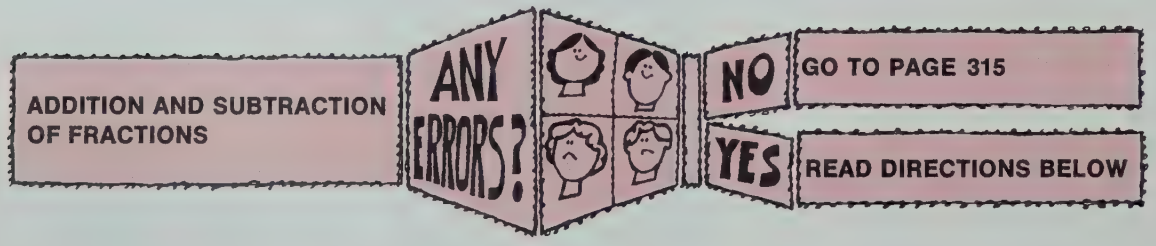
See how well you can do.

WILSON'S WAREHOUSE APPLICATION page 2

Please complete these problems on a sheet of paper. We need to see how you work these problems.

1. $\frac{1}{3} + \frac{2}{3}$
2. $\frac{3}{4} - \frac{1}{4}$
3. $\frac{2}{3} + \frac{1}{9}$
4. $\frac{3}{8} - \frac{1}{4}$
5. $2\frac{4}{5} + 3\frac{1}{10}$
6. $2\frac{7}{9} - 1\frac{2}{3}$

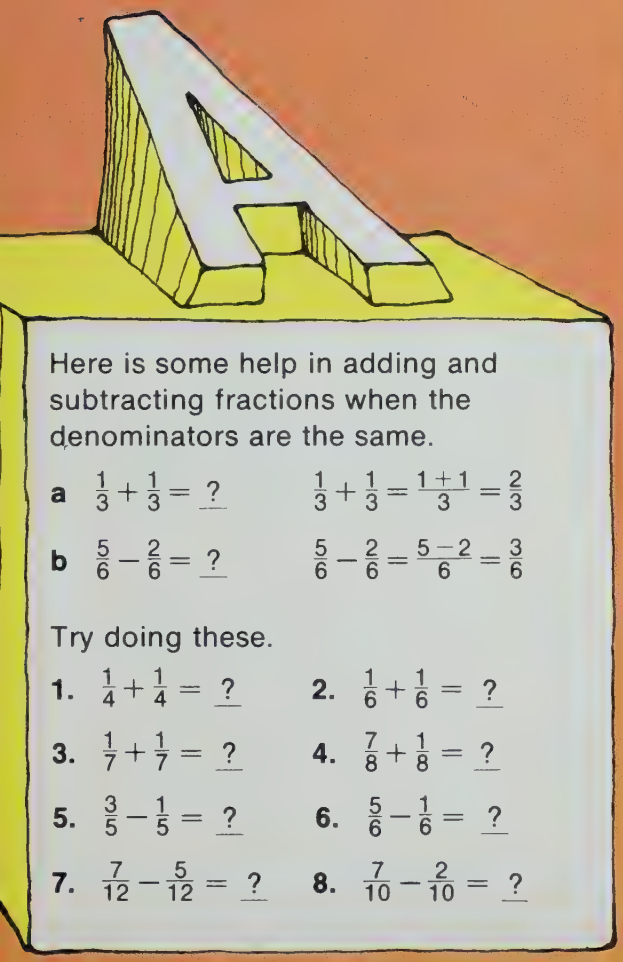
Wilson's Warehouse wants no errors. Would you get the job? Check your answers with the key on page 336.



If you missed problem 1 or 2, do section A on the next page.

If you missed problem 3 or 4, do section B on the next page.

If you missed problem 5 or 6, do section C on page 314.



Here is some help in adding and subtracting fractions when the denominators are the same.

a $\frac{1}{3}+\frac{1}{3}=\frac{?}{}$ $\quad\frac{1}{3}+\frac{1}{3}=\frac{1+1}{3}=\frac{2}{3}$

b $\frac{5}{6}-\frac{2}{6}=\frac{?}{}$ $\quad\frac{5}{6}-\frac{2}{6}=\frac{5-2}{6}=\frac{3}{6}$

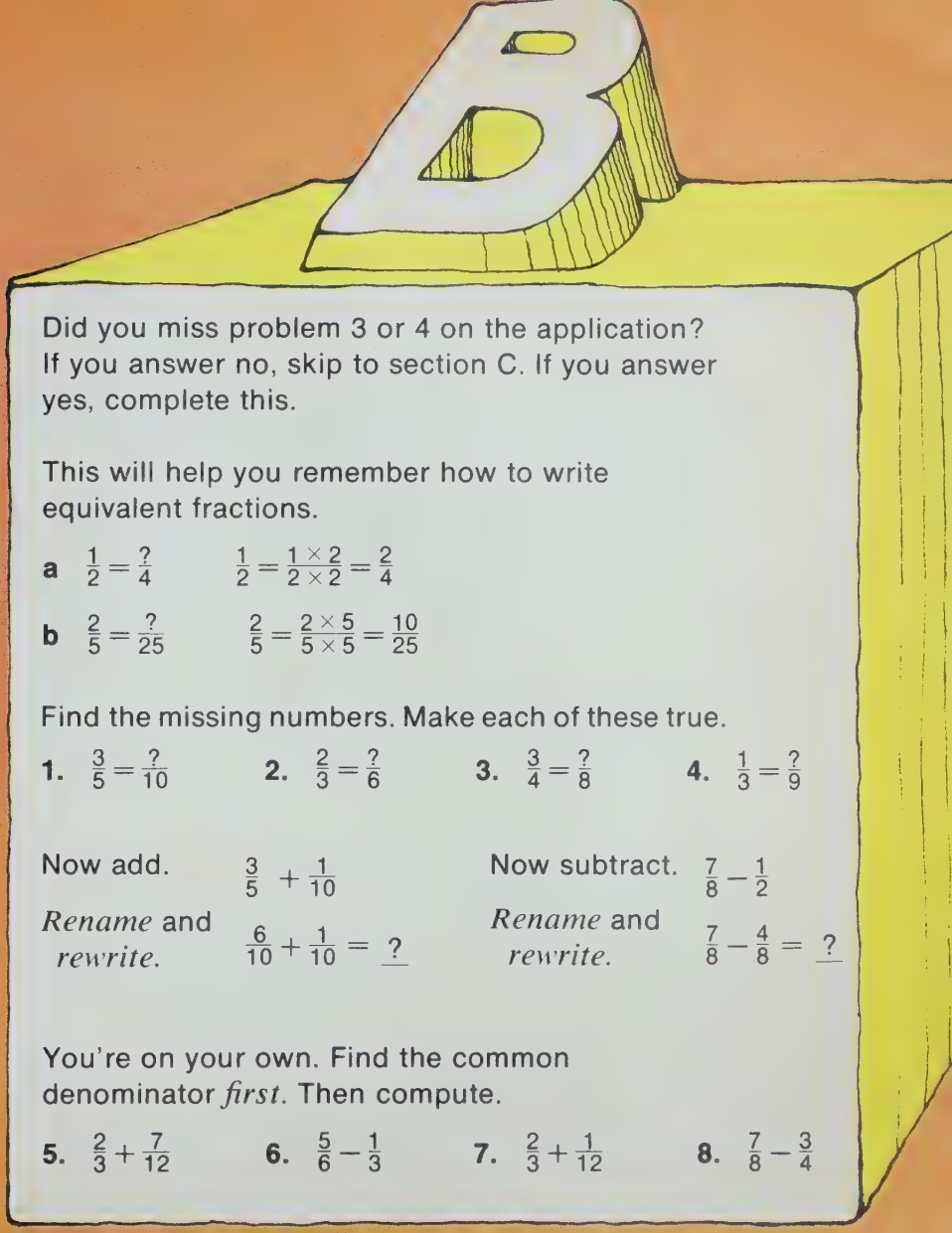
Try doing these.

1. $\frac{1}{4}+\frac{1}{4}=\frac{?}{}$ **2.** $\frac{1}{6}+\frac{1}{6}=\frac{?}{}$

3. $\frac{1}{7}+\frac{1}{7}=\frac{?}{}$ **4.** $\frac{7}{8}+\frac{1}{8}=\frac{?}{}$

5. $\frac{3}{5}-\frac{1}{5}=\frac{?}{}$ **6.** $\frac{5}{6}-\frac{1}{6}=\frac{?}{}$

7. $\frac{7}{12}-\frac{5}{12}=\frac{?}{}$ **8.** $\frac{7}{10}-\frac{2}{10}=\frac{?}{}$



Did you miss problem 3 or 4 on the application? If you answer no, skip to section C. If you answer yes, complete this.

This will help you remember how to write equivalent fractions.

a $\frac{1}{2}=\frac{?}{4}$ $\quad\frac{1}{2}=\frac{1\times2}{2\times2}=\frac{2}{4}$

b $\frac{2}{5}=\frac{?}{25}$ $\quad\frac{2}{5}=\frac{2\times5}{5\times5}=\frac{10}{25}$

Find the missing numbers. Make each of these true.

1. $\frac{3}{5}=\frac{?}{10}$ **2.** $\frac{2}{3}=\frac{?}{6}$ **3.** $\frac{3}{4}=\frac{?}{8}$ **4.** $\frac{1}{3}=\frac{?}{9}$

Now add. $\frac{3}{5}+\frac{1}{10}$

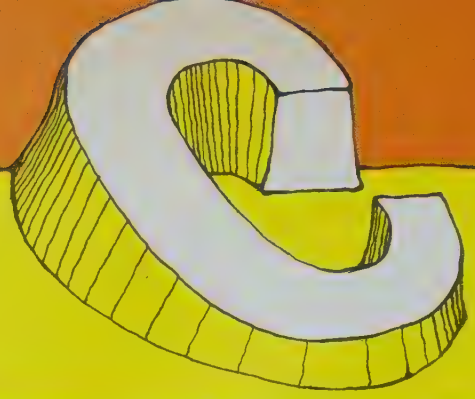
Rename and *rewrite.* $\frac{6}{10}+\frac{1}{10}=\frac{?}{}$

Now subtract. $\frac{7}{8}-\frac{1}{2}$

Rename and *rewrite.* $\frac{7}{8}-\frac{4}{8}=\frac{?}{}$

You're on your own. Find the common denominator *first*. Then compute.

5. $\frac{2}{3}+\frac{7}{12}$ **6.** $\frac{5}{6}-\frac{1}{3}$ **7.** $\frac{2}{3}+\frac{1}{12}$ **8.** $\frac{7}{8}-\frac{3}{4}$



Sometimes you must rename when working with mixed numbers.

Study these examples. **a** $3\frac{1}{6} \rightarrow 3\frac{1}{6}$
$+ 1\frac{1}{2} \quad + 1\frac{3}{6}$
$4\frac{4}{6}$ or $4\frac{2}{3}$

b $5\frac{1}{3} \rightarrow 4\frac{4}{3}$
$- 1\frac{2}{3} \quad - 1\frac{2}{3}$
$3\frac{2}{3}$

c $5 \rightarrow 4\frac{2}{2}$
$- 1\frac{1}{2} \quad - 1\frac{1}{2}$
$3\frac{1}{2}$

This one is started. You finish it.

d $4\frac{1}{2} \rightarrow 4\frac{2}{4} \rightarrow 3\frac{6}{4}$
$- 2\frac{3}{4} \quad - 2\frac{3}{4} \quad - 2\frac{3}{4}$
?

Here are some practice problems.

1. $5\frac{1}{2}$
$+ 2\frac{1}{4}$

2. $3\frac{5}{6}$
$+ 1\frac{1}{12}$

3. $2\frac{2}{3}$
$- 1\frac{2}{9}$

4. $7\frac{4}{5}$
$- 2\frac{4}{5}$

5. $9\frac{11}{100}$
$- 7\frac{3}{100}$

6. $1\frac{1}{8}$
$- \frac{3}{4}$

7. $5\frac{7}{10}$
$+ 4\frac{7}{100}$

8. $9\frac{1}{6}$
$- 3\frac{1}{3}$

9. 6 (Hint: $1 = \frac{3}{3}$)
$- 1\frac{1}{3}$

10. 4
$- 3\frac{2}{7}$

11. 8
$- 5\frac{3}{4}$

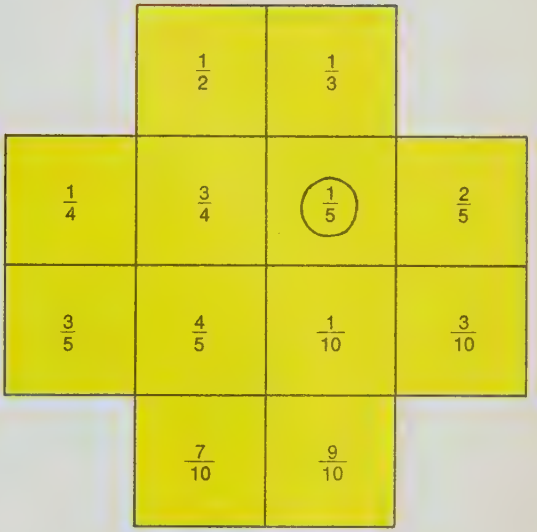
Mixed Bag

Draw twelve 2 cm squares to form the figure at the right. Copy the fraction printed in each square.

Each answer in the exercise below needs to be renamed to its simplest form. Compute and rename the answer. Look for the final answer on your drawing. Circle the fraction that is your final answer.

When you complete the exercise, each fraction on your drawing should be circled. The final answer to the first problem is already circled for you.

Recopy the questions in vertical form if you wish. Watch for the operation signs.



1. $\frac{9}{10} - \frac{7}{10}$

2. $\frac{19}{100} - \frac{9}{100}$

3. $\frac{27}{40} + \frac{3}{40}$

4. $\frac{2}{15} + \frac{7}{15}$

5. $\frac{5}{6} - \frac{1}{3}$

6. $\frac{3}{4} - \frac{1}{20}$

7. $\frac{3}{4} + \frac{1}{20}$

8. $\frac{1}{3} + \frac{17}{30}$

9. $\frac{1}{5} + \frac{1}{20}$

10. $\frac{3}{4} - \frac{9}{20}$

11. $\frac{5}{6} - \frac{13}{30}$

12. $\frac{1}{20} + \frac{17}{60}$

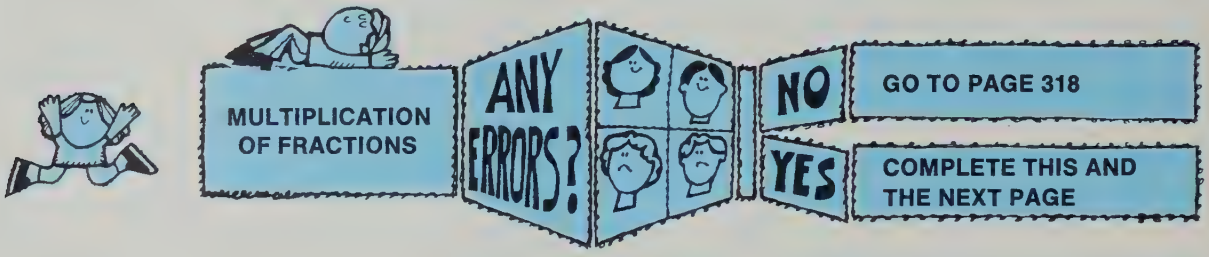
Very Important Problems

You have just finished practice in addition and subtraction with fractions.

Try some multiplication

1. $3 \times \frac{1}{5} = \underline{?}$
2. $\frac{3}{8} \times 4 = \underline{?}$
3. $\frac{1}{3} \times \frac{1}{4} = \underline{?}$
4. $\frac{2}{5} \times \frac{1}{2} = \underline{?}$
5. $2 \times 1\frac{2}{3} = \underline{?}$
6. $3\frac{1}{2} \times 4\frac{3}{4} = \underline{?}$

Check your answers with the answer key on page 336.



Reminder! Here's how you multiply with fractions

a $3 \times \frac{2}{7} = \underline{?}$ $\quad 3 \times \frac{2}{7} = \frac{3 \times 2}{7} = \frac{6}{7}$

b $\frac{2}{5} \times 2 = \underline{?}$ $\quad \frac{2}{5} \times 2 = \frac{2 \times 2}{5} = \frac{4}{5}$

c $4 \times \frac{5}{6} = \underline{?}$ $\quad 4 \times \frac{5}{6} = \frac{4 \times 5}{6} = \frac{20}{6} = 3\frac{2}{6} = \underline{?}$

d $\frac{1}{2} \times \frac{1}{4} = \underline{?}$ $\quad \frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \underline{?}$

e $\frac{2}{3} \times \frac{4}{5} = \underline{?}$ $\quad \frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \underline{?}$

f $\frac{5}{9} \times \frac{3}{4} = \underline{?}$ $\quad \frac{5}{9} \times \frac{3}{4} = \frac{5 \times 3}{9 \times 4} = \underline{?}$

Complete for practice

1. $2 \times \frac{1}{3} = \underline{?}$
2. $4 \times \frac{1}{6} = \underline{?}$
3. $\frac{1}{3} \times \frac{1}{2} = \underline{?}$
4. $\frac{3}{10} \times \frac{1}{2} = \underline{?}$
5. $3 \times \frac{2}{5} = \underline{?}$
6. $\frac{2}{3} \times \frac{1}{3} = \underline{?}$
7. $6 \times \frac{3}{4} = \underline{?}$
8. $\frac{3}{4} \times \frac{2}{5} = \underline{?}$
9. $\frac{2}{3} \times \frac{5}{6} = \underline{?}$

Do you remember how to multiply mixed numbers? If you know how to multiply fractions, mixed numbers will be no problem. Here is what you do.

A Change each mixed number to a fraction.
B Multiply the fractions.
C Write the product as a mixed number.

Here is an example:

$3\frac{2}{3} \times 2\frac{3}{5} = \underline{?}$

Can you explain each step in finding the answer?

$3\frac{2}{3} \times 2\frac{3}{5} = \frac{11}{3} \times \frac{13}{5} = \frac{143}{15} = 9\frac{8}{15}$

Try doing these exercises now

1. $1\frac{1}{2} \times 1\frac{2}{3} = \underline{?}$
2. $2\frac{1}{2} \times 1\frac{3}{4} = \underline{?}$
3. $3\frac{2}{5} \times 1\frac{2}{3} = \underline{?}$
4. $2\frac{2}{3} \times 2\frac{2}{3} = \underline{?}$
5. $1\frac{5}{6} \times 5\frac{1}{2} = \underline{?}$
6. $4\frac{3}{4} \times 1\frac{3}{5} = \underline{?}$

FRANTIC FRACTIONS GAME

Up to four can play. Everyone sits on the same side of the playing board.

Each player has a token. Place your token on a vacant START.

Everyone plays in turn.

First turn. Move your token straight forward onto the first square.

Other turns. Move your token one square in any direction. (No jumps.) You must move to a vacant square.

After you move each time, challenge any other player who has a token on a square to give the product of your number and his number.

Imagine you have just challenged someone. Your token is on $1\frac{1}{2}$.

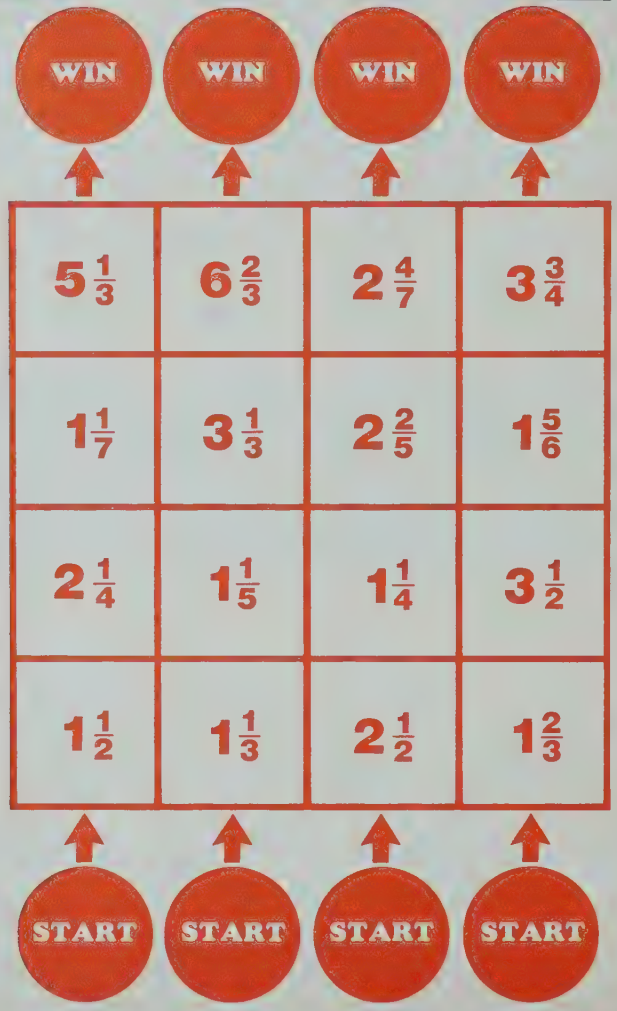
His token is on $1\frac{1}{3}$. He must give the product of $1\frac{1}{2} \times 1\frac{1}{3}$.

If his answer is right, your turn is over.

Suppose his answer is wrong. He goes back to START (any vacant START).

Then your turn is over. It's the next person's move.

Have fun!



Our number system is put together in such a way that there is no "biggest number" and no "smallest number." The numbers just go on and on and on.

$1\frac{1}{10}$ can be written 1.1

$1\frac{1}{100}$ can be written 1.01

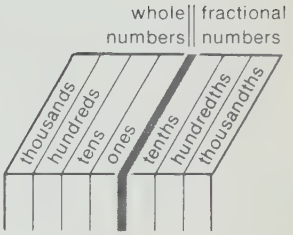
$1\frac{1}{1000}$ can be written 1.001

1 can be written as $\frac{1}{1}$ or $\frac{10}{10}$ or $\frac{100}{100}$ or $\frac{1000}{1000}$ or 1.0.

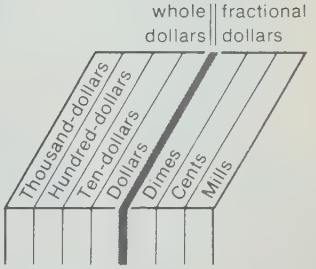
10 means 1 *ten;* 0.1 means 1 *tenth.*

100 means 1 *hundred;* 0.01 means 1 *hundredth.*

1000 means 1 *thousand;* 0.001 means 1 *thousandth.*



You have worked with money for a long time. Look at the chart. Is there anything there that you don't know about? We don't hear much about a mill, but it is $\frac{1}{1000}$, or 1 thousandth, of one dollar. Use the place-value chart to help answer these questions.



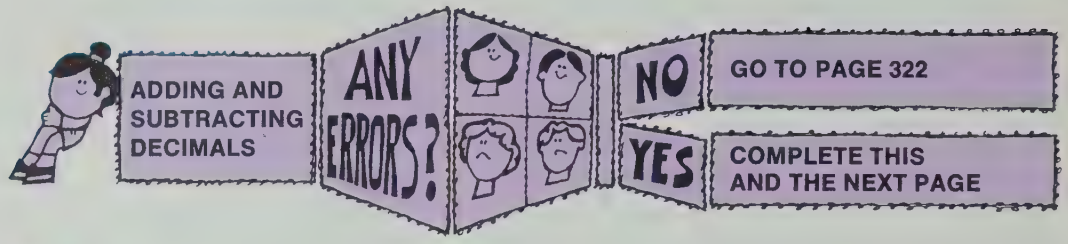
1. One dime is what fractional part of one dollar?
2. One cent is what fractional part of one dollar?
3. How many mills would be the same as one cent?
4. How many cents would be the same as one dime?
5. How many dimes would be the same as one dollar?
6. How many dimes would be the same as ten dollars?
7. How many dimes would be the same as one hundred dollars?

Very Important Problems

Compute. Watch the signs.

1.	$4.24	**2.**	10.78	**3.**	$5.79	**4.**	8.28
	+ 1.63		+ 8.22		− 2.53		− 3.79

Check your answers on page 336.



Look back at the problems you missed. Find out *why* you made a mistake. Use these questions as a checkout list.

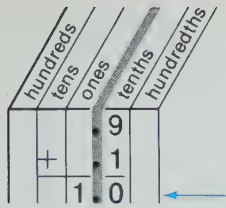
1. Did your answer have the dollar sign and the decimal in the correct place? If it didn't, why didn't it?
   Think about this.
   You gave a clerk $5 to pay for a fifty-cent item.

   $$\begin{array}{r} \$5.00 \\ -\ 0.50 \\ \hline \$4\ 50 \end{array}$$

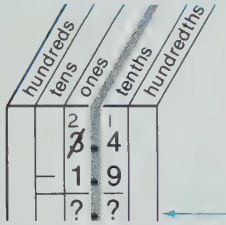
   The clerk gave you $450 in change. Would the clerk get in trouble? Would you stand for it if the clerk gave you 45 cents in change? The decimal is important.

2. Did you make an error in adding or subtracting? If you did, you need practice with whole numbers. You won't have success with decimals until you get good with whole numbers.

Before you start, think about place value.



You are using an addition fact.
But remember the place value in tenths.



You must rename. Same idea that you know for whole numbers.
You rename 1 one as tenths.

Complete these.

Add.

1. 0.7 + 0.9	**2.** 9.5 + 1.2	**3.** 3.7 + 6.8	**4.** 27.3 + 13.6	**5.** 47.89 + 12.10

Subtract.

6. 1.8 − 0.7	**7.** 2.3 − 0.9	**8.** 51.6 − 13.7	**9.** 10.1 − 9.7	**10.** 20.00 − 4.91

Copy and compute. Watch the signs.

11. 1.2 + 3.56 **12.** 21.5 − 17.2 **13.** 25 − 1.25

Mixed Bag

Water freezes at 0°C. (Remember ° stands for degrees; C stands for Celsius.) It boils at 100°C at sea level.

Oven temperatures for baking are almost always higher than the boiling point of water. The chart on this page gives metric baking temperatures and the time needed for several products.

1. What is the lowest oven temperature on the chart? What can be baked at that temperature? How many hours will it take?

2. What is the highest oven temperature? What two products may require that temperature? Which of the two needs less time?

3. What is the range of baking temperatures on the chart? (Remember that the *range* is the difference between lowest and highest.)

4. Name two egg, meat, milk, or cheese dishes that require the same oven temperature. If the times differ, which dish would you put in the oven first?

Courtesy of Maple Leaf Mills Limited

Our system of measure is really very simple.
It is organized the same way as our number system is.
The language of our measurement system is as simple as its organization.
It uses the same prefixes in measurements of length, mass, and capacity.

Do these charts look like a place-value chart?
Use these charts to help you answer these questions.

LENGTH

1. How many metres in a decametre? a hectometre? a kilometre?
2. A millimetre is what part of a centimetre? a decimetre? a metre?

MASS

1. How many grams in a decagram? a hectogram? a kilogram?
2. How many milligrams in 1 gram?
3. How many centigrams in 1 gram?

CAPACITY

1. How many litres in a kilolitre?
2. Is a centilitre larger or smaller than a decalitre?

The units of length you need to remember are centimetre, metre, and kilometre.
The units of mass you should remember are gram and kilogram.
The units of capacity to remember are millilitre, litre, and kilolitre.

Very Important Problems

Replace the ? by the correct measurement.

1. 7.5 cm = ? mm **2.** 7.555 kg = ? g **3.** 4584 ml = ? ℓ **4.** 6.08 m = ? cm

5. 844 cm = ? m **6.** 3.075 km = ? m **7.** 4448 mm = ? m **8.** 4448 mm = ? cm

Check your answers with the key on page 336.

If you made an error on these problems, you may not know measurement units well enough. You may also need practice on the "sliding" decimal point.

Copy and complete the tables.

1.

mm	cm
75	?
?	9.4
904	?

2.

cm	m
608	?
3057	?
?	45.98

3.

ml	ℓ
2995	?
?	3.750
?	5.117

4.

g	kg
7555	?
?	9.080
9530	?

5.

m	km
9007	?
?	9.405
9999	?

Tell which of the units of measure is probably the better one.

1.

You are told that a man is 2 ? (centimetres or metres) tall.

2.

A cat at your friend's house has a mass of 2 ? (grams or kilograms).

3.

Your mother's sewing needle is 5 ? (centimetres or metres) long.

4.

A large bottle of pop holds about 5 ? (decilitres or litres).

5.

The race driver said his top speed was 200 ? (metres or kilometres) per hour.

6.

Your finger is about 1.5 ? (centimetres or millimetres) wide.

7.

A pill contains 250 ? (milligrams or grams) of vitamin C.

8.

One of the events at the Olympics was a 5000 ? (metre or kilometre) run.

9.

The border between the United States and Canada is 6416 ? (metres or kilometres) long.

10.

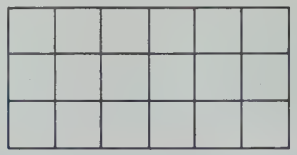
The fuel tank of a small compact car holds about 5 ? (decalitres or litres).

You need a break from all that computation you've been doing. Here is a new topic for you.

The whole number 18 can be shown by three different arrays.

A

B



C

Now try these yourself. Make other arrays for each number shown, if you can.

1.

2.

3.

4.

5.

6.

7.

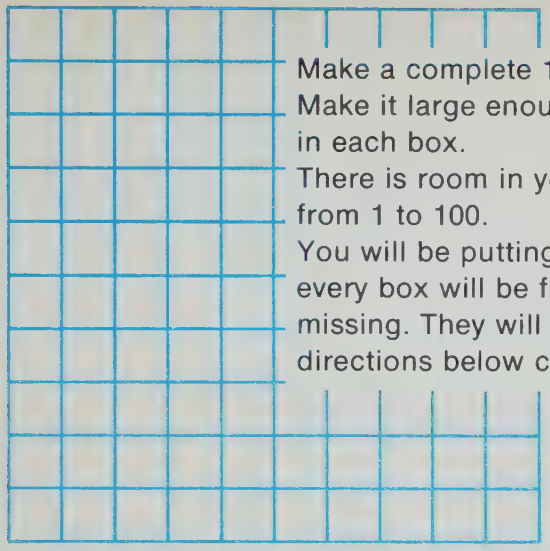
8.

9. Which numbers could you make other arrays for? Which ones could be shown only by one array?

Whole numbers that can be shown only by a one-row array are called **prime numbers.**

Whole numbers that can be shown with more than one array are called **composite numbers.**

10. Are the numbers 2, 3, 4, and 5 prime or composite? Draw arrays to prove it.



Make a complete 10x10 grid like this.
Make it large enough so that you can enter 2-digit numbers in each box.
There is room in your grid for 100 numbers written in order from 1 to 100.
You will be putting numbers in order into the grid, but *not* every box will be filled. There will be some numbers missing. They will be the important ones. Follow the directions below carefully.

a Enter the number 1 in the first box of the first row.

b Enter the number 2 in the second box of the first row. The proper order of numbers has started. Make sure it continues.

c Enter every number that is a multiple of 2 in its correct location.

d Enter the number 3 in the third box in the first row.

e Enter every multiple of 3 in its correct location.

f 4 has already been entered. So enter 5 in the fifth box.

g Enter every multiple of 5 in its correct location.

h 6 has been entered. So enter 7 in the seventh box.

i Enter every multiple of 7 in its correct location.

j Have 8, 9, and 10 been entered? They should have been.

k Make a list of the numbers that should be in the empty boxes.

1.

Look at your list of numbers.

a The number 1 isn't on your list. 2 isn't there either.

b Can you divide any number on your list by 2 and not have a remainder? A number like that does not belong on your list!

c Is any number on your list divisible by 3?

d Is any number on your list divisible by 5?

e Is any number on your list divisible by 7?

f Is any number on your list divisible by 11? Which one? What is the quotient? 11 belongs on your list. But no multiple of 11 belongs there.

g Is any number on your list divisible by 13, other than 13? No multiples of 13 belong on your list.

h Check the rest of your numbers in the same way. No multiples allowed.

Any number that is not divisible by any number other than itself and 1 is a prime number. All other whole numbers are composite numbers.

2.

How many prime numbers are there between 1 and 100?

3.

Is 100 a prime or composite number?

4.

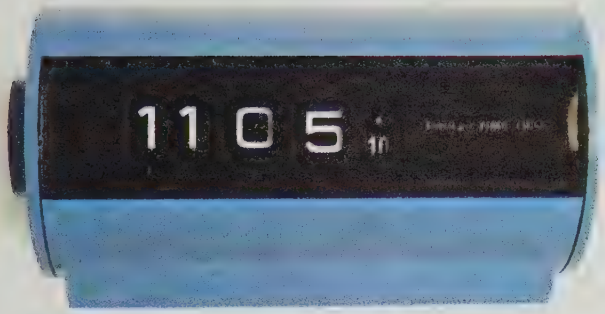
Is 101 a prime or composite number?

5.

How many prime numbers do you think there are?

More than 2000 years ago a Greek mathematician called Eratosthenes did just about the same thing you just did. He crossed out multiples from a grid of numbers. But he was the first to do it. His grid is called the sieve of Eratosthenes by those people who can pronounce it. Since his grid was named after him, you can name *your* list after *you*. Fair enough?

Mixed Bag

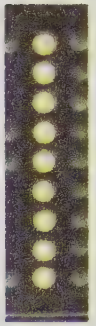


Have you ever seen a digital clock or watch?

Only one digit shows in each of the openings. How many different digits have to be available for any of the openings?

Does 0 have to be available for the first opening on the left?

Huge outdoor clocks are often digital clocks. The numerals are formed by a series of light bulbs flashing on.



From a distance these lighted bulbs would look like the numeral 1.

Many times all the bulbs that are needed to show any digit are arranged in an array. The bulbs that are turned on to show any given numeral are controlled by a machine.

Your job is to figure out one array of bulbs that could show any one of the numerals from 0 through 9. The numerals sometimes have a bit different shape than the ones we write. Be sure to show what each numeral would look like.

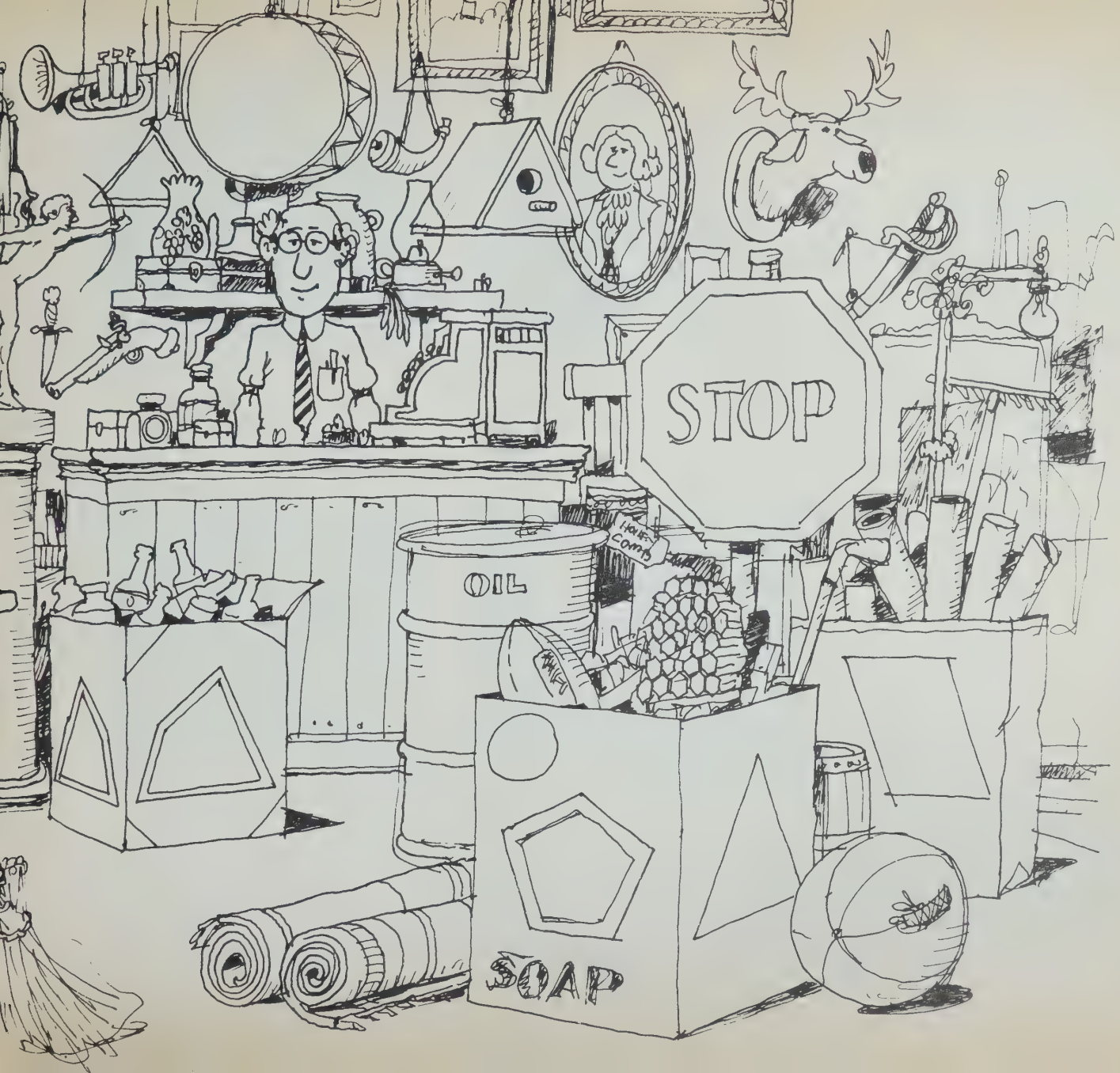
Cracker Barrel Session

Find the following shapes if you can. Tell what object reminded you of each of them. Start with some plane figures.

1. Squares or rectangles
2. Triangles
3. Parallelograms or quadrilaterals that are not rectangles
4. Circles
5. Pentagons or hexagons or octagons

Now look for some solid shapes. Be sure to name the object that reminded you of the shape.

6. Cubes or rectangular prisms
7. Cylinders
8. Spheres


STOP
OIL
HALF-CORN
SOAP

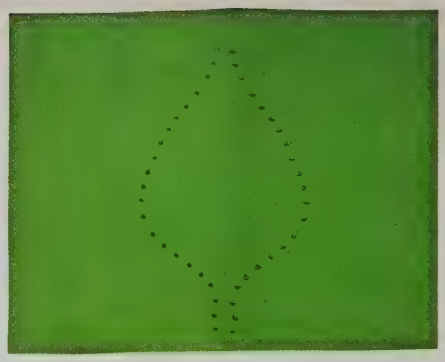
Very Important Problems

Fold a piece of paper in half. Keep it folded.
Find something with a sharp point.
The end of a ballpoint pen will do.

Make a number of holes through the paper.
Make them about 1 cm apart.
The holes should begin and end near the fold.

1.

2.

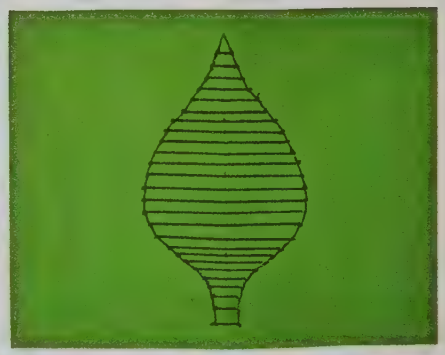


Open the paper.

3.

Connect each pair of holes.

4.



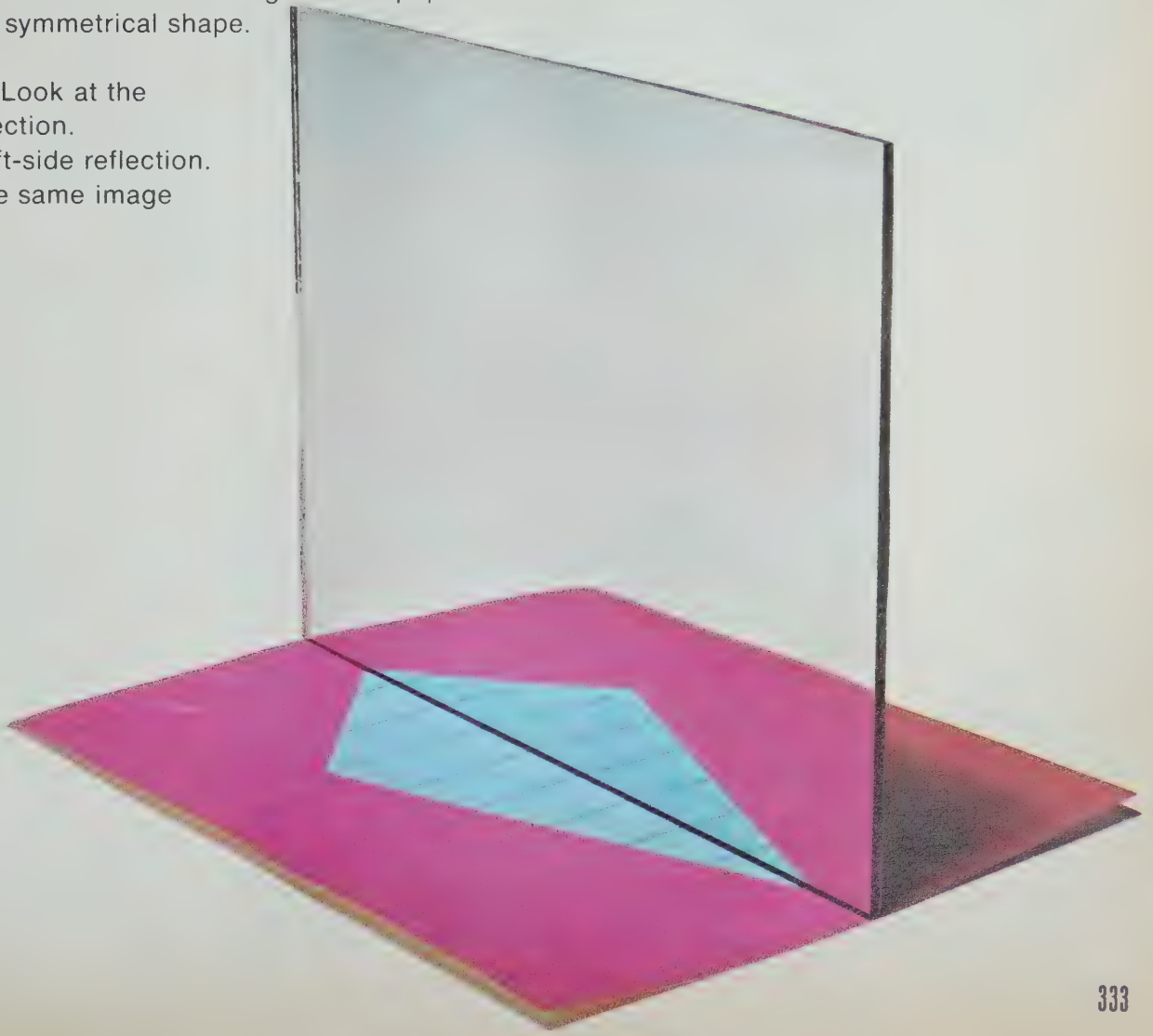
Then connect each point.

5. Does the left half of your picture match the right half?
Where is a line of symmetry?
Is there another line of symmetry?

6. Repeat this activity. This time make up a shape of your own. Do you know in advance if it will be symmetrical? Why?

There is another way to see symmetrical shapes.
Put a mirror up to part of a shape.
The edge of the mirror becomes the line of symmetry.
You will see a reflection of the image on the paper.
You will see a symmetrical shape.

Find a mirror. Look at the
right-side reflection.
Look at the left-side reflection.
Do you see the same image
both times?



Mixed Bag

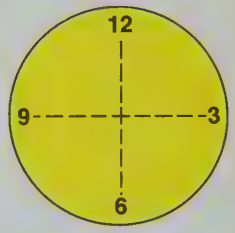
Look at the clock in your room. Most clockfaces have the shape of a circle. How many equal parts is the circular clockface divided into? You'd better say 12 or you're in trouble.

You're pretty good with folding. Trace around the bottom of a can or jar. Cut out the circular region. Divide it into 12 equal parts. It's not hard.

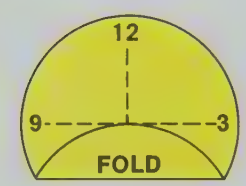
Fold in half.

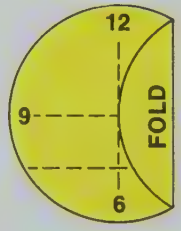
Then fold in half again.

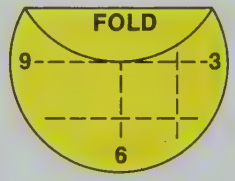
Open. Mark 3, 6, 9, and 12.

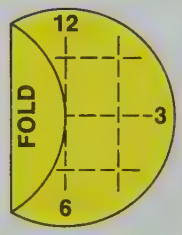


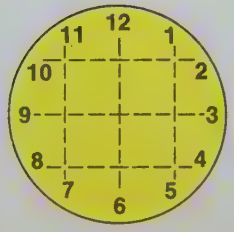
Four hours marked. How many to go?





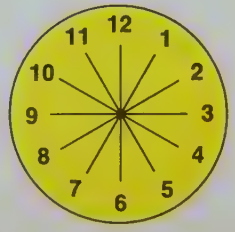






Finish marking the hours. Your clock is complete. But our goal was to divide a circular region into 12 parts. So finish. Draw lines from the center to each number.

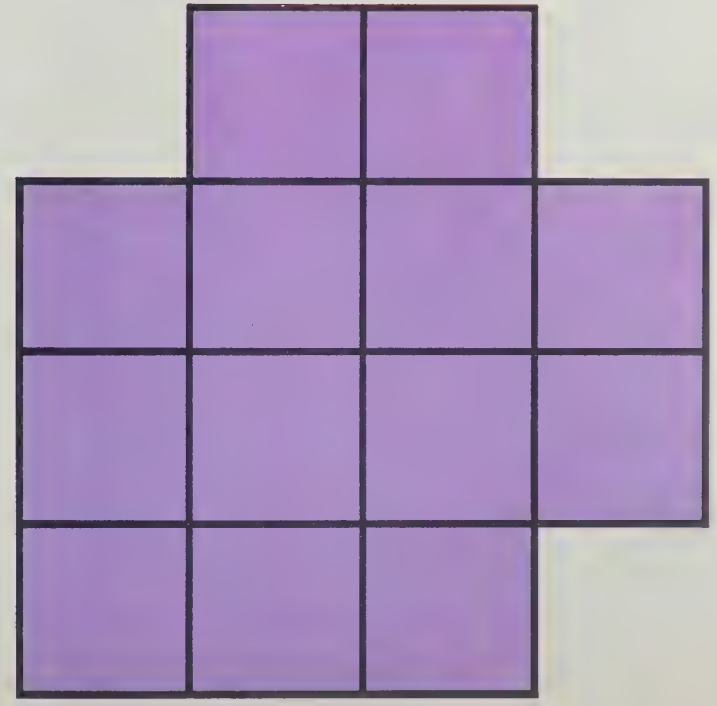
All 12 hours are marked.



Go on to the next page if you have time.

Another Mixed Bag

How many squares can you find?



ANSWERS

294 **a** 8950; 6960; 5900 **b** 8900; 7000; 5900

c 9000; 7000; 6000

298 **a** 600; 570 **b** 7000; 7223 **c** 1500; 1534

300 **a** 200; 209 **b** 3000; 3164 **c** 3000; 3263

303 **a** 800 **b** 648 **c** 14 000 **d** 17 064

e 60 000 **f** 83 232

307 **a** 3 **b** 3 **c** 12 **d** 12 **e** 10 or 11 **f** 11

312 **1.** $\frac{3}{3}$ **2.** $\frac{2}{4}$ **3.** $\frac{7}{9}$ **4.** $\frac{1}{8}$ **5.** $5\frac{9}{10}$ **6.** $1\frac{1}{9}$

316 **1.** $\frac{3}{5}$ **2.** $1\frac{1}{2}$ **3.** $\frac{1}{12}$ **4.** $\frac{1}{5}$ **5.** $3\frac{1}{3}$ **6.** $16\frac{5}{8}$

320 **1.** $5.87 **2.** 19.00 **3.** $3.26 **4.** 4.49

324 **1.** 75 mm **2.** 7555 g **3.** 4.584 ℓ **4.** 608 cm

5. 8.44 m **6.** 3075 m **7.** 4.448 m **8.** 444.8 cm

GLOSSARY

arithmetic mean An average

Add the numbers. $10 + 3 + 5 = 18$

Divide by number of numbers. $3\overline{)18}$ $\quad 6$

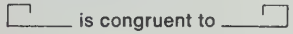
6 is the mean of 10, 3, and 5.

composite number A number with a pair of factors other than 1 and the number

$6 = 3 \times 2$

6 is composite.

congruent Two figures are congruent if they are the same size and shape



curve A figure drawn without lifting the pencil

data Numbers collected from events by counting or observing

decimal Another name for some fractions

$\frac{23}{100} = 0.23$ $\quad 1.3 = 1\frac{3}{10}$

decimal point $\quad$ decimals

division

quotient → 22 R2 ← remainder

2

20

divisor → $13\overline{)288}$ ← dividend

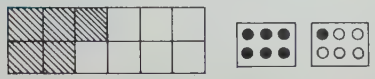
260 (20×13)

28

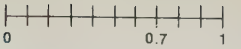
26 (2×13)

2

fraction A number that tells how much



$\frac{5}{12}$ $\quad 1\frac{1}{6}$



$\frac{5}{7}$ ← numerator / denominator

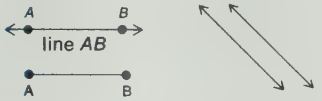
$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ equivalent fractions

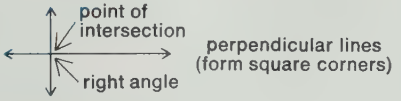
$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$ simplest name

$\frac{1}{5} = \frac{2}{10}$

$\frac{7}{10} = \frac{7}{10}$ common denominator

line





math sentence

true: $3 + 6 < 10$

open: $6 + \square = 10$

$6 + 5 = 10$ ← replacement for □

false: $6 - 3 = 4$

open: $6 + \square = 10$

true: $6 + 4 = 10$ ← solution

mixed number Another name for some fractions

$2.4 = 2\frac{4}{10}$ $\quad 3\frac{1}{3} = \frac{10}{3}$ mixed numbers

multiplication

$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

factors $\quad$ product

$3 \times 7 = 21$

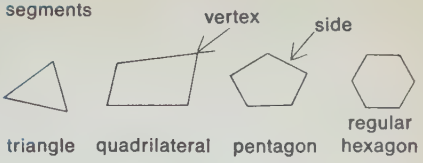
$4 \times 7 = 28$

21 and 28 are multiples of 7.

operation Using a pair of numbers to get another number

addition	multiplication
+ (plus)	× (times)
subtraction	division
− (minus)	÷ (divided by)

polygon A simple closed curve made with line segments



triangle $\quad$ quadrilateral $\quad$ pentagon $\quad$ hexagon

All sides and angles are congruent in a regular polygon.

prime number A whole number greater than 1 whose only pair of factors is 1 and itself. 2, 3, 5, 7, 11 are some prime numbers.

probability A way of finding how likely something is to happen

quadrilateral A four-sided polygon

parallelogram $\quad$ rectangle $\quad$ square

ray A part of a line that starts at a point and continues without end in one direction

relation symbols $=, \neq, >, <$

$7 > 3$ — 7 is greater than 3

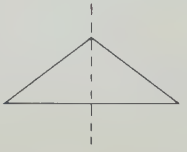
$2 + 4 = 6$ — 2 + 4 equals 6

$6 < 8$ — 6 is less than 8

$4 \neq 6$ — 4 is not equal to 6

statistics Computing with data, such as finding the arithmetic mean for a set of numbers

symmetry When a plane figure can be folded on a line to match exactly



line of symmetry

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TABLES OF MEASURES

METRIC

LENGTH

10 millimetres (mm) = 1 centimetre
10 centimetres (cm) = 1 decimetre
10 decimetres (dm) = 1 metre
10 metres (m) = 1 decametre
10 decametres (dam) = 1 hectometre
10 hectometres (hm) = 1 kilometre (km)

AREA

100 square millimetres (mm^2) = 1 square centimetre
10 000 square centimetres (cm^2) = 1 square metre
10 000 square metres (m^2) = 1 hectare
100 hectares (ha) = 1 square kilometre (km^2)

MASS

10 milligrams (mg) = 1 centigram
10 centigrams (cg) = 1 decigram
10 decigrams (dg) = 1 gram
10 grams (g) = 1 decagram
10 decagrams (dag) = 1 hectogram
10 hectograms (hg) = 1 kilogram
1000 kilograms (kg) = 1 tonne (t)

VOLUME

1000 cubic millimetres (mm^3) = 1 cubic centimetre
1000 cubic centimetres (cm^3) = 1 cubic decimetre
1000 cubic decimetres (dm^3) = 1 cubic metre (m^3)

CAPACITY

Liquid and Dry

10 millilitres (ml) = 1 centilitre
10 centilitres (cl) = 1 decilitre
10 decilitres (dl) = 1 litre
10 litres (ℓ) = 1 decalitre
10 decalitres (dal) = 1 hectolitre
10 hectolitres (hl) = 1 kilolitre (kl)

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